

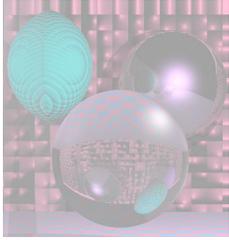
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# CS4310

# Graduate Computer Graphics

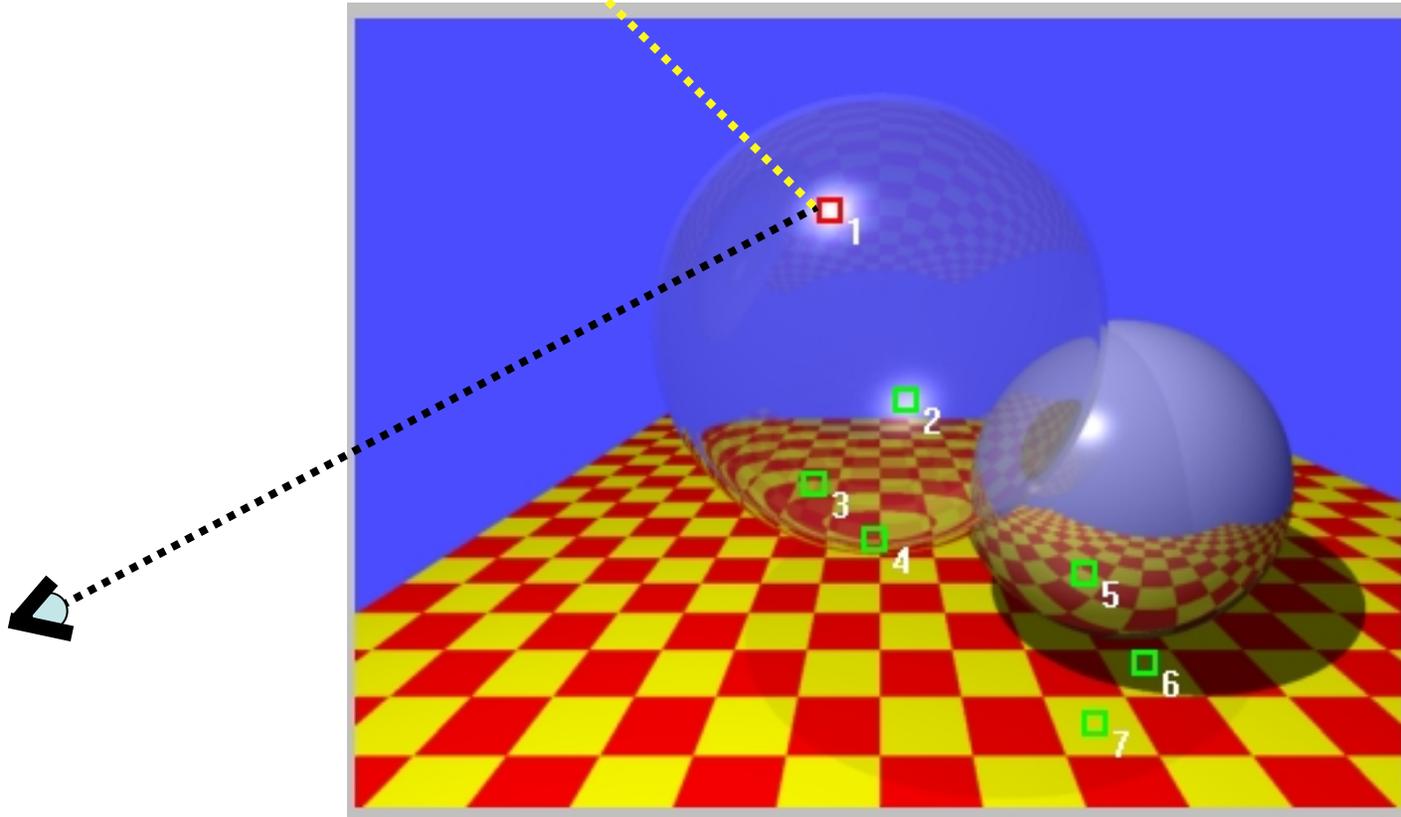
Prof. Harriet Fell  
Fall 2012

Lecture 27 – November 5, 2012

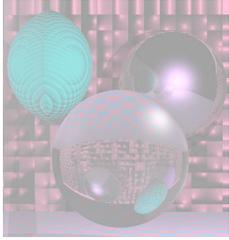


