

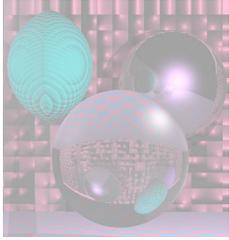
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# CS 4300

# Computer Graphics

Prof. Harriet Fell  
Fall 2012

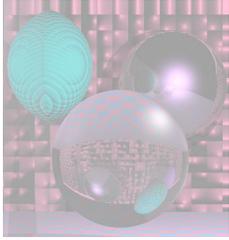
Lecture 28 – November 8, 2012



# Today's Topics

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- Fractals
  - Mandelbrot Set
    - Julia Sets
  - L-Systems



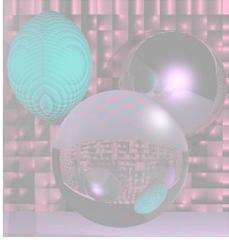
# Fractals

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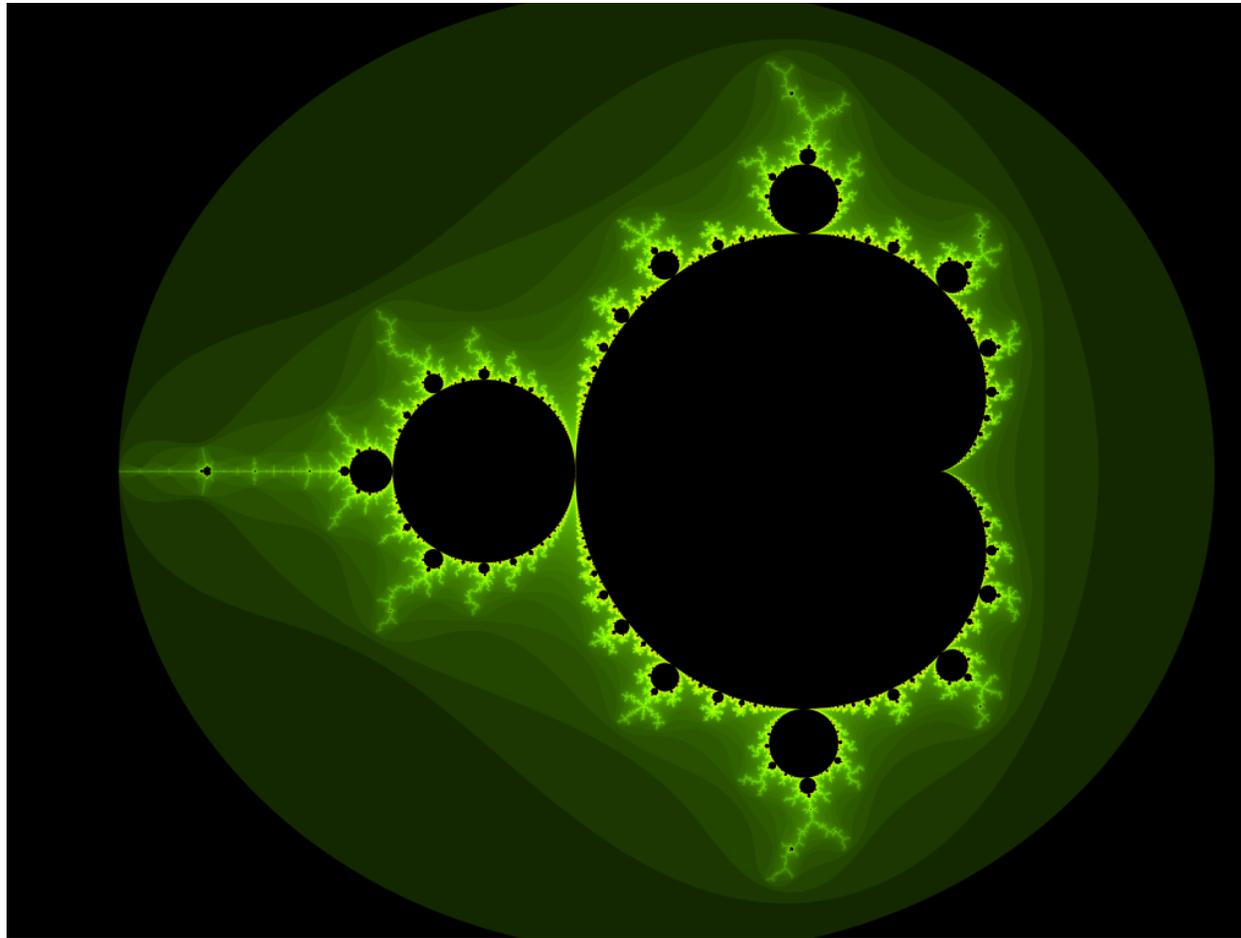
The term *fractal* was coined in 1975 by Benoît Mandelbrot, from the Latin *fractus*, meaning "broken" or "fractured".

