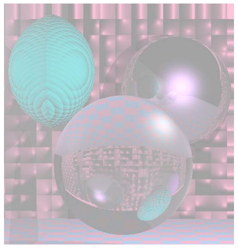
CS 4300
Computer Graphics

Prof. Harriet Fell

Fall 2012

Lecture 5 – September 13, 2012



Today’s Topics

- Vectors – review Shirley *et al.* 2.4
- Rasters Shirley *et al.* 3.0 - 3.2.1
- Rasterizing Lines
    - Shirley *et al.*

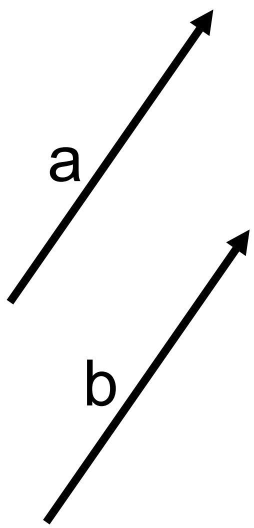
      8.0 - 8.1.1

      Implicit 2D lines pp. 30-35

      Parametric Lines p. 41
  - Antialiasing
  - Line Attributes

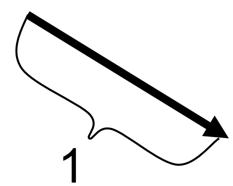
Vectors

- A *vector* describes a length and a direction.



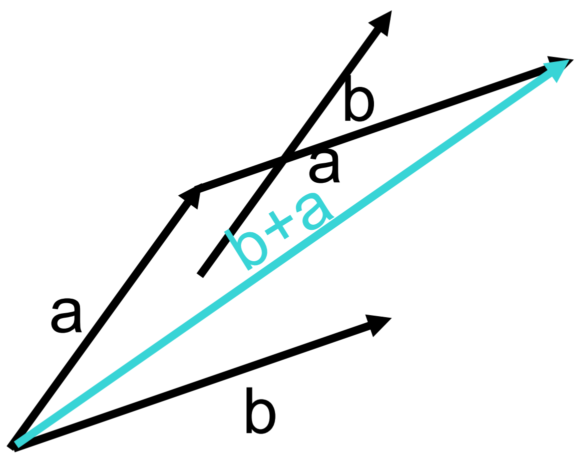
a = b

- a zero length vector

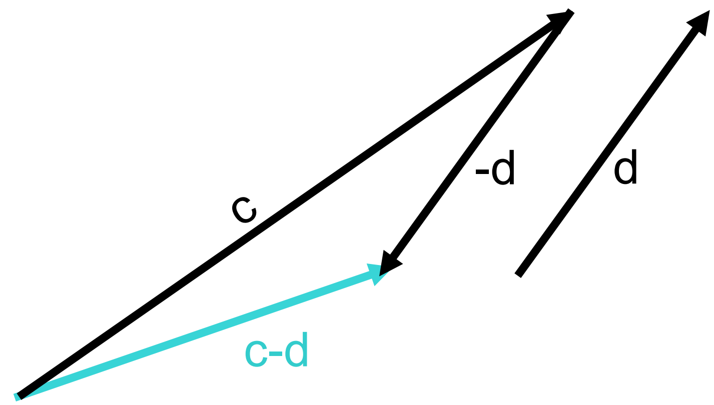


a unit vector

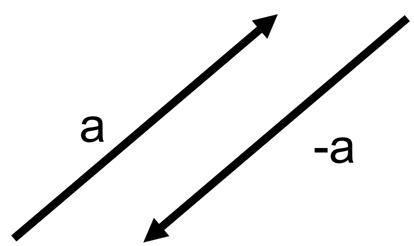
Vector Operations

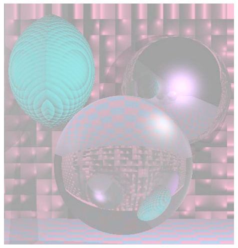


Vector Sum



Vector Difference





Cartesian Coordinates

- Any two non-zero, non-parallel 2D vectors form a *2D basis*.
- Any 2D vector can be written uniquely as a linear combination of two 2D basis vectors.
- **x** and **y** (or **i** and **j**) denote unit vectors parallel to the *x-axis* and *y-axis.*
- **x** and **y** form an *orthonormal* 2D basis.

$$a = x_a\mathbf{x} + y_a\mathbf{y}$$

$$a = (x_a, y_a) \quad \text{or} \quad a = \begin{bmatrix} x_a \\ y_a \end{bmatrix}$$

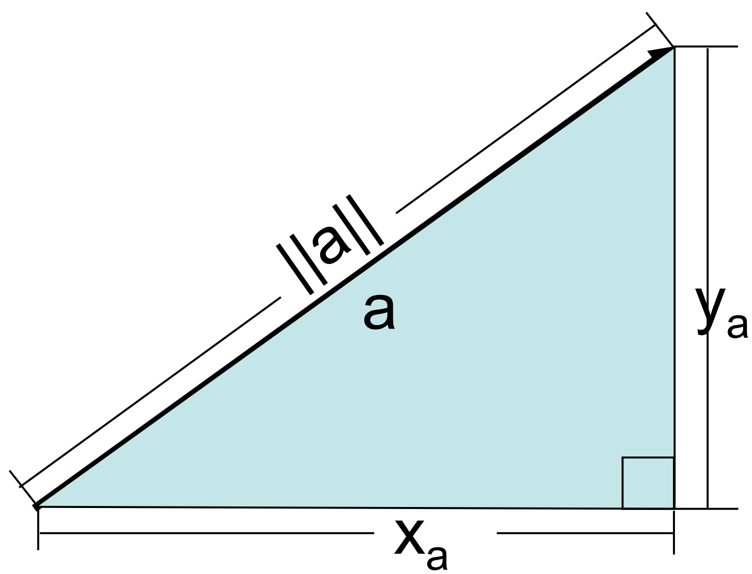
$$\text{or } a = (a_x, a_y)$$

- **x, y** and **z** form an *orthonormal* 3D basis.

Vector Length

Vector **a** =(x_a, y_a)

$$Length(\mathbf{a}) = Norm(\mathbf{a}) = \|\mathbf{a}\| = \sqrt{x_a^2 + y_a^2}$$



Dot Product

Dot Product

a =(x_a, y_a)　　　　**b** =(x_b, y_b)

$$\mathbf{a} \bullet \mathbf{b} = x_a x_b + y_a y_b$$

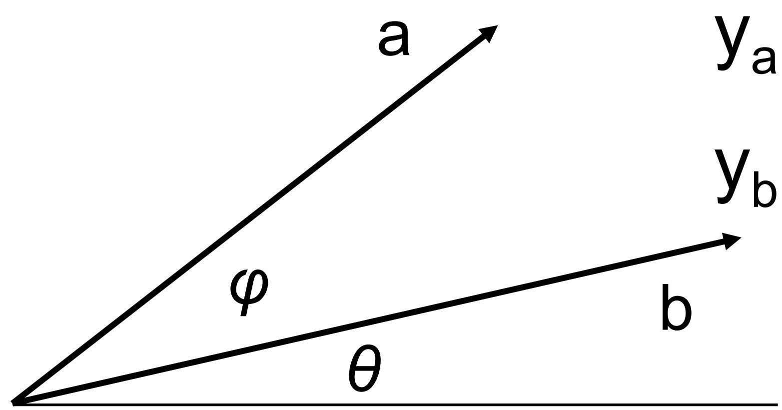
$$\mathbf{a} \bullet \mathbf{b} = ||\mathbf{a}|| \bullet ||\mathbf{b}|| \cos(\varphi)$$