# Recursive Ray Tracing

Adventures of the 7 Rays - Watt

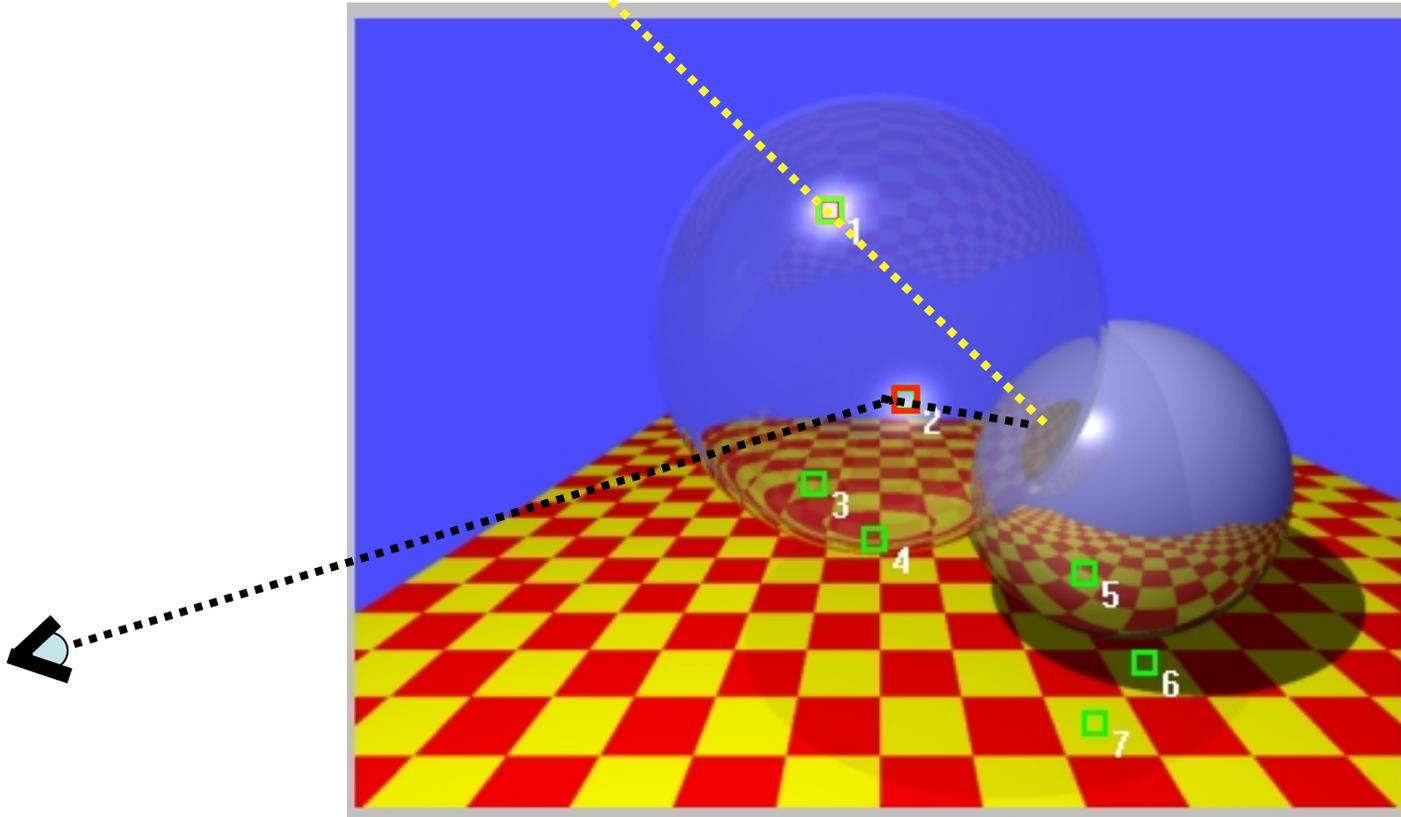


Specular Highlight on Outside of Sphere

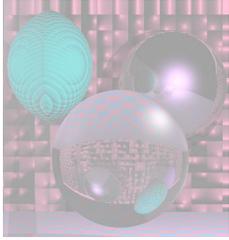


# Recursive Ray Tracing

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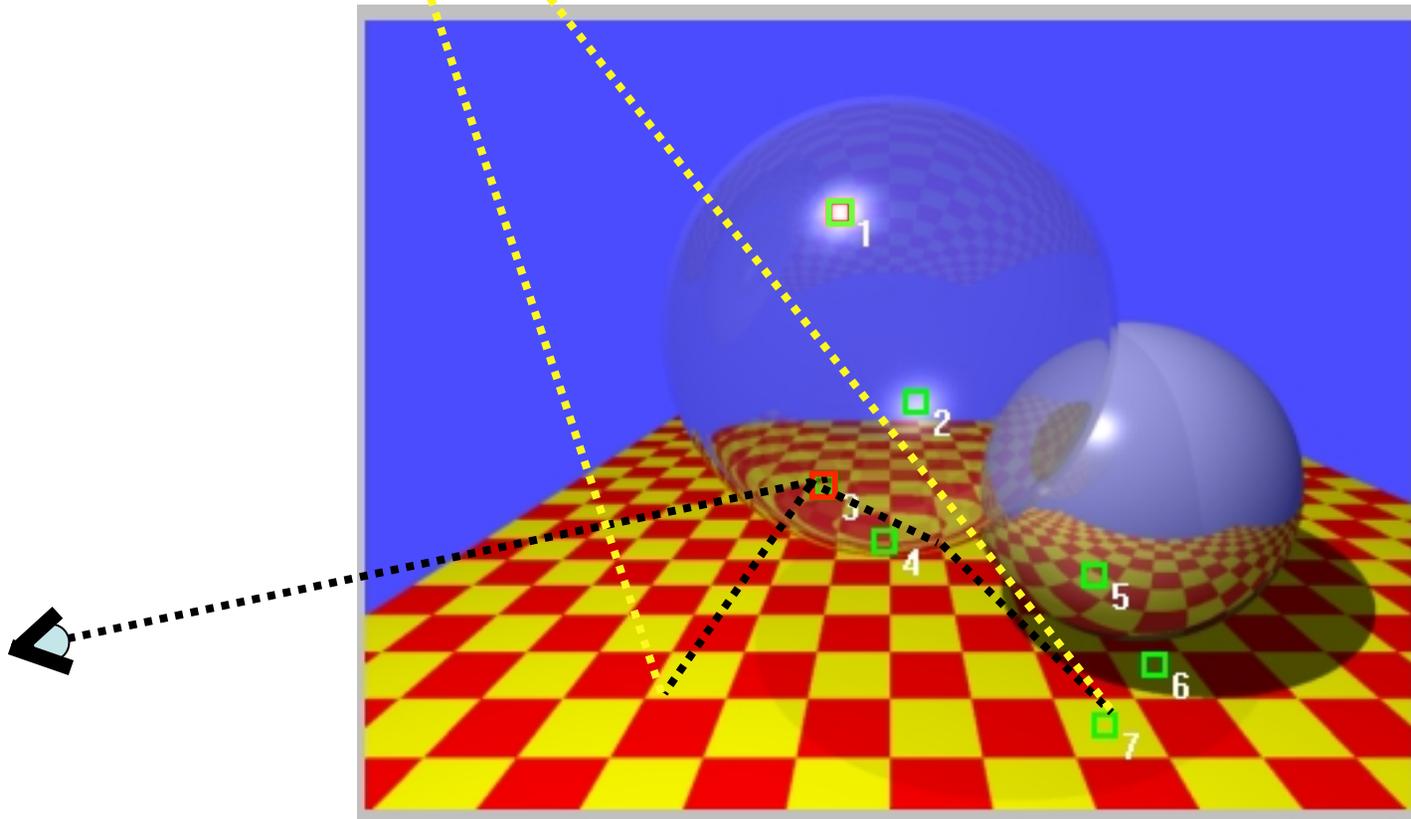