(colloquial) a shape that is recursively constructed or self-similar, that is, a shape that appears similar at all scales of magnification.

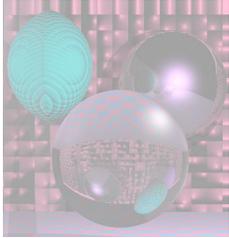
(mathematics) a geometric object that has a Hausdorff dimension greater than its topological dimension.



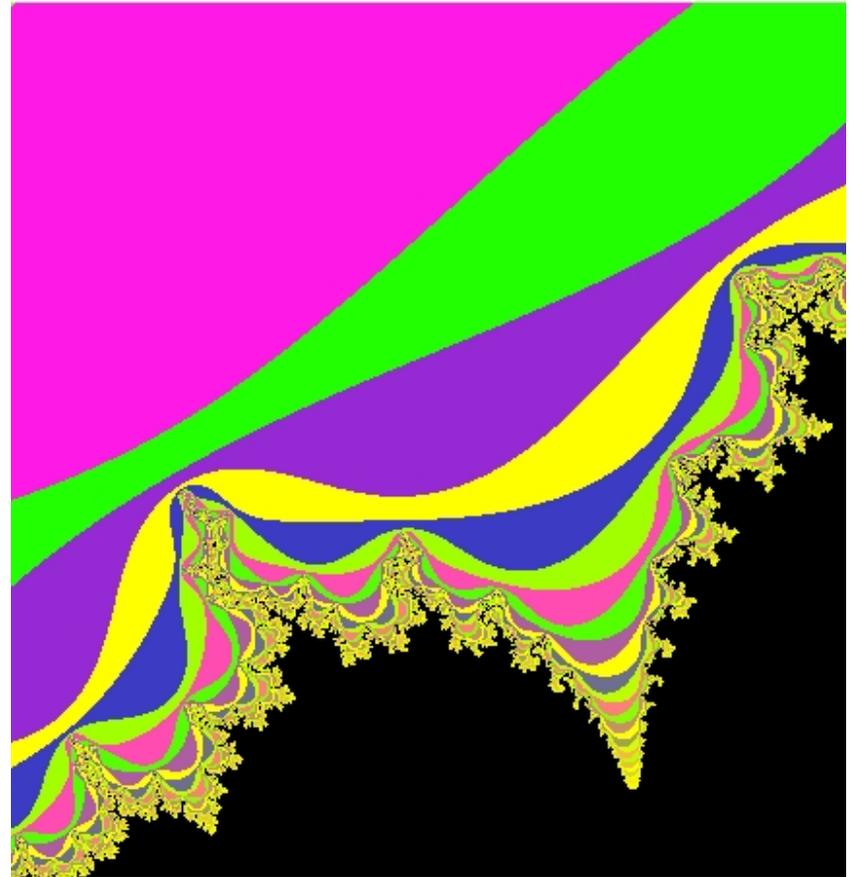
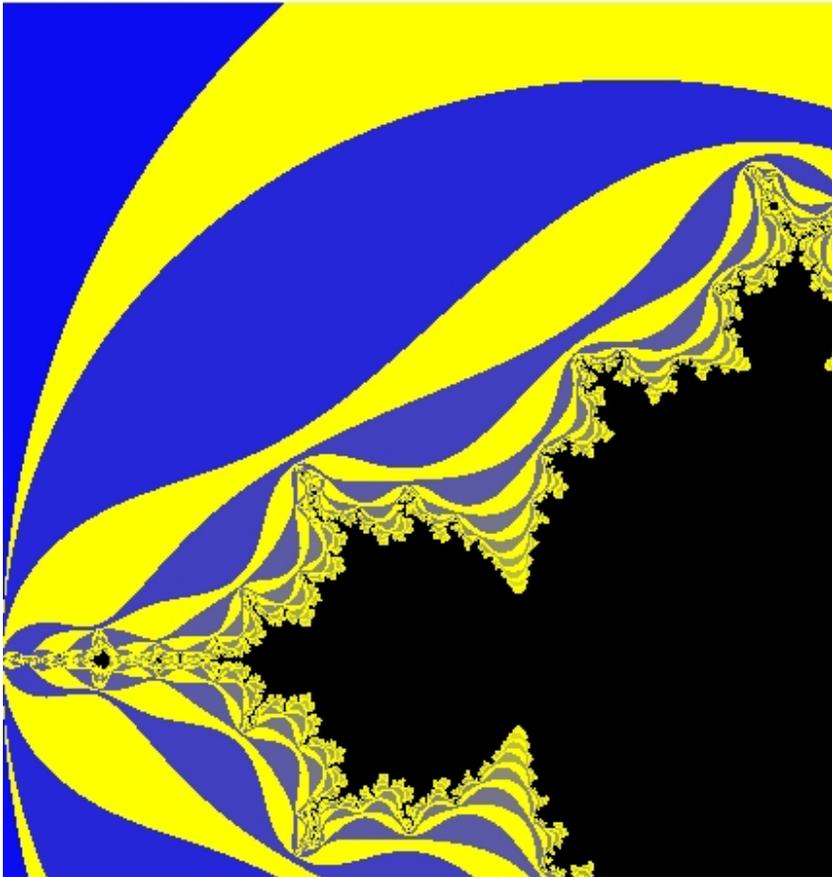
# Mandelbrot Set