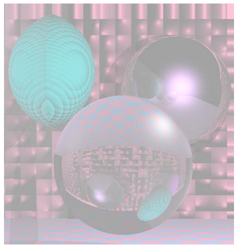
$$x_a = ||a||\cos(\theta+\varphi)$$

$$x_b = ||b||\cos(\theta)$$

$$y_a = ||a||\sin(\theta+\varphi)$$

$$y_b = ||b||\sin(\theta)$$

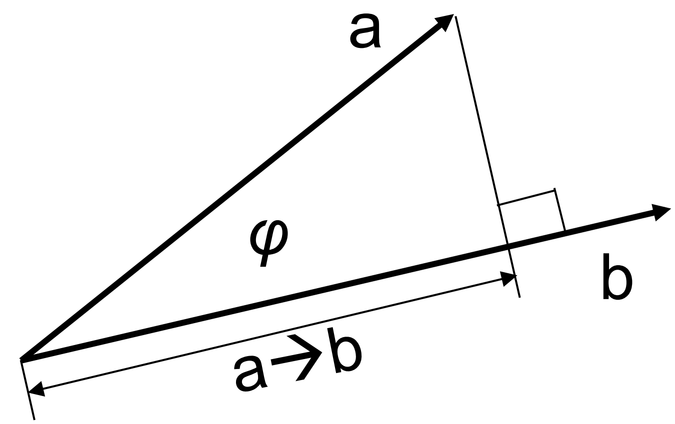




Projection

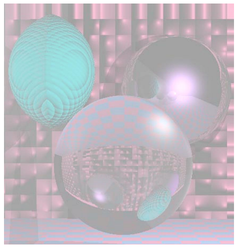
a =(x_a, y_a)   **b** =(x_b, y_b)

a•b = ||**a**||•||**b**||cos(φ)



The length of the projection of **a** onto **b** is given by

$$a \rightarrow b = \|a\| \cos(\varphi) = \frac{a \bullet b}{\|b\|}$$



Output Devices

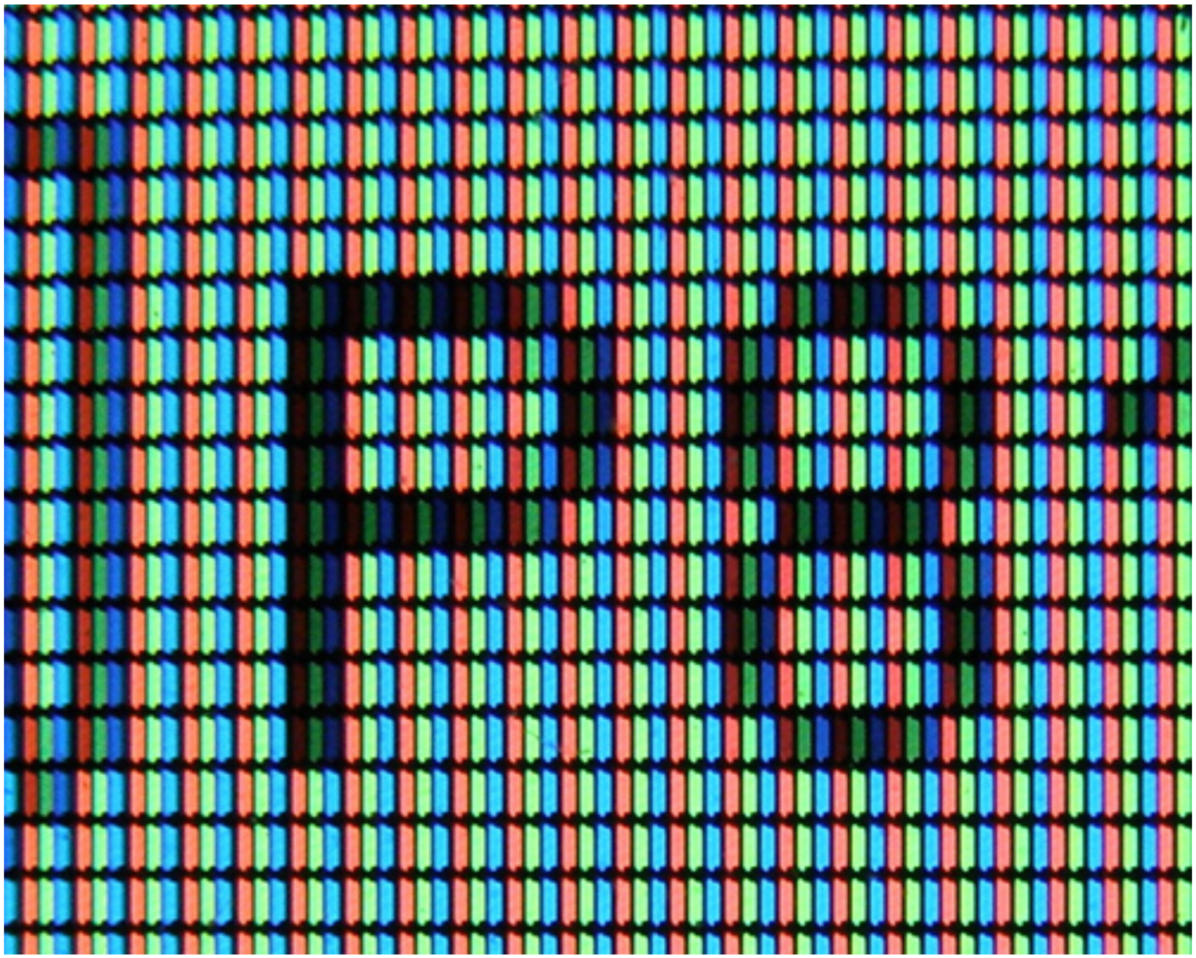
- *a raster is a rectangular array of pixels (picture elements)*
- *common raster output devices include CRT and LCD monitors, ink jet and laser printers*
- *typically considered as top-to-bottom array of left-to-right rows, because that is how CRTs are (were) typically scanned*
- *for this reason, device (e.g. on-screen) coordinate frame typically has origin in upper left, axis aims to right, and axis aims down*

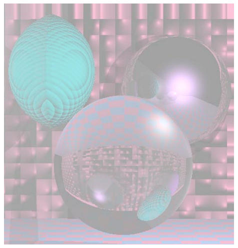
Device Resolution

- (native) *resolution of the device is the dimensions (note this is reverse of typical way we write matrix dimensions) of its raster output hardware*
- *typical resolutions for monitors are 640x480 (VGA, the archaic but celebrated Video Graphics Array), 800x600, 1024x768, 1280x1024, 1600x1200, etc*
- *higher resolution is generally "better" because finer detail can be represented*
  - *more computation required for more pixels though, and more pixels makes the display hardware more expensive*
  - *however monitors usually can display lower or higher (within some limits) resolution images than their native resolution by scaling (we will study how to scale images later in the course)*

Sub-pixel Display

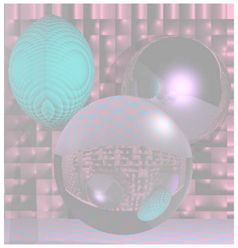
http://en.wikipedia.org/wiki/Pixel





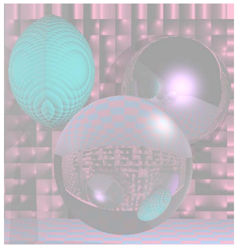
How are Rasters Represented?