## Specular Highlight on Inside of Sphere

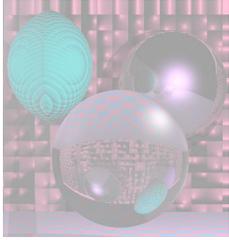


# Recursive Ray Tracing

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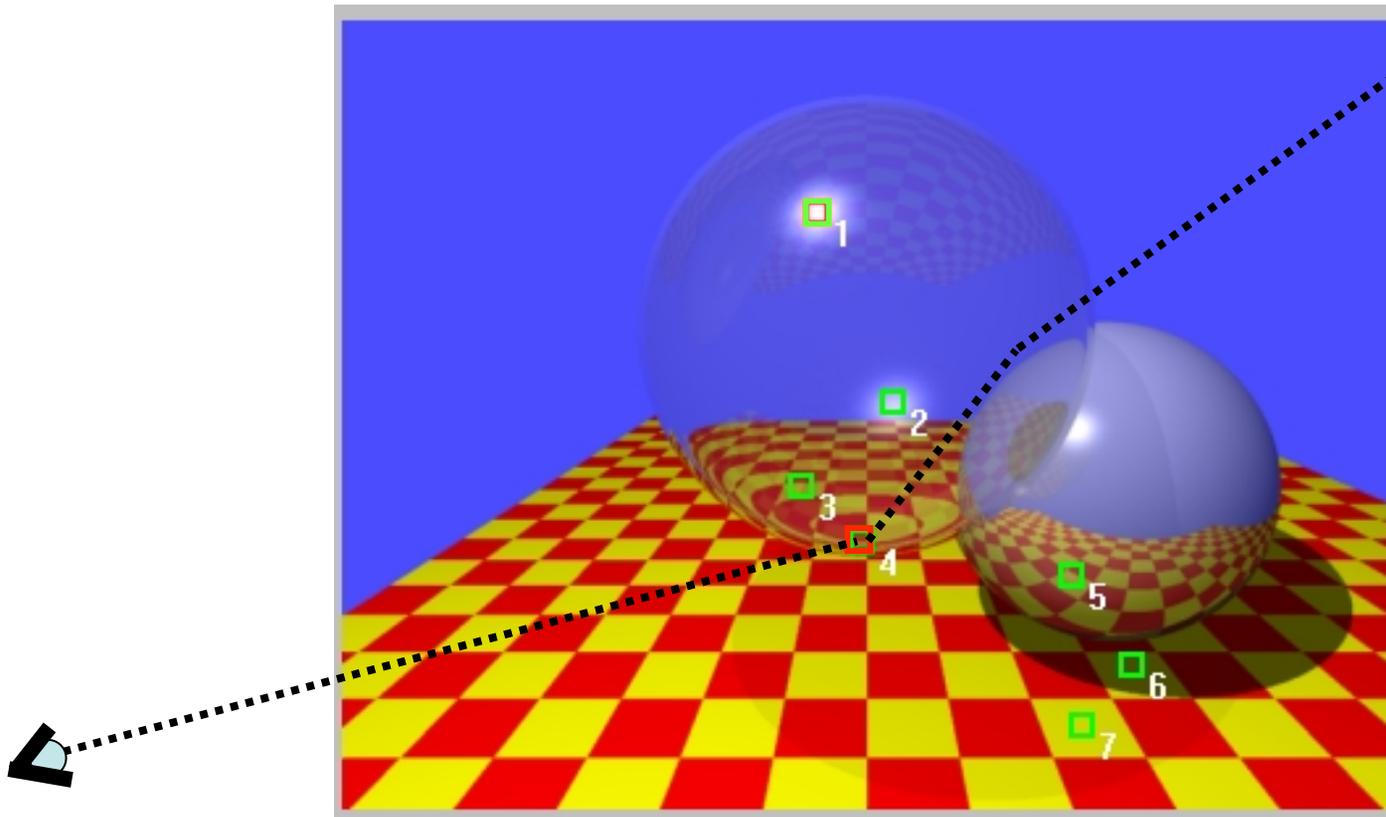