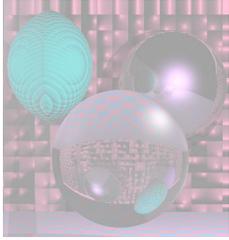


Mandelbrotset, rendered with [Evercat](#)'s program.



# Mandelbrot Set





# What is the Mandelbrot Set?

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We start with a quadratic function on the complex numbers.

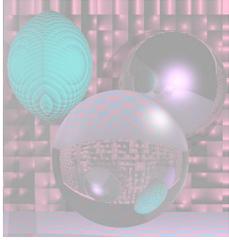
$$f_c : \mathbb{C} \rightarrow \mathbb{C}$$

$$z \mapsto z^2 + c$$

The *Mandelbrot Set* is the set of complex  $c$  such that

$$f_c^n(0) \not\rightarrow \infty$$

where  $f_c^n$  is the  $n$ -fold composition of  $f_c$  with itself.



# Example

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$$f(z) = z^2 - 1$$

$$f(0) = -1 \qquad f^2(0) = f(-1) = 1 - 1 = 0$$

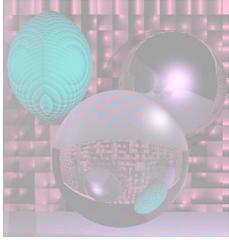
$$f^n(0) = \begin{cases} -1 & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$f(2) = 4 - 1 = 3 \qquad f^2(2) = f(3) = 9 - 1 = 8$$

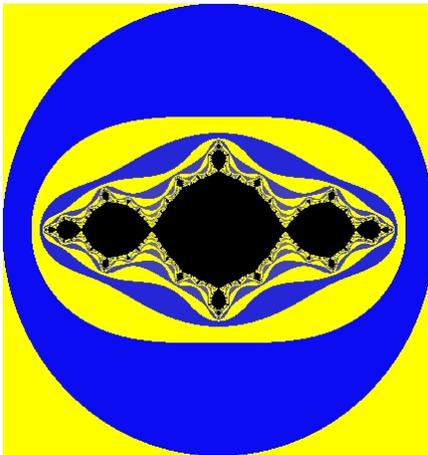
$f^n(2)$  tend to  $\infty$  as  $n$  tends to  $\infty$ .

$$f(i) = i^2 - 1 = -2 \qquad f^2(i) = f(-2) = 4 - 1 = 3$$

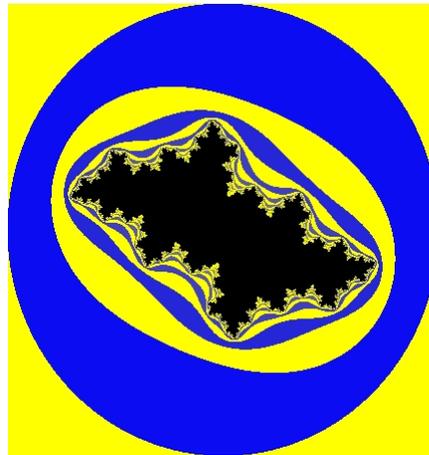
$f^n(i)$  tend to  $\infty$  as  $n$  tends to  $\infty$ .