- for a *monochrome image, each pixel corresponds to one bit (also called a binary image)*
- *typically in graphics we use at least greyscale images, where bits are used to represent the intensity at each pixel. The number of gray levels at each pixel is usually a multiple of 8.*
- *for a color image, compose multiple greyscale images, where each corresponds to a different color component. Three images corresponding to red, green, and blue color components are one typical arrangement. The images can be stored as independent planes or they may be interleaved.*



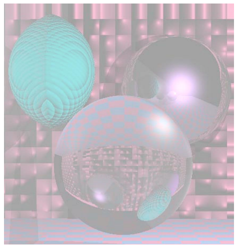
in-memory representation of a raster

- monochrome image is typically a linear array of r x c x $\mathcal{B}$ bytes, where r and c are the number of rows and columns in the raster, and $\mathcal{B}$ is the number of bytes per pixel
- value of pixel at location (i, j) is thus stored in the $\mathcal{B}$ bytes at memory location $(ic + j)\mathcal{B}$ relative to the beginning of the array
  - the order of bytes within the pixel value is determined by the *byte order of the computer, which may be little-endian (least significant byte first) or big-endian (most significant byte first).*
  - *Nowadays, little-endian is more common (e.g. Intel x86). Big-endian may still be encountered on e.g. PowerPC architectures (which is what Apple used in Mac computers up to around 2006).*



Color Image Representation

- for color images, either store as (typically three) separate monochrome rasters (planes), or interleave by packing all color components for a pixel into a contiguous block of memory (interleaved is more common now)
- the order of the color components, as well as the number of bits per component, is called the *pixel format*

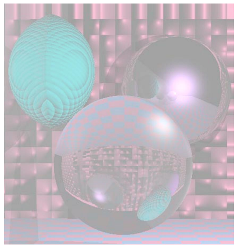


Common Pixel Formats

- *common pixel formats today include*
  - *24-bit RGB ($b_r = b_g = b_b = 8$) ("over 16 million colors!")*
  - *32-bit RGB (like 24 bit but with one byte of padding)*
  - *16-bit 5:6:5 RGB ($b_r = 5$, $b_g = 6$, $b_b = 5$) (human eye is most sensitive to green; common for lower-quality video because it looks ok for images of real-world scenes and uses 2 bytes per pixel, reducing file size)*
- $(ic + j)\mathcal{B}$ works with
  - $\mathcal{B} = \{(b_r + b_g + b_b + padding)\}/8$
- byte ordering (little- vs big-endian) only matters *within each color component and if some $b_r > 8$*

Frame Buffer

- In-memory raster is called a *frame buffer when hardware is set up so that changes to memory contents drive pixel colors on the display itself. Most modern display hardware has a such a frame buffer.*
  - *in fact, generally more than one, and can switch among them*
  - *a common way to produce a smooth-looking animation is to use two buffers: the front buffer is rendered to the screen, and the back buffer is not*
  - *this is called double buffering*
  - *Each new frame of the animation is drawn onto the back buffer. Because it can take some time (hopefully not too long) to draw, this avoids seeing a "partial frame".*
  - *once the drawing is complete, the buffers are swapped*



Rasterization

- how to *render images of geometry, say line segments or triangles, onto a raster display?*
- *need to figure out what pixels to "light up" to draw the shape*
- *this is the process of rasterization*
- *will study line segment rasterization now and triangles later in the course*