## Reflection and Refraction of Checkerboard

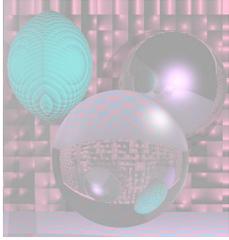


# Recursive Ray Tracing

Adventures of the 7 Rays - Watt

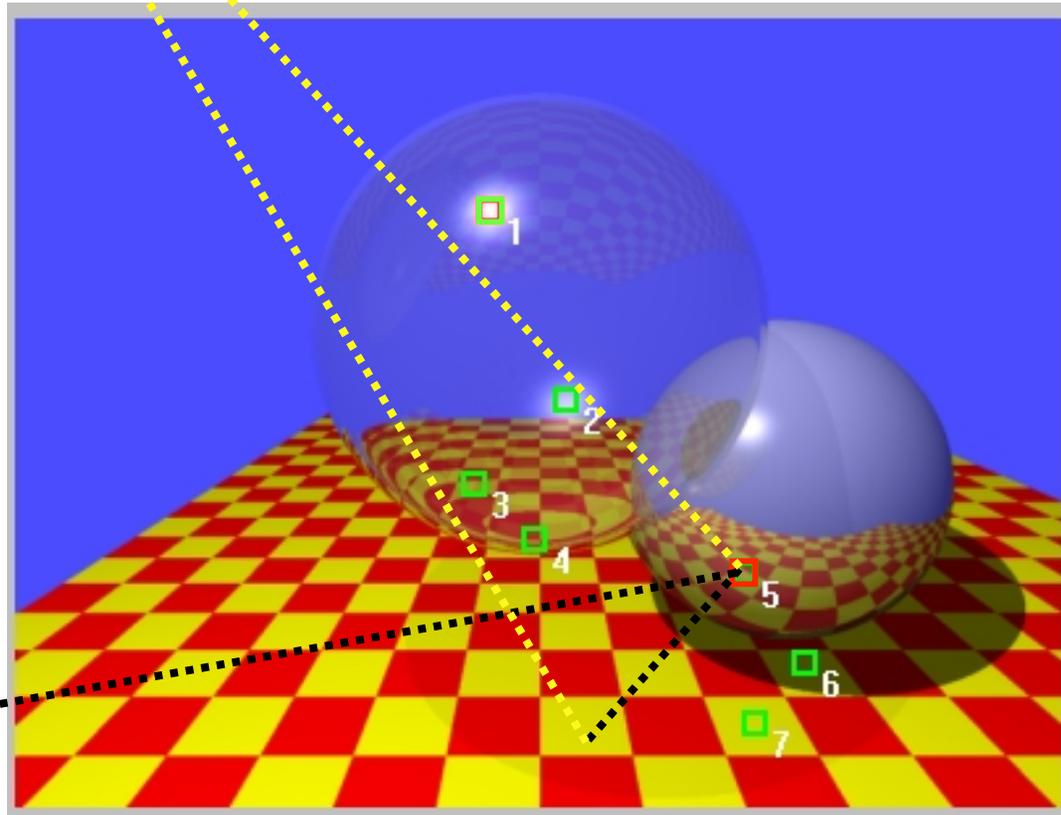


## Refraction Hitting Background

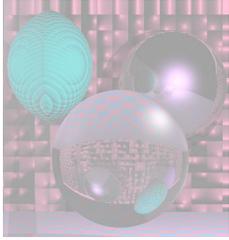


# Recursive Ray Tracing

Adventures of the 7 Rays - Watt

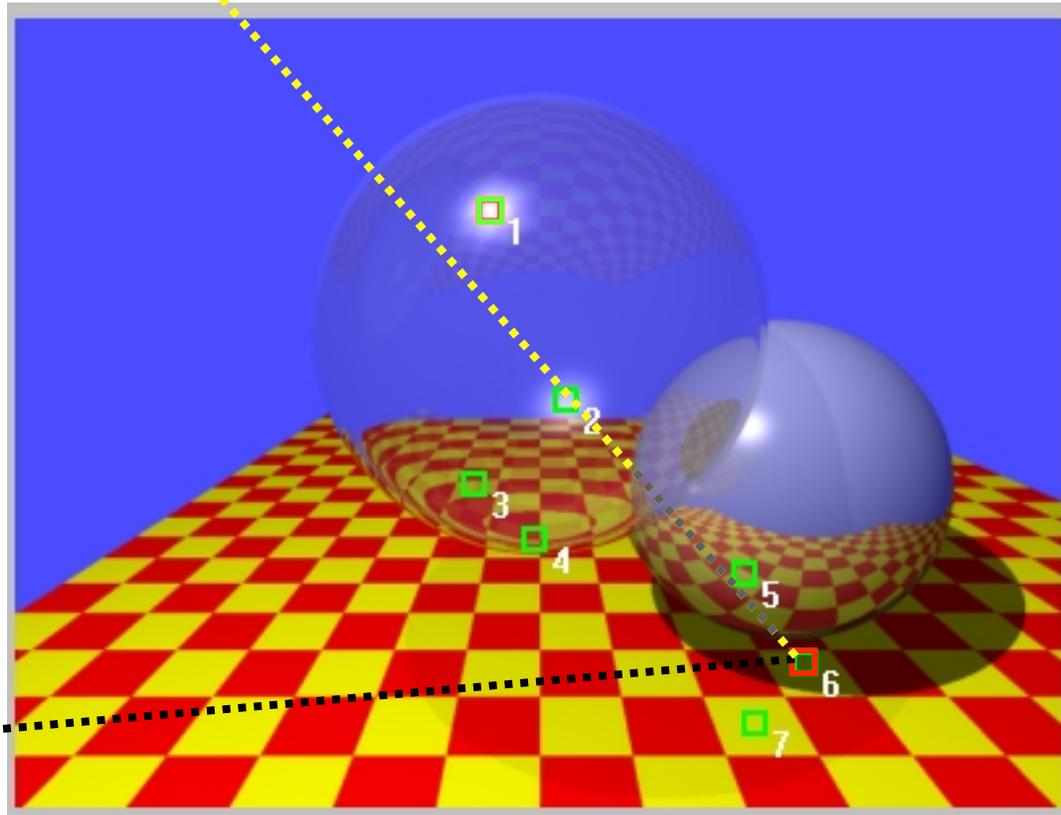


## Local Diffuse Plus Reflection from Checkerboard

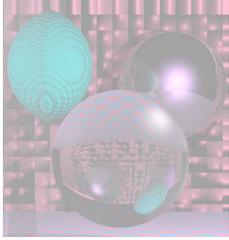


# Recursive Ray Tracing

Adventures of the 7 Rays - Watt

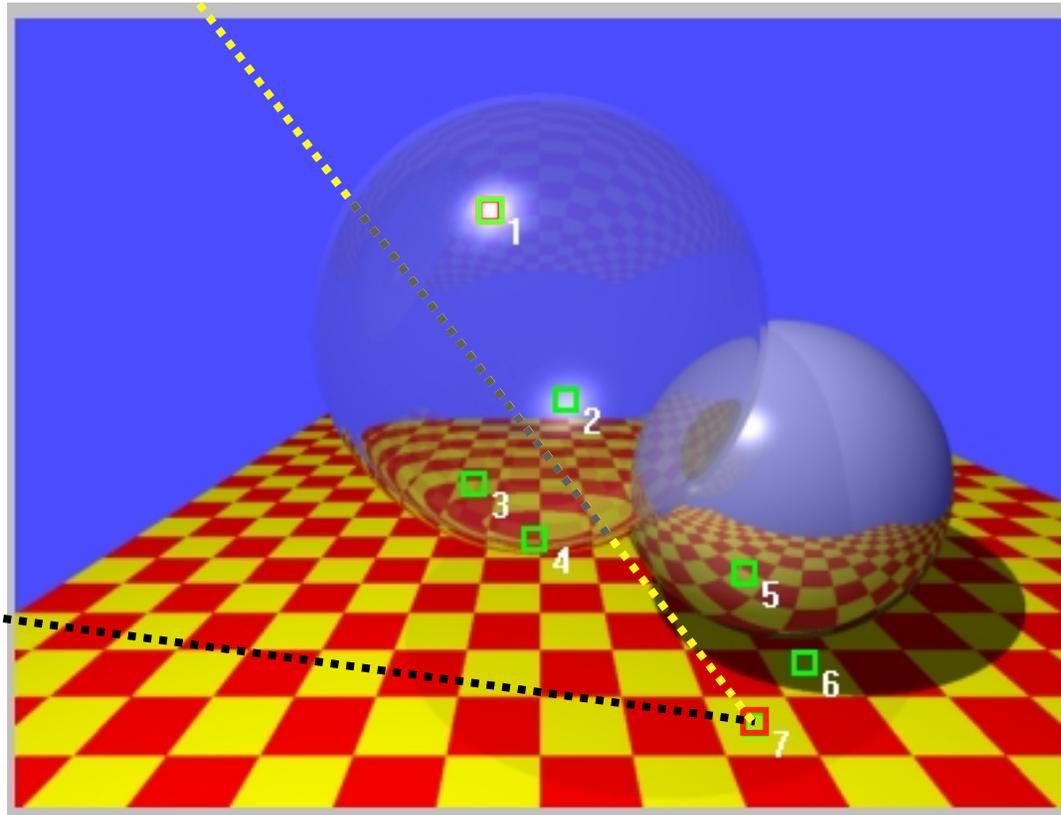


Local Diffuse in Complete Shadow

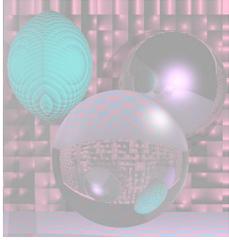


# Recursive Ray Tracing

Adventures of the 7 Rays - Watt



Local Diffuse in Shadow from Transparent Sphere



# Recursive Ray-Tracing

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- How do we know which rays to follow?
- How do we compute those rays?
- How do we organize code so we can follow all those different rays?

```
select center of projection(cp) and window on view plane;  
for (each scan line in the image) {  
  for (each pixel in scan line) {  
    determine ray from the cp through the pixel;  
    pixel = RT_trace(ray, 1);}}
```

```
// intersect ray with objects; compute shade at closest intersection  
// depth is current depth in ray tree
```

```
RT_color RT_trace (RT_ray ray; int depth){  
  determine closest intersection of ray with an object;  
  if (object hit) {  
    compute normal at intersection;  
    return RT_shade (closest object hit, ray, intersection, normal,  
                    depth);}  
  
  else  
    return BACKGROUND_VALUE;  
}
```

```

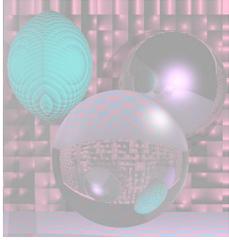
// Compute shade at point on object,
// tracing rays for shadows, reflection, refraction.
RT_color RT_shade (
    RT_object object, // Object intersected
    RT_ray ray,       // Incident ray
    RT_point point,   // Point of intersection to shade
    RT_normal normal, // Normal at point
    int depth )       // Depth in ray tree
{
    RT_color color; // Color of ray
    RT_ray rRay, tRay, sRay; // Reflected, refracted, and shadow ray
    color = ambient term ;
    for ( each light ) {
        sRay = ray from point to light ;
        if ( dot product of normal and direction to light is positive ){
            compute how much light is blocked by opaque and
            transparent surfaces, and use to scale diffuse and specular
            terms before adding them to color;}}