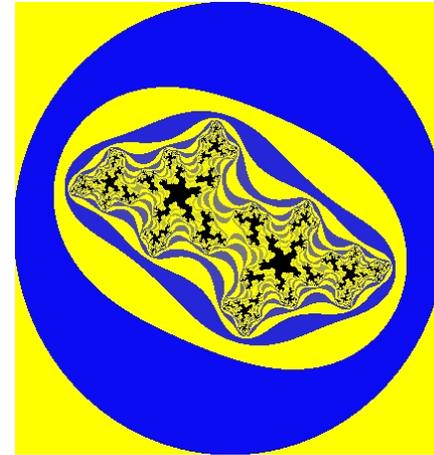
# (Filled-in) Julia Sets



$$c = -1$$



$$c = -.5 + .5i$$



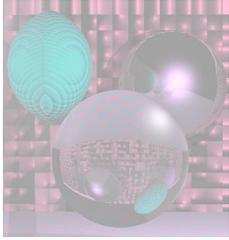
$$c = -5 + .5i$$

$$f_c : \mathbb{C} \rightarrow \mathbb{C}$$

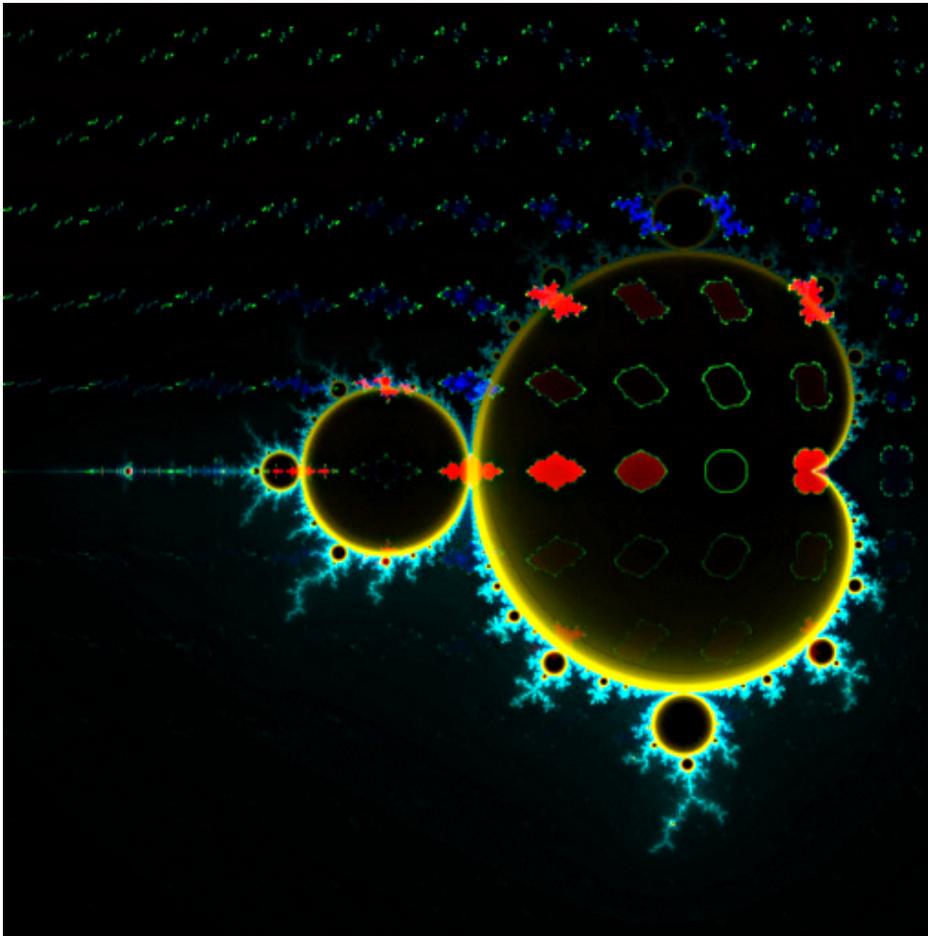
$$z \mapsto z^2 + c$$

The *Julia Set* of  $f_c$  is the set of points with 'chaotic' behavior under iteration.

The *filled-in Julia set* (or Prisoner Set), is the set of all  $z$  whose orbits do not tend towards infinity. The "normal" Julia set is the boundary of the filled-in Julia set.