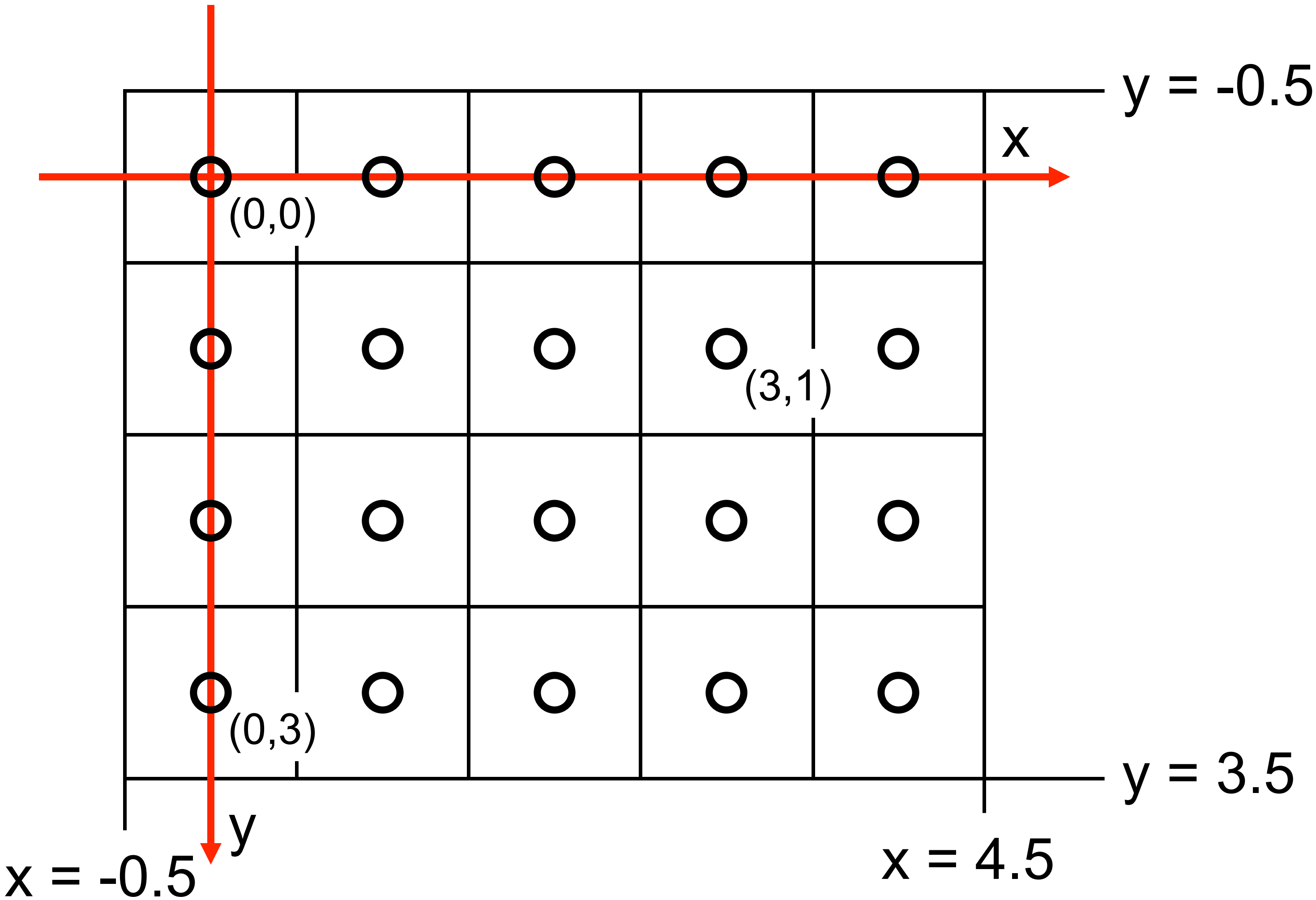
Vector Output

- historically, vector displays were developed first
- a CRT is made to scan line segments by steering an electron beam from start to end of each segment (can generalize to curves)
- potentially more efficient because only need to scan along the actual line segments vs always scanning a raster across the whole screen
- but hard to draw images of real-world scenes, and how to deal with color?
- nowadays, vector output is sometimes still encountered on a *pen plotter, but even these are mostly antiques*

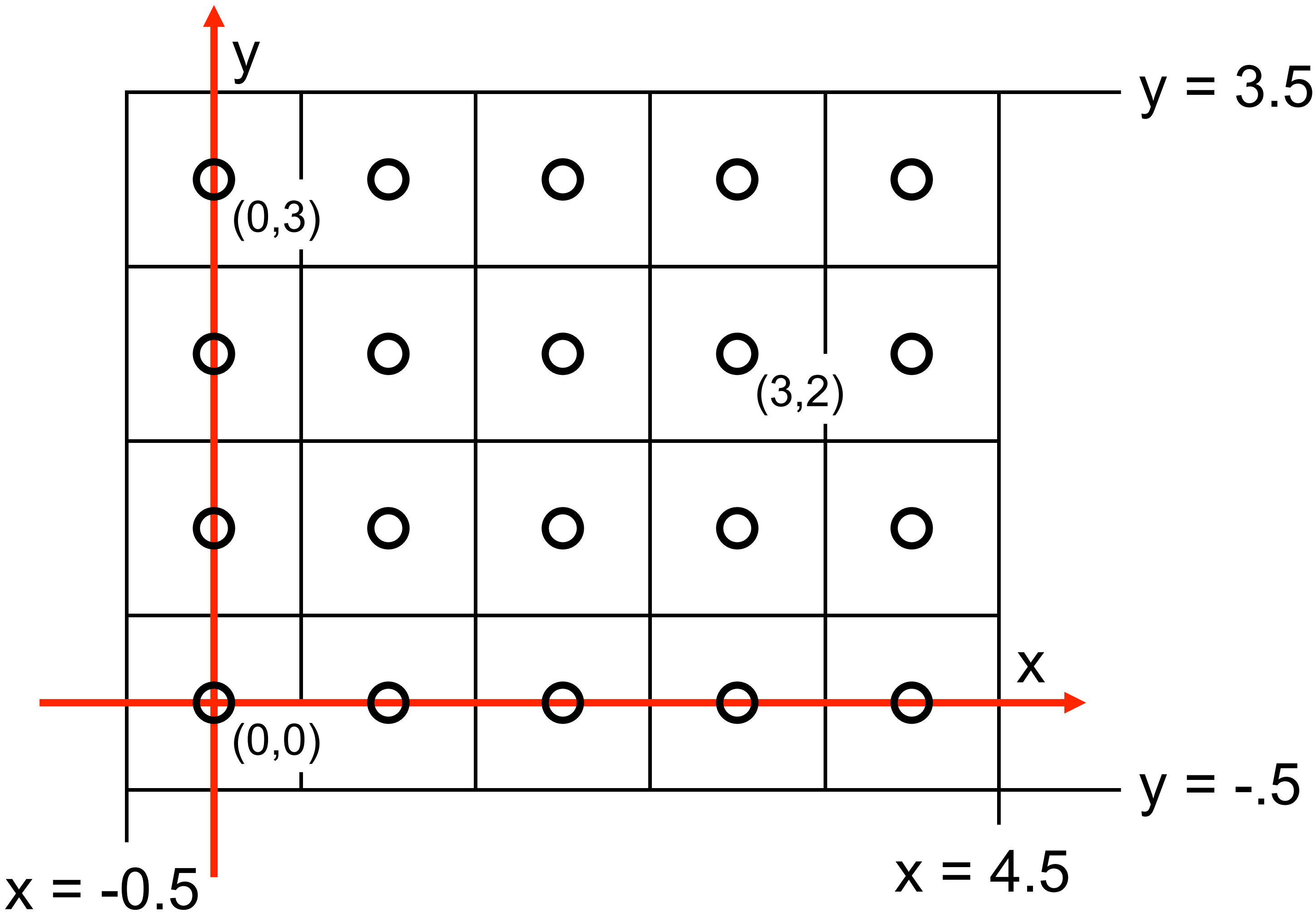
Vector Representation

- *some software systems represent graphics in a vector form. PostScript, PDF (portable document format), and SVG (scalable vector graphics)*
- *in a vector format, a picture is stored not as an array of pixels, but as a list of instructions about how to draw it*
  - *vector format is "better" for some kinds of images, particularly line drawings and images (e.g. cartoons or computer art)*
- *since the actual geometry, vs a sampling of it, is stored, vector images can generally be scaled to larger or smaller sizes without any loss of quality*
  - *vector images may also require less memory to store, and may be more compressible*