```

```

if ( depth < maxDepth ) { // return if depth is too deep
    if ( object is reflective ) {
        rRay = ray in reflection direction from point;
        rColor = RT_trace(rRay, depth + 1);
        scale rColor by specular coefficient and add to color;
    }
    if ( object is transparent ) {
        tRay = ray in refraction direction from point;
        if ( total internal reflection does not occur ) {
            tColor = RT_trace(tRay, depth + 1);
            scale tColor by transmission coefficient
            and add to color;
        }
    }
}
return color; // Return the color of the ray
}

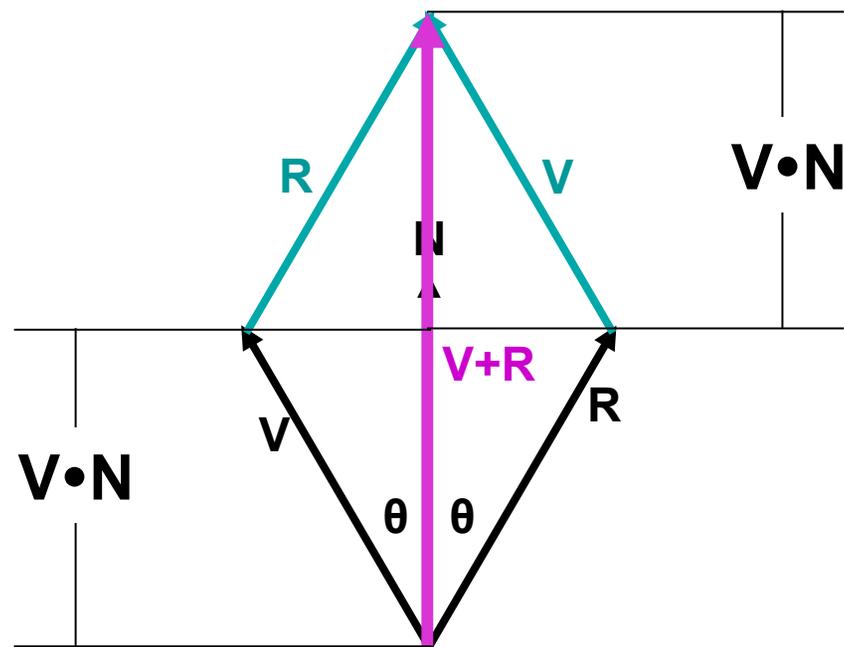
```

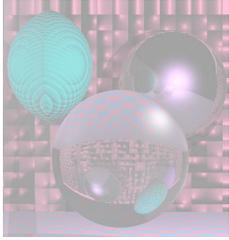


# Computing R

$$\mathbf{V} + \mathbf{R} = (2 \mathbf{V} \cdot \mathbf{N}) \mathbf{N}$$

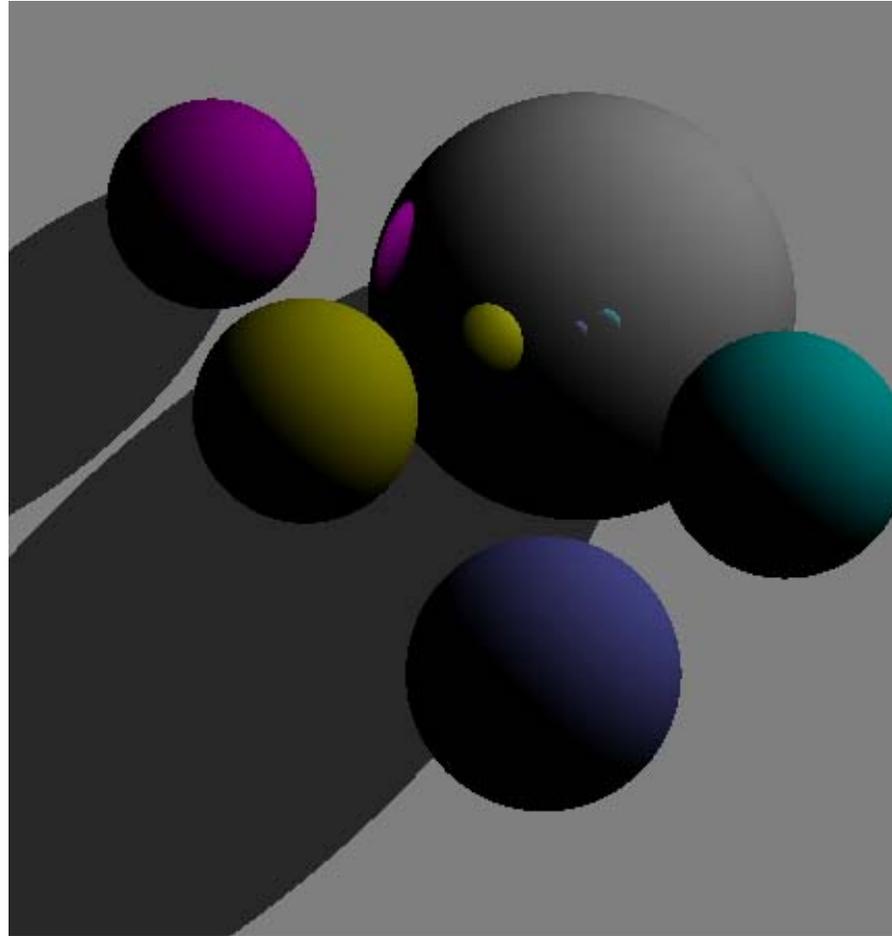
$$\mathbf{R} = (2 \mathbf{V} \cdot \mathbf{N}) \mathbf{N} - \mathbf{V}$$

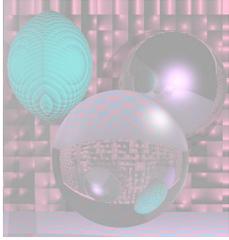




# Reflections, no Highlight

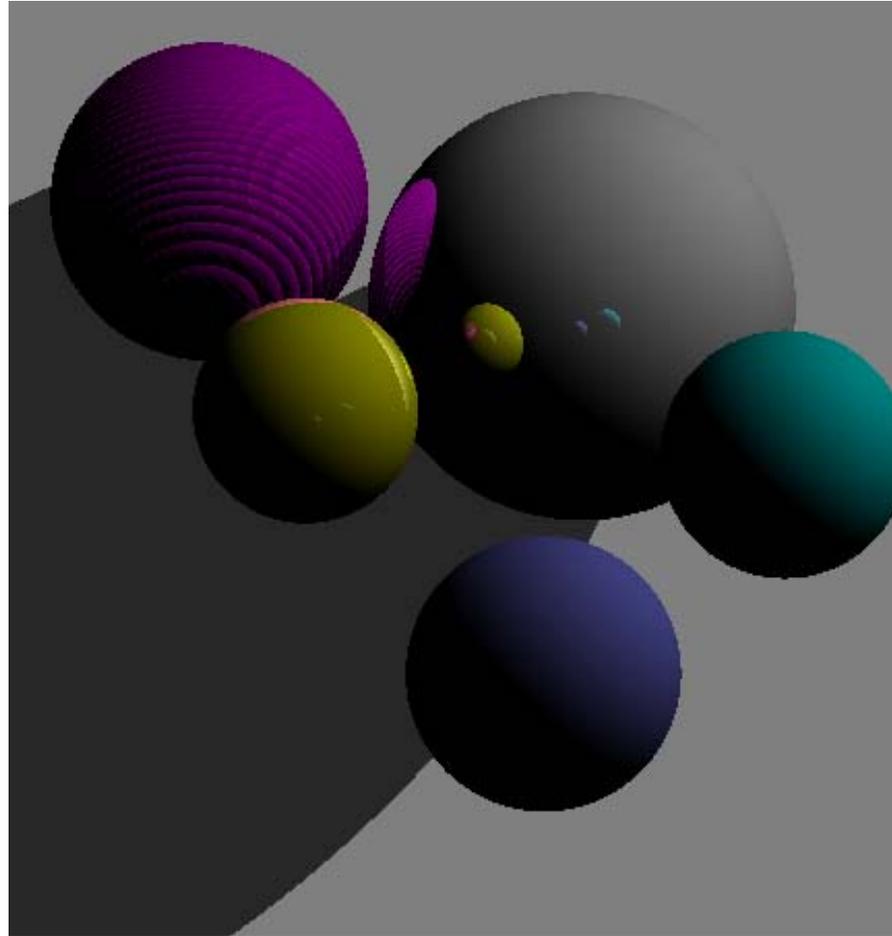
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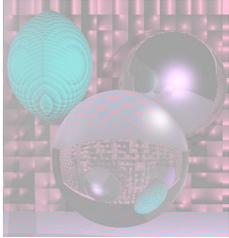