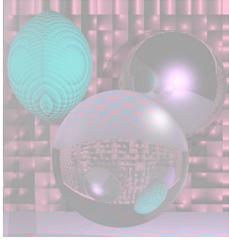
# Julia Sets and the Mandelbrot Set



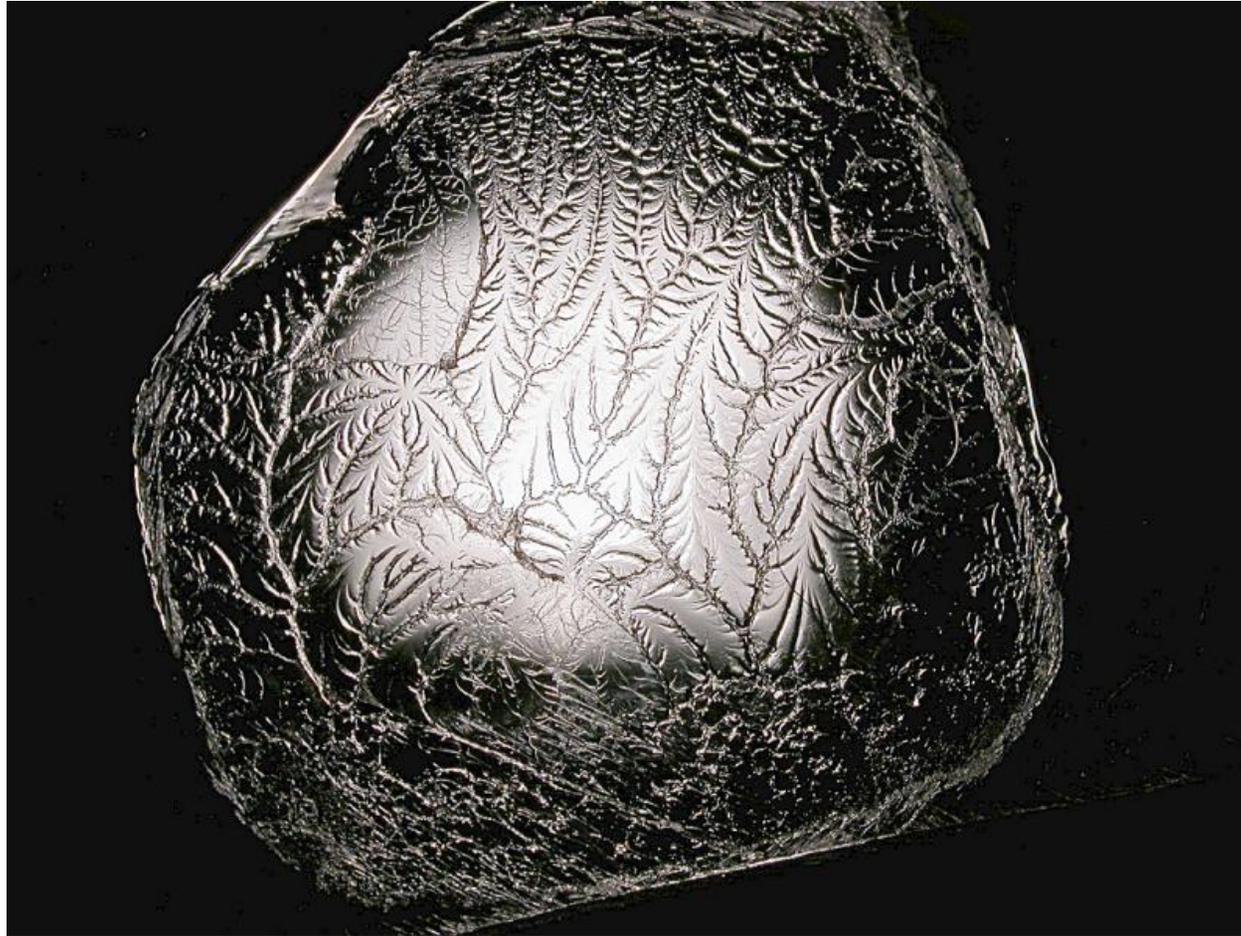
Some Julia sets are connected others are not.

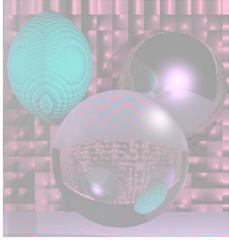
The Mandelbrot set is the set of Complex numbers  $c$  for which the Julia set of  $f_c(z) = z^2 + c$  is connected.

Map of 121 Julia sets in position over the Mandelbrot set (wikipedia)



A fractal is formed when pulling apart two glue-covered acrylic sheets.



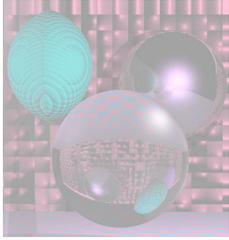


# Fractal Form of a Romanesco Broccoli

photo by Jon Sullivan

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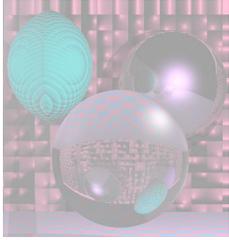




# L-Systems

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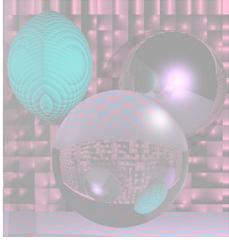
- An *L-system* or *Lindenmayer system*, after Aristid Lindenmayer (1925–1989), is a formal grammar (a set of rules and symbols) most famously used to model the growth processes of plant development, though able to model the morphology of a variety of organisms.
- L-systems can also be used to generate self-similar fractals such as iterated function systems.