Pixel Coordinates



Pixel Coordinates

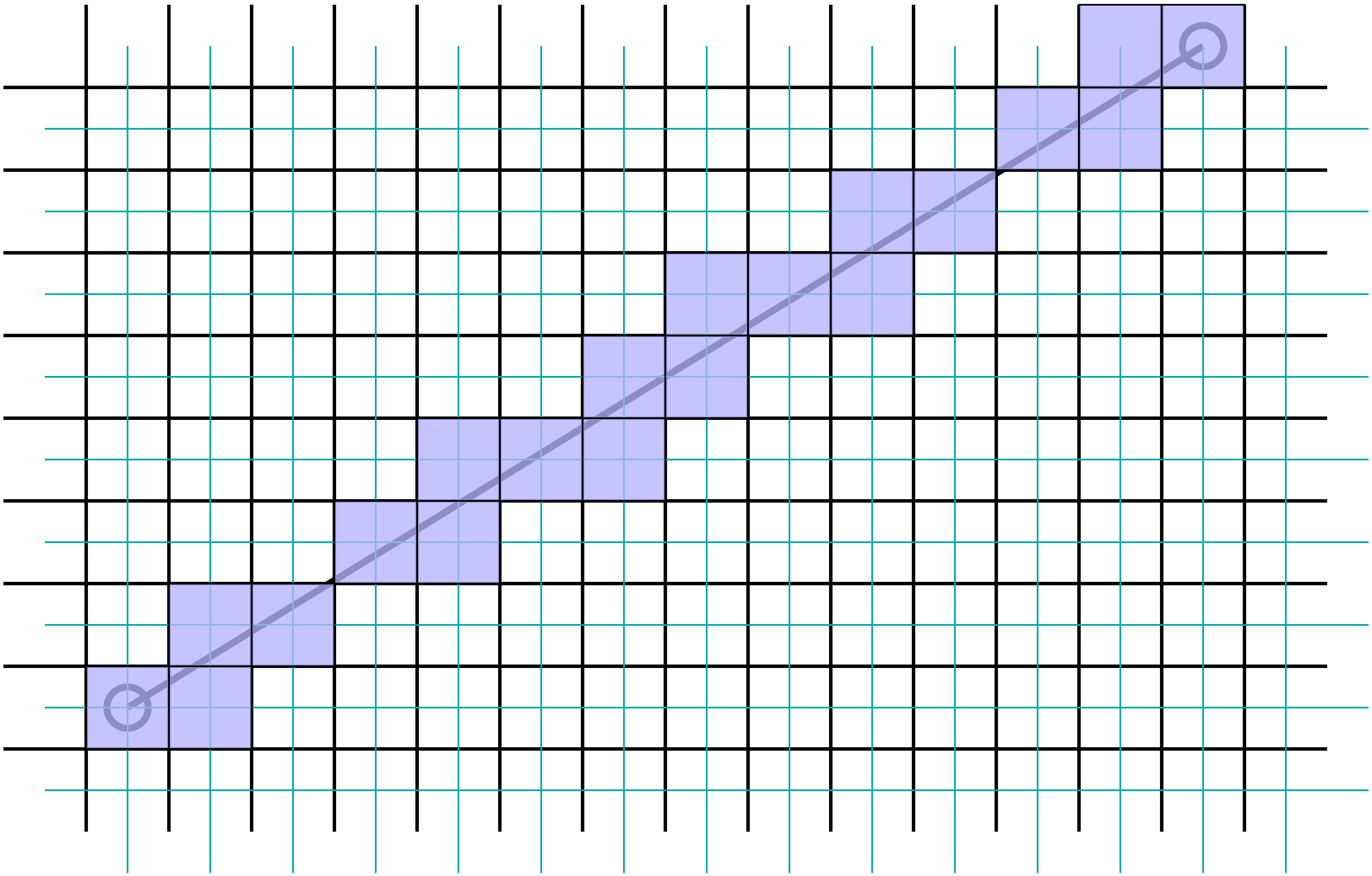


What Makes a Good Line?

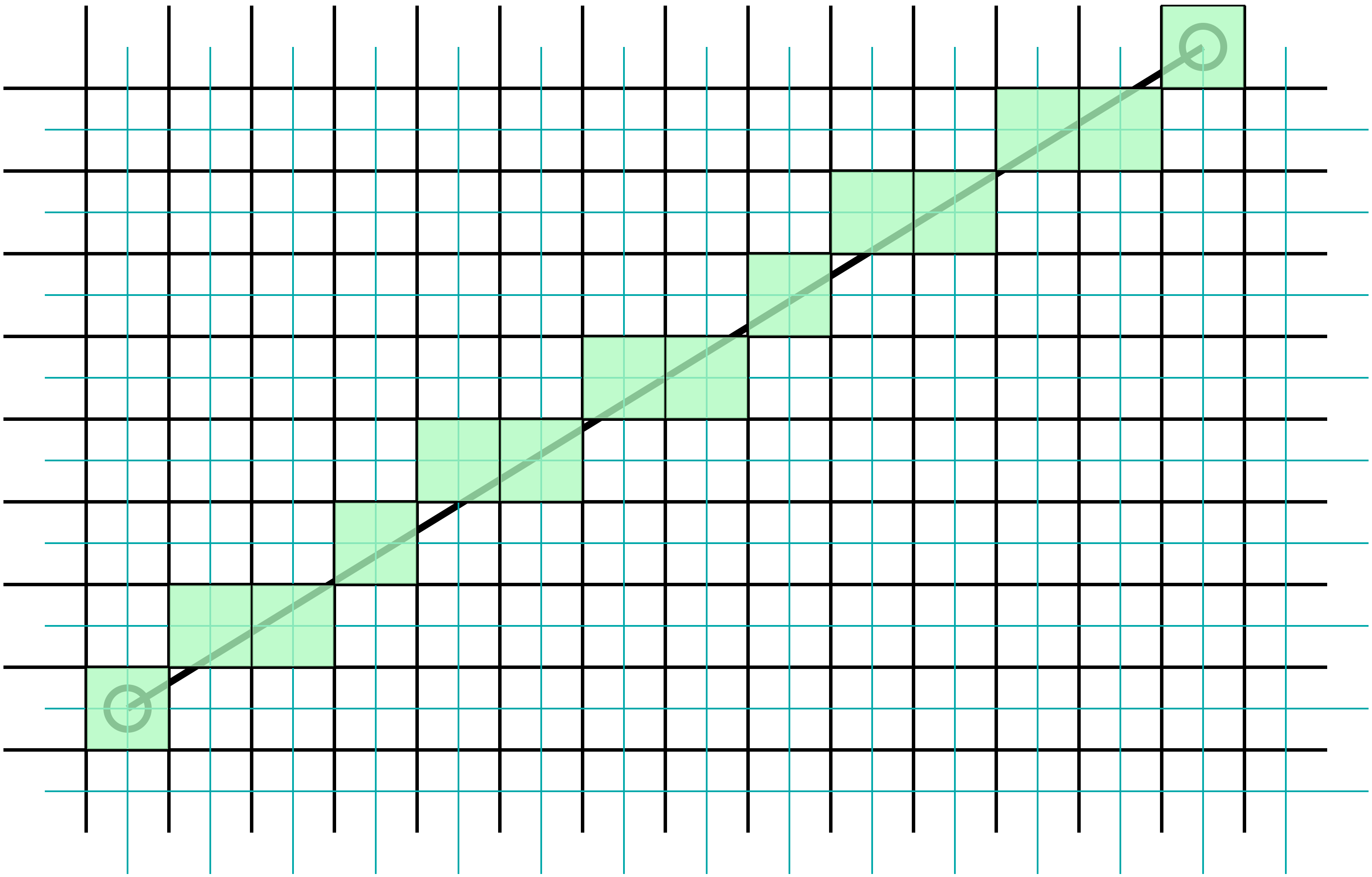
- Not too jaggy
- Uniform thickness along a line
- Uniform thickness of lines at different angles
- Symmetry, Line(P,Q) = Line(Q,P)

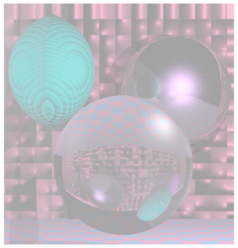
- A good line algorithm should be fast.

Line Drawing



Line Drawing





Which Pixels Should We Color?