# Second Order Reflection

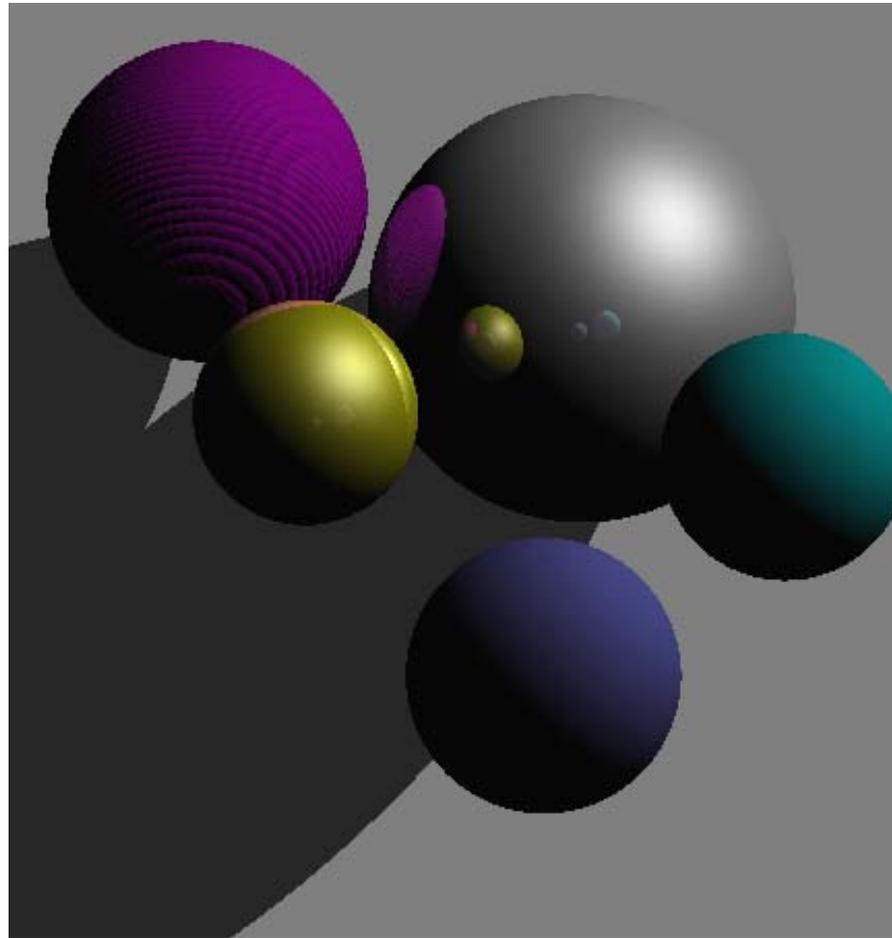
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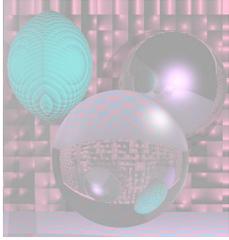




# Refelction with Highlight

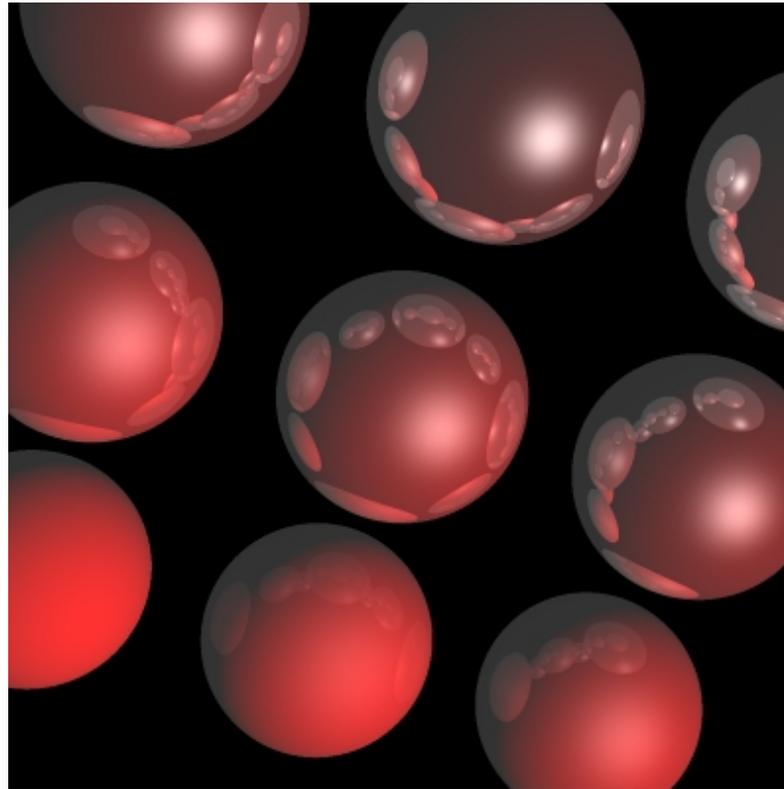
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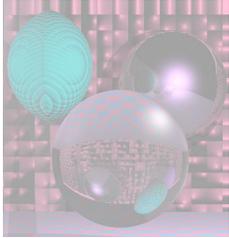