# L-System References

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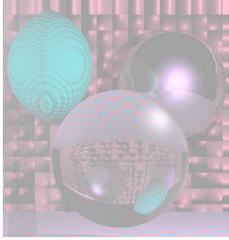
- Przemyslaw Prusinkiewicz & Aristid Lindenmayer, “The Algorithmic Beauty of Plants,” Springer, 1996.
- <http://en.wikipedia.org/wiki/L-System>



# L-System Grammar

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- $\mathbf{G} = \{V, S, \omega, P\}$ , where
  - $\mathbf{V}$  (the *alphabet*) is a set of variables
  - $\mathbf{S}$  is a set of constant symbols
  - $\omega$  (*start, axiom or initiator*) is a string of symbols from  $\mathbf{V}$  defining the initial state of the system
  - $\mathbf{P}$  is a set of *rules or productions* defining the way variables can be replaced with combinations of constants and other variables.
    - A production consists of two strings - the *predecessor* and the *successor*.



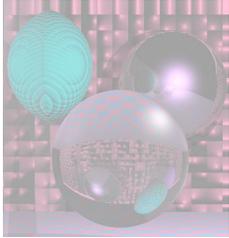
# L-System Examples

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- **Koch curve** (from wikipedia)
- A variant which uses only right-angles.
  - **variables** : F
  - **constants** : + -
  - **start** : F
  - **rules** :  $(F \rightarrow F+F-F-F+F)$
- Here,  $F$  means "draw forward", + means "turn left  $90^\circ$ ", and - means "turn right  $90^\circ$ " (see [turtle graphics](#)).

# Turtle Graphics

```
class Turtle {  
    double angle;    // direction of turtle motion in degrees  
    double X;       // current x position  
    double Y;       // current y position  
    double step;    // step size of turtle motion  
    boolean pen;    // true if the pen is down  
  
    public void forward(Graphics g)  
    // moves turtle forward distance step in direction angle  
  
    public void turn(double ang)  
    // sets angle = angle + ang;  
  
    public void penDown(), public void penUp()  
    // set pen to true or false  
}
```



# My L-System Data Files

Koch Triangle Form

4

90

F

F:F+F-F-F+F

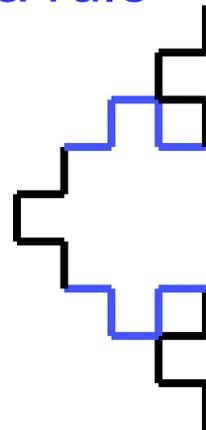
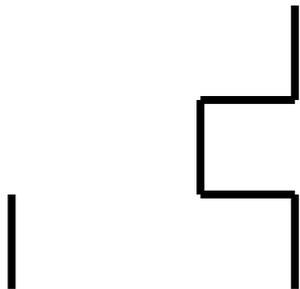
// title

// number of levels to iterate

// angle to turn

// starting shape

// a rule

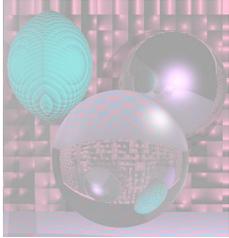


[Go to Eclipse](#)

F

F+F-F-F+F

F+F-F-F+F+F+F-F-F+F-F+F-F-F+F-F  
+F-F-F+F+F+F-F-F+F



# More Variables

Dragon

10

90

L

L:L+R+

R:-L-R

When drawing, treat L and R just like F.



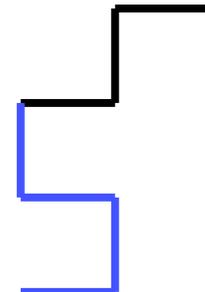
L



L+R+

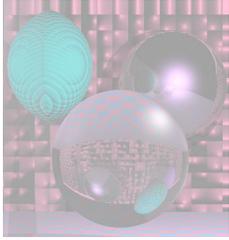


L+R+ + -L-R +



L+R+ + -L-R + + -L

+R+ - -L-R +



# A Different Angle

Sierpinski Gasket

6

60

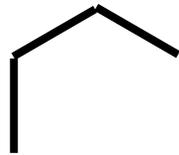
R

L:R+L+R

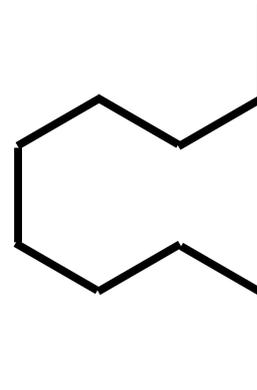
R:L-R-L



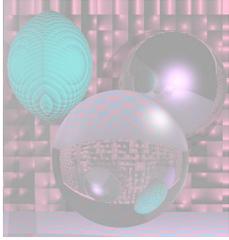
R



L-R-L



R+L+R- L-R-L -R+L+R



# Moving with Pen Up

Islands and Lakes

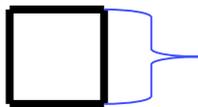
2

90

F+F+F+F

F:F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF

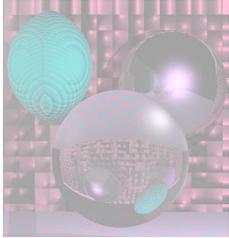
f:ffffff // f means move forward with the pen up