- Given $P_0 = (x_0, y_0)$, $P_1 = (x_1, y_1)$
- We could use the equation of the line:
  - $y = mx + b$
  - $m = (y_1 – y_0)/(x_1 – x_0)$
  - $b = y_1 - mx_1$
- And a loop

for $x = x_0$ to x_1
  $y = mx + b$
  draw (x, y)

This calls for real multiplication for each pixel

This only works if $x_0 <= x_1$ and $|m| <= 1$.

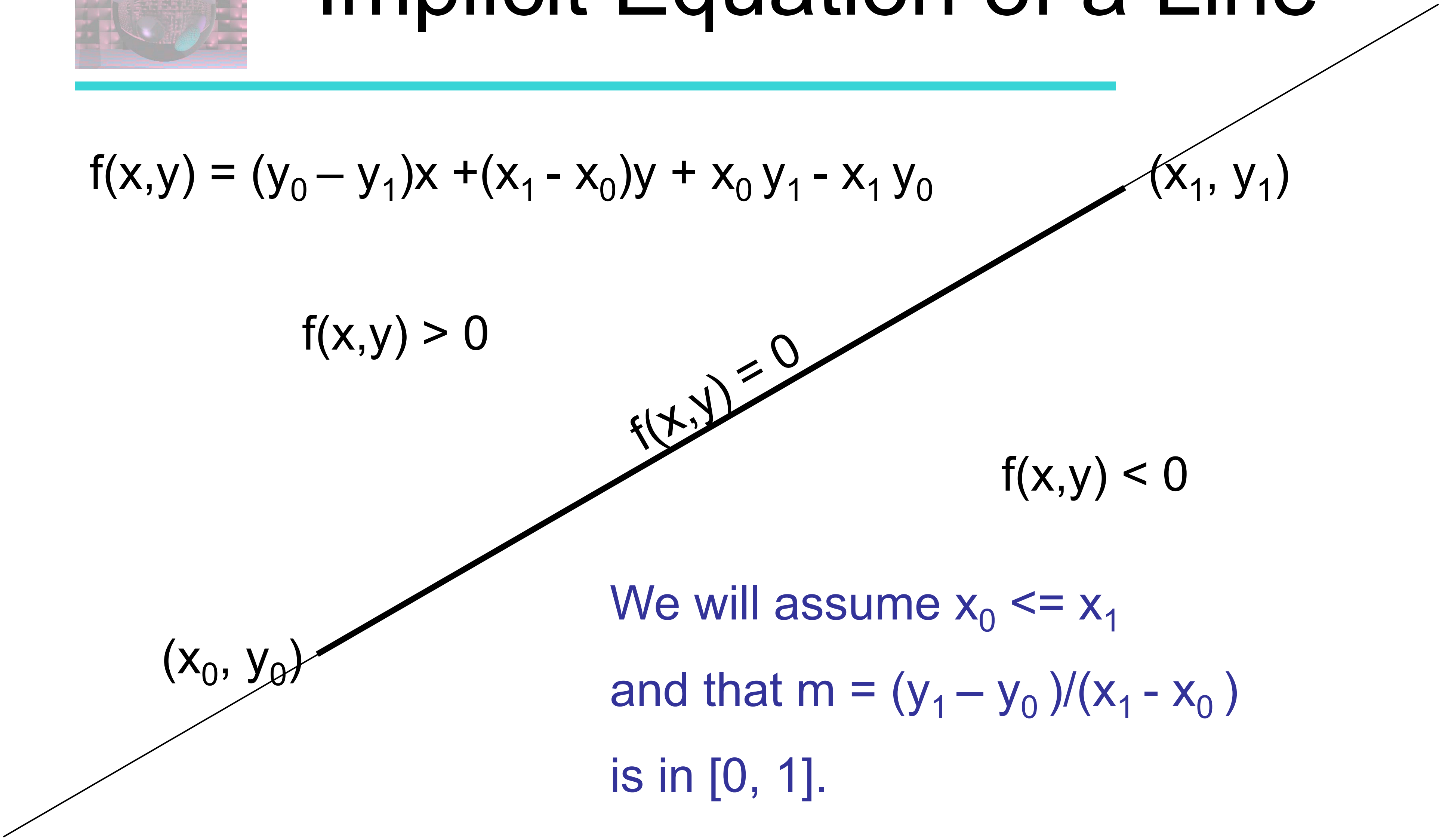
Midpoint Algorithm

- Pitteway 1967
- Van Aiken abd Nowak 1985
- Draws the same pixels as *Bresenham Algorithm* 1965.
- Uses integer arithmetic and incremental computation.
- Uses a decision function to decide on the next point
- Draws the thinnest possible line from

  (x_0, y_0) to (x_1, y_1) that has no gaps.
- A diagonal connection between pixels is not a gap.

Implicit Equation of a Line

$f(x,y) = (y_0 – y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0$



We will assume $x_0 <= x_1$

and that $m = (y_1 – y_0)/(x_1 - x_0)$

is in [0, 1].

Basic Form of the Algotithm

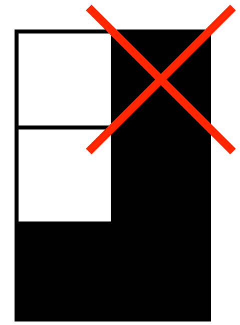
$y = y_0$

for $x = x_0$ to x_1 **do**

    draw (x, y)

    if (some condition) **then**

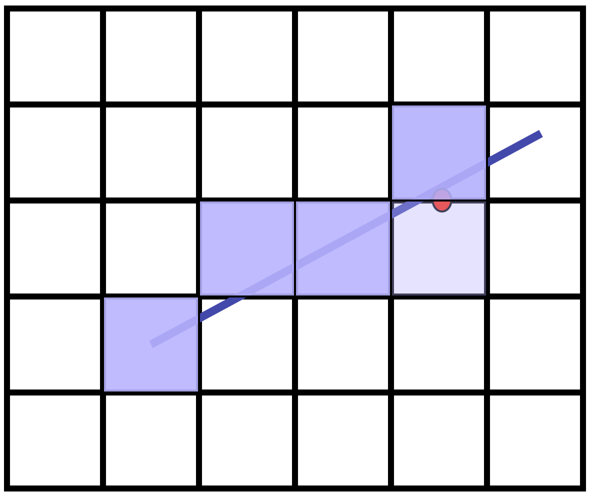
        $y = y + 1$

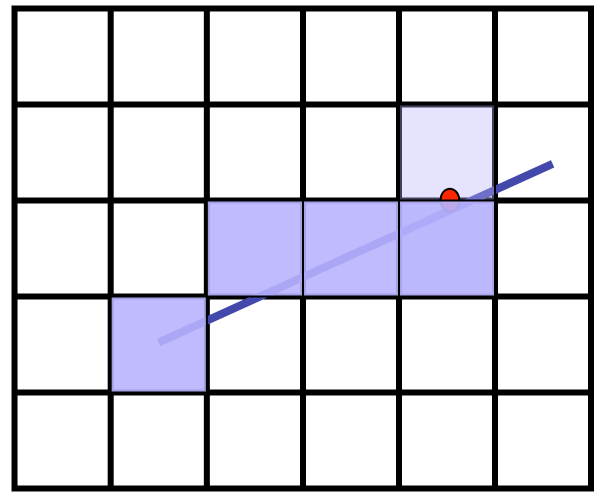


We want to compute this condition efficiently.