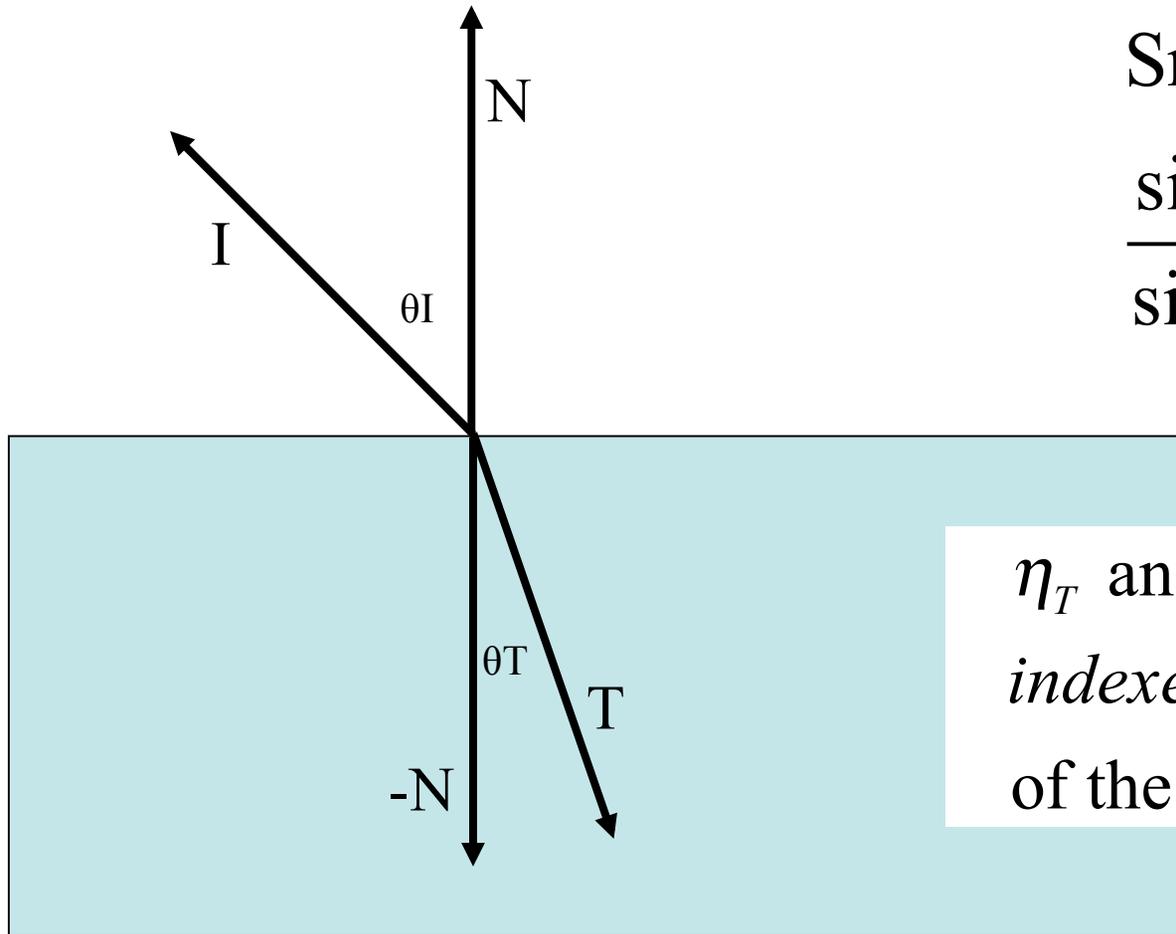
# Nine Red Balls

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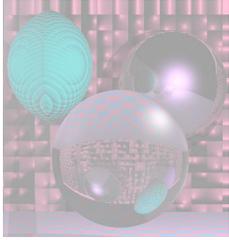
# Refraction



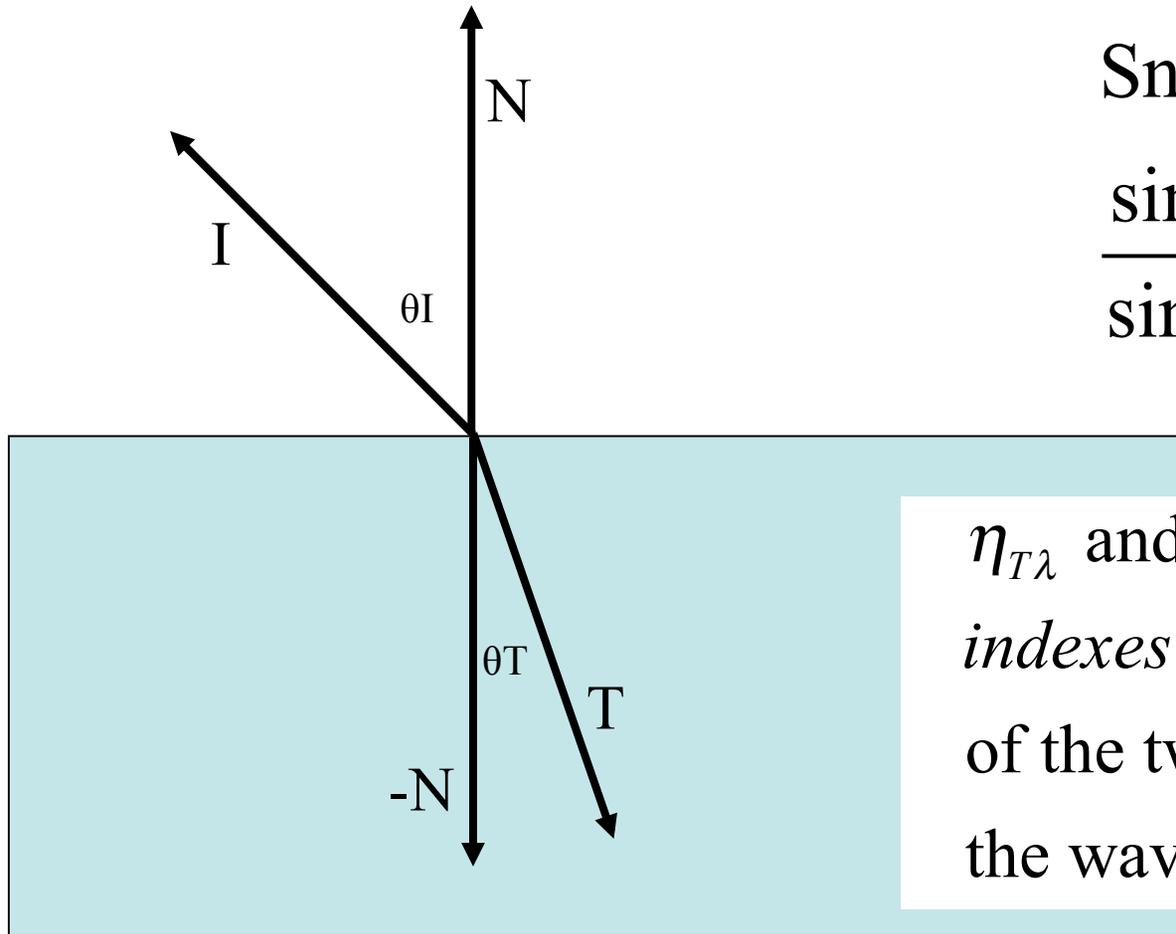
Snell's Law

$$\frac{\sin(\theta_I)}{\sin(\theta_T)} = \frac{\eta_T}{\eta_I}$$

$\eta_T$  and  $\eta_I$  are the  
*indexes of refraction*  
of the two mediums.