next slide

F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF

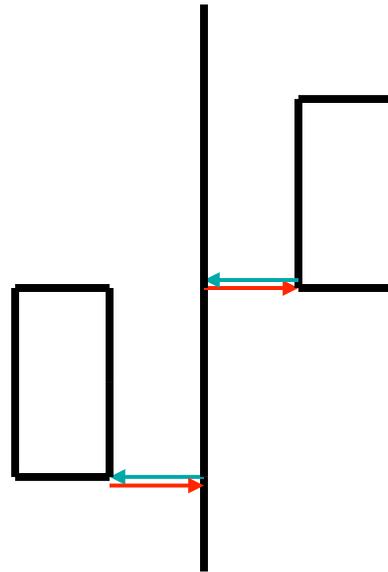
F+F+F+F



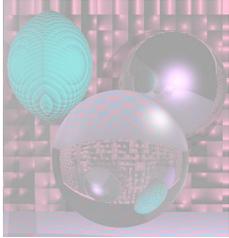
# Islands and Lakes

## One Side of the Box

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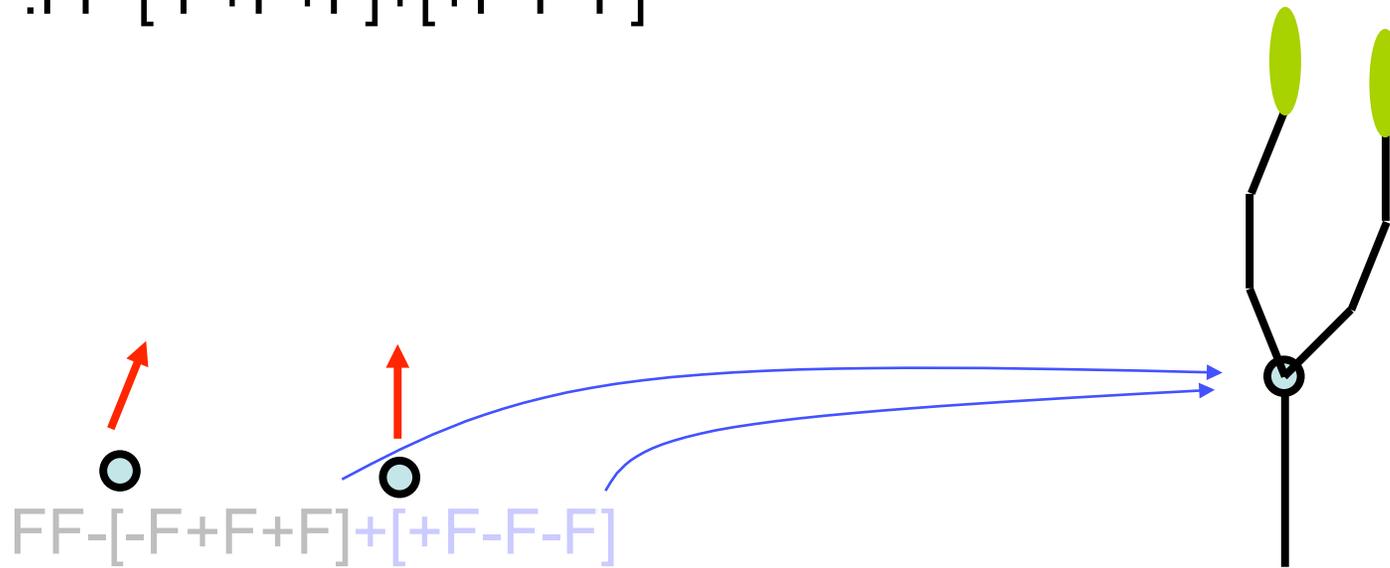