Since m is in [0, 1], as we move from x to x+1, the y value stays the same or goes up by 1.

Above or Below the Midpoint?





Finding the Next Pixel

Assume we just drew (x, y).

For the next pixel, we must decide between

(x+1, y) and (x+1, y+1).

The midpoint between the choices is

(x+1, y+0.5).

If the line passes below (x+1, y+0.5), we draw the bottom pixel.

Otherwise, we draw the upper pixel.

The Decision Function

if f(x+1, y+0.5) < 0

    // midpoint below line

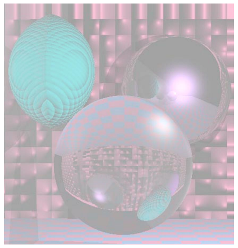
    y = y + 1

$f(x,y) = (y_0 – y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0$

How do we compute f(x+1, y+0.5)

    incrementally?

    using only integer arithmetic?



Incremental Computation

$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0$

$f(x + 1, y) = f(x, y) + (y_0 - y_1)$

$f(x + 1, y + 1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$

$y = y_0$

$d = f(x_0 + 1, y + 0.5)$

for $x = x_0$ to x_1 **do**

draw (x, y)

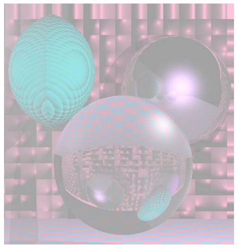
if $d < 0$ **then**

$y = y + 1$

$d = d + (y_0 - y_1) + (x_1 - x_0)$

else

$d = d + (y_0 - y_1)$



Integer Decision Function

$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0$

$f(x_0 + 1, y + 0.5)$

$= (y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(y + 0.5) + x_0 y_1 - x_1 y_0$

$2f(x_0 + 1, y + 0.5)$

$= 2(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2y + 1) + 2x_0 y_1 - 2x_1 y_0$

2f(x, y) = 0 if (x, y) is on the line.
< 0 if (x, y) is below the line.
> 0 if (x, y) is above the line.

Midpoint Line Algorithm

$y = y_0$

$d = 2(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2y_0 + 1) + 2x_0 y_1 - 2x_1 y_0$

for $x = x_0$ to x_1 **do**

    draw (x, y)

    if $d < 0$ **then**

        $y = y + 1$

        $d = d + 2(y_0 - y_1) + 2(x_1 - x_0)$

    else

        $d = d + 2(y_0 - y_1)$

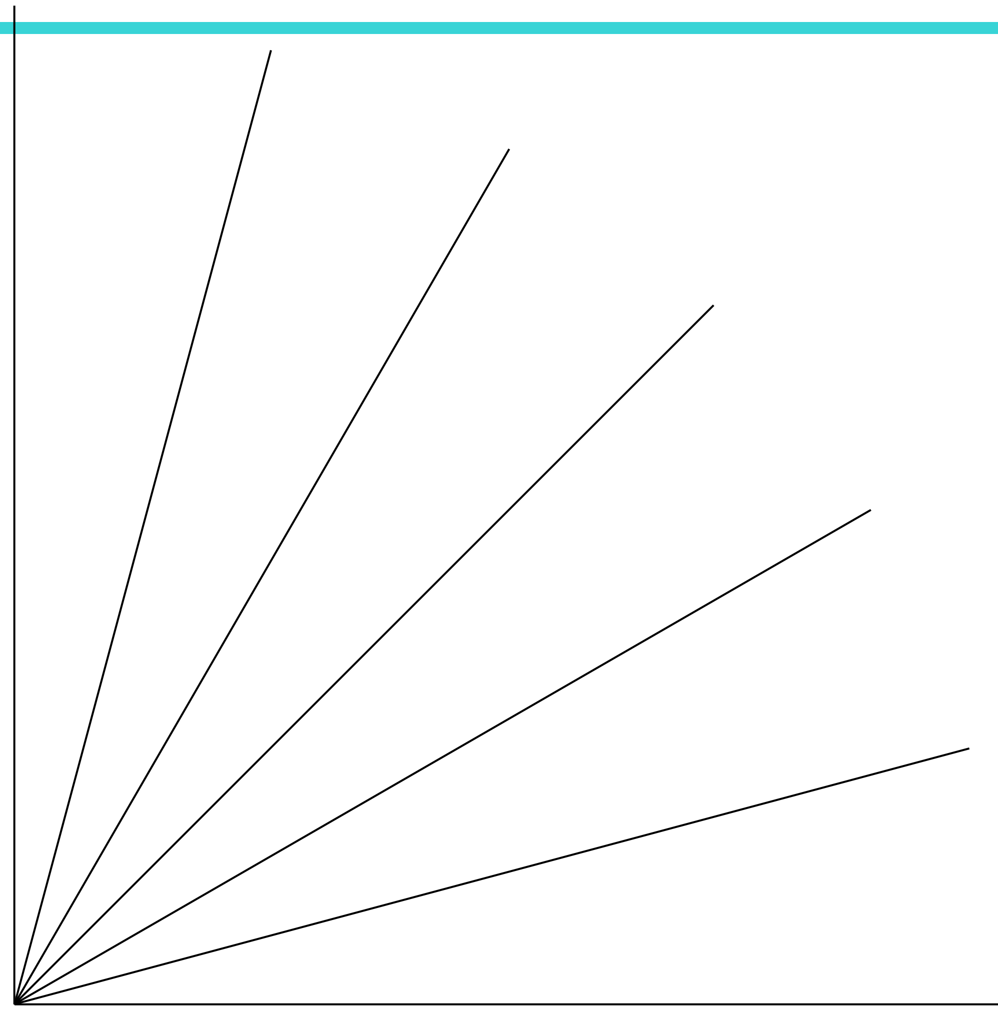
These are constants and can be computed before the loop.