F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF



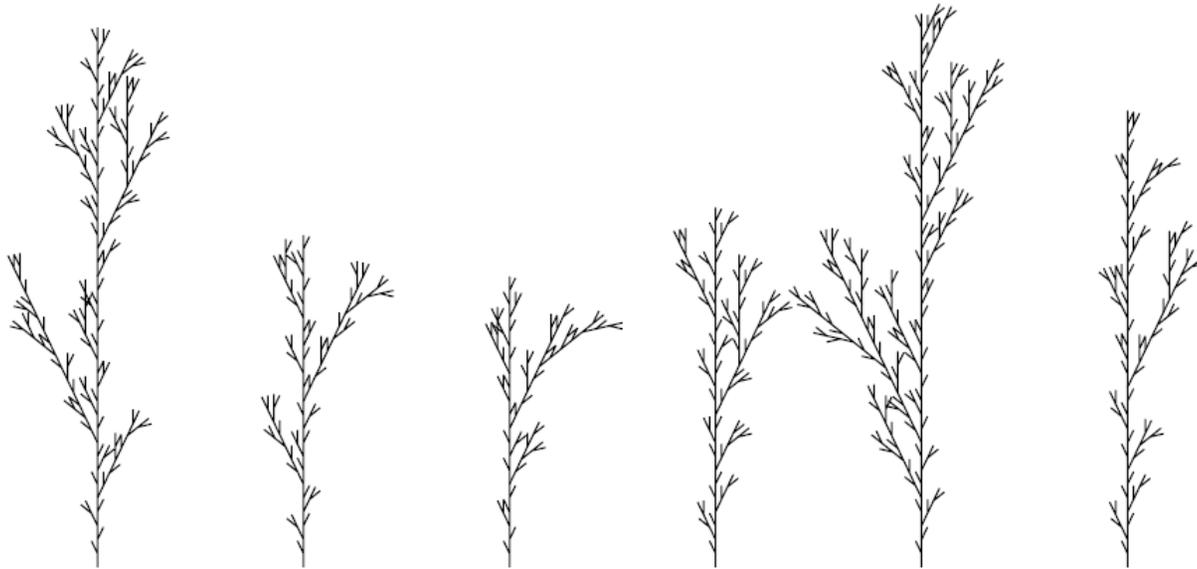
# Using a Stack to Make Trees

Tree1                    [    push the turtle state onto the stack  
4                            ]    pop the turtle state from the stack  
22.5                        and I add leaves here  
F  
F:FF-[-F+F+F]+[+F-F-F]



# Stochastic L-Systems

<http://algorithmicbotany.org/lstudio/CPFGman.pdf>



seed: 2454 // different seeds for different trees

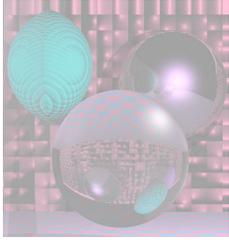
derivation length: 3

axiom: F

$F \rightarrow F[+F]F[-F]F : 1/3$

$F \rightarrow F[+F]F : 1/3$

$F \rightarrow F[-F]F : 1/3$



# 3D Turtle Rotations

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Heading, Left, or, Up vector tell turtle direction.

$+(\theta)$  Turn left by angle  $\theta^\circ$  around the  $U$  axis.

$-(\theta)$  Turn right by angle  $\theta^\circ$  around the  $U$  axis.

$\&(\theta)$  Pitch down by angle  $\theta^\circ$  around the  $L$  axis.

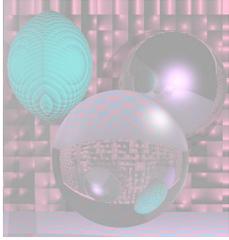
$\wedge(\theta)$  Pitch up by angle  $\theta^\circ$  around the  $L$  axis.

$\backslash(\theta)$  Roll left by angle  $\theta^\circ$  around the  $H$  axis.

$/(\theta)$  Roll right by angle  $\theta^\circ$  around the  $H$  axis.

$|$  Turn around  $180^\circ$  around the  $U$  axis.

$@v$  Roll the turtle around the  $H$  axis so that  $H$  and  $U$  lie in a common vertical plane with  $U$  closest to up.



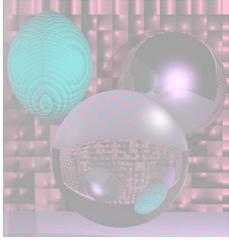
# A Mint

<http://algorithmicbotany.org/papers/>

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A model of a member of the mint family that exhibits a basipetal flowering sequence.

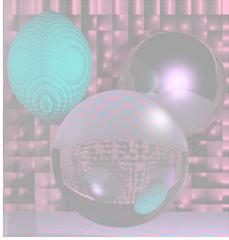


# Flow

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*“Flow means we need a certain amount of time to load our knowledge and skills into our brain RAM. And the more big or small interruptions we have, the less likely we are to ever get there.”*

*-- Kathy Sierra*



## Slow Down, Brave Multitasker, and Don't Read This in Traffic

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- Check e-mail messages once an hour, at most.
- In a recent study, a group of [Microsoft](#) workers took, on average, 15 minutes to return to serious mental tasks, like writing reports or computer code, after responding to incoming e-mail or instant messages.