Line Attributes

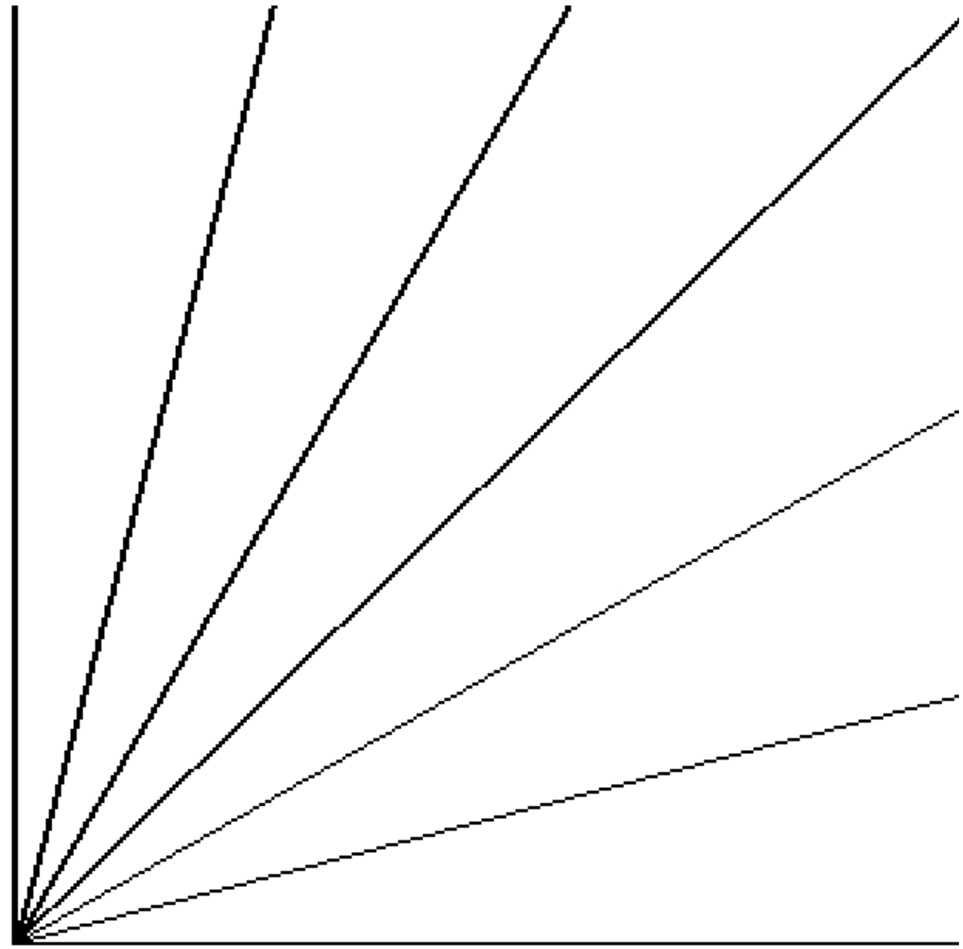
- line width
- dash patterns
- end caps: butt, round, square

Joins: round, bevel, miter

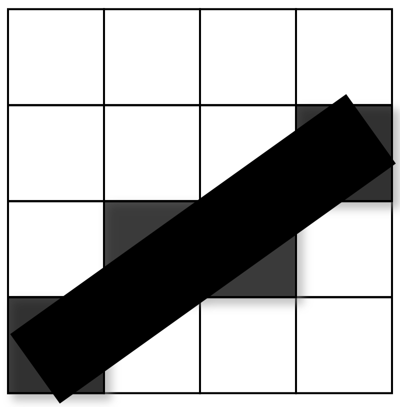
Some Lines

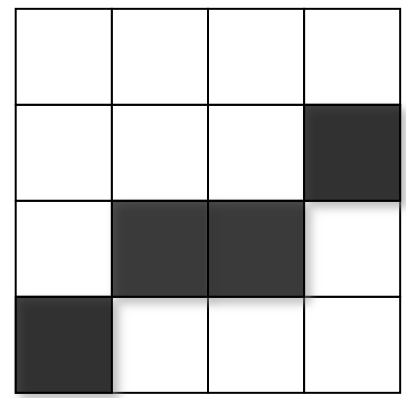


Some Lines Magnified

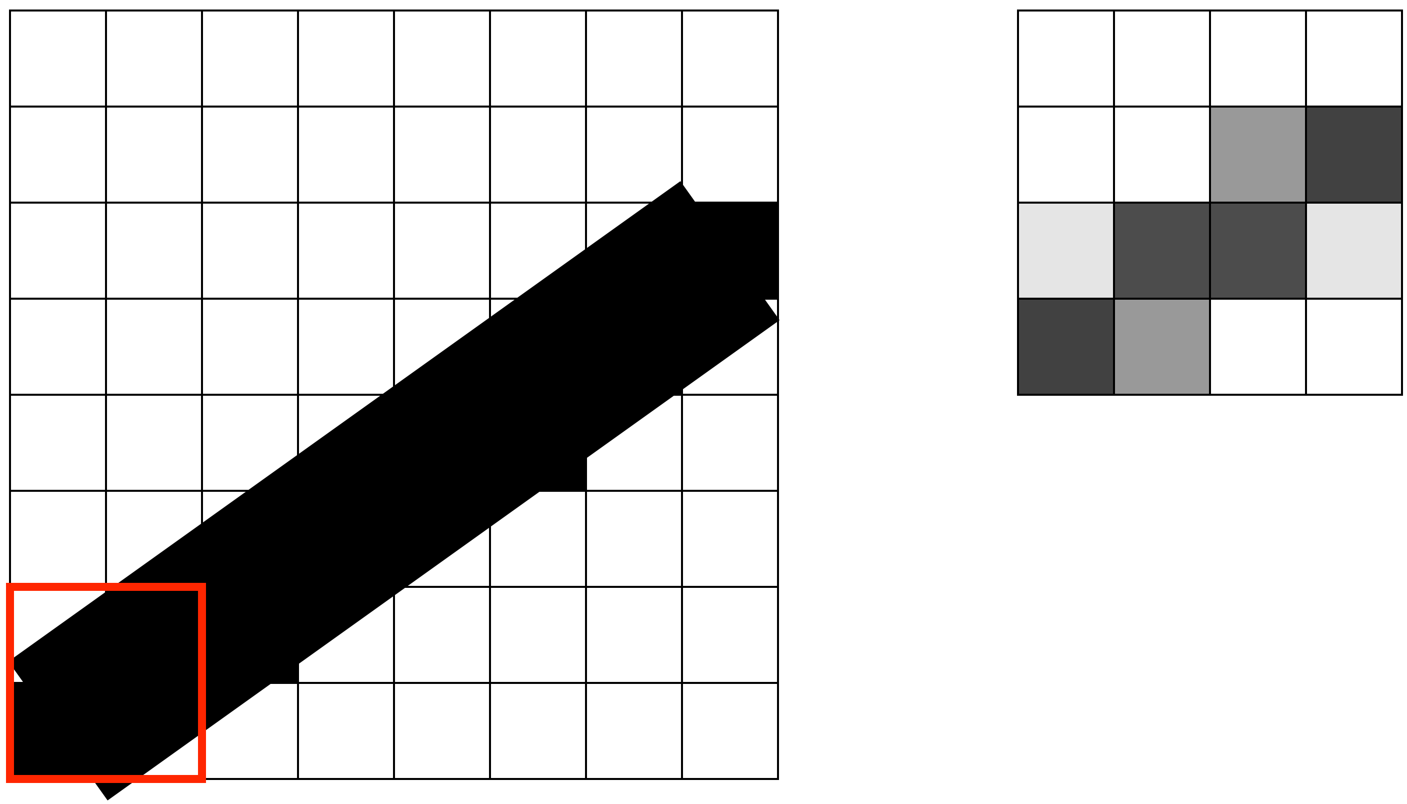


Antialiasing by Downsampling





Antialiasing by Downsampling



Antialiasing by Downsampling