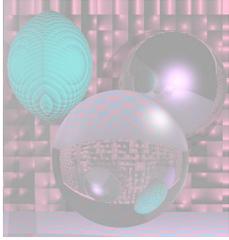
# Refraction and Wavelength



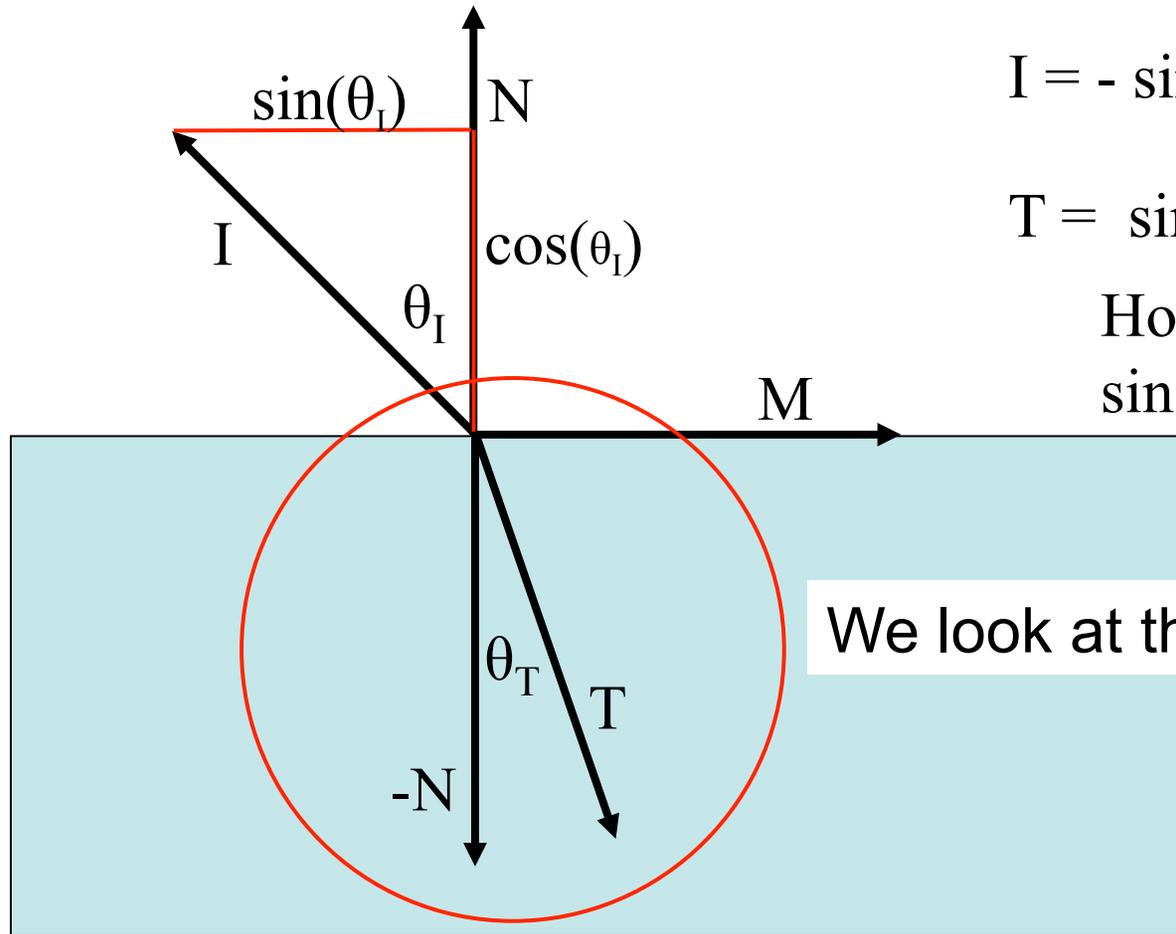
Snell's Law

$$\frac{\sin(\theta_I)}{\sin(\theta_T)} = \frac{\eta_{T\lambda}}{\eta_{I\lambda}}$$

$\eta_{T\lambda}$  and  $\eta_{I\lambda}$  are the *indexes of refraction* of the two mediums for the wavelength of light  $\lambda$ .



# Computing T

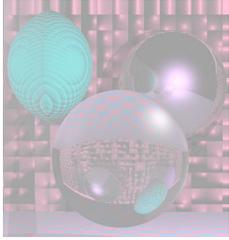


$$I = -\sin(\theta_I)M + \cos(\theta_I)N$$

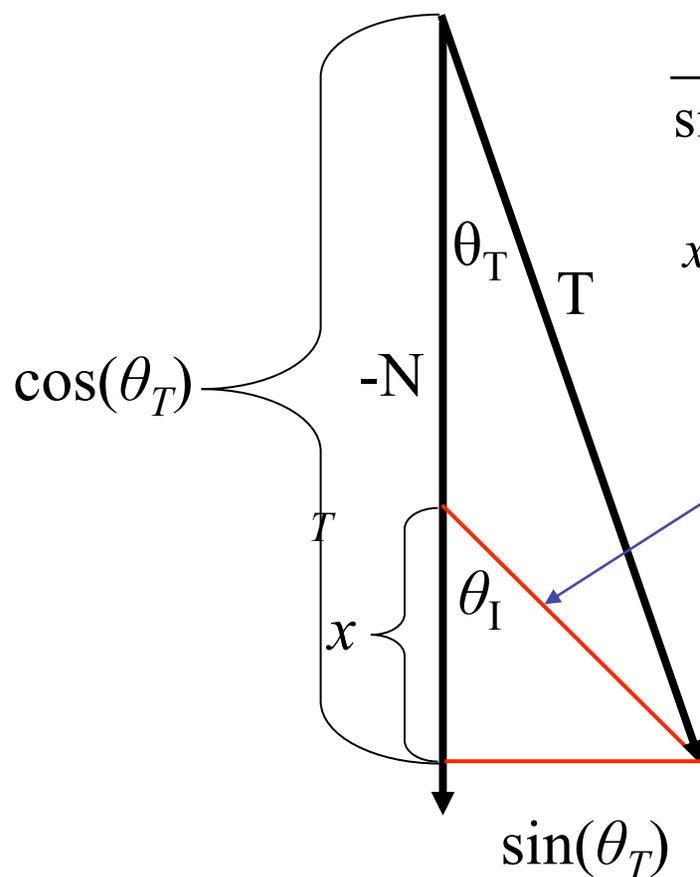
$$T = \sin(\theta_T)M - \cos(\theta_T)N$$

How do we compute  $M$ ,  $\sin(\theta_T)$ , and  $\cos(\theta_T)$ ?

We look at this in more detail



# Computing T



$$\frac{x}{\sin(\theta_T)} = \cot(\theta_I) = \frac{\cos(\theta_I)}{\sin(\theta_I)}$$

$$x = \cos(\theta_I) \frac{\sin(\theta_T)}{\sin(\theta_I)} = \frac{\eta_I}{\eta_T} \cos(\theta_I) \quad \text{by Snell's law.}$$

$$= \frac{x}{\cos(\theta_I)} = \frac{\eta_I}{\eta_T}$$

# Computing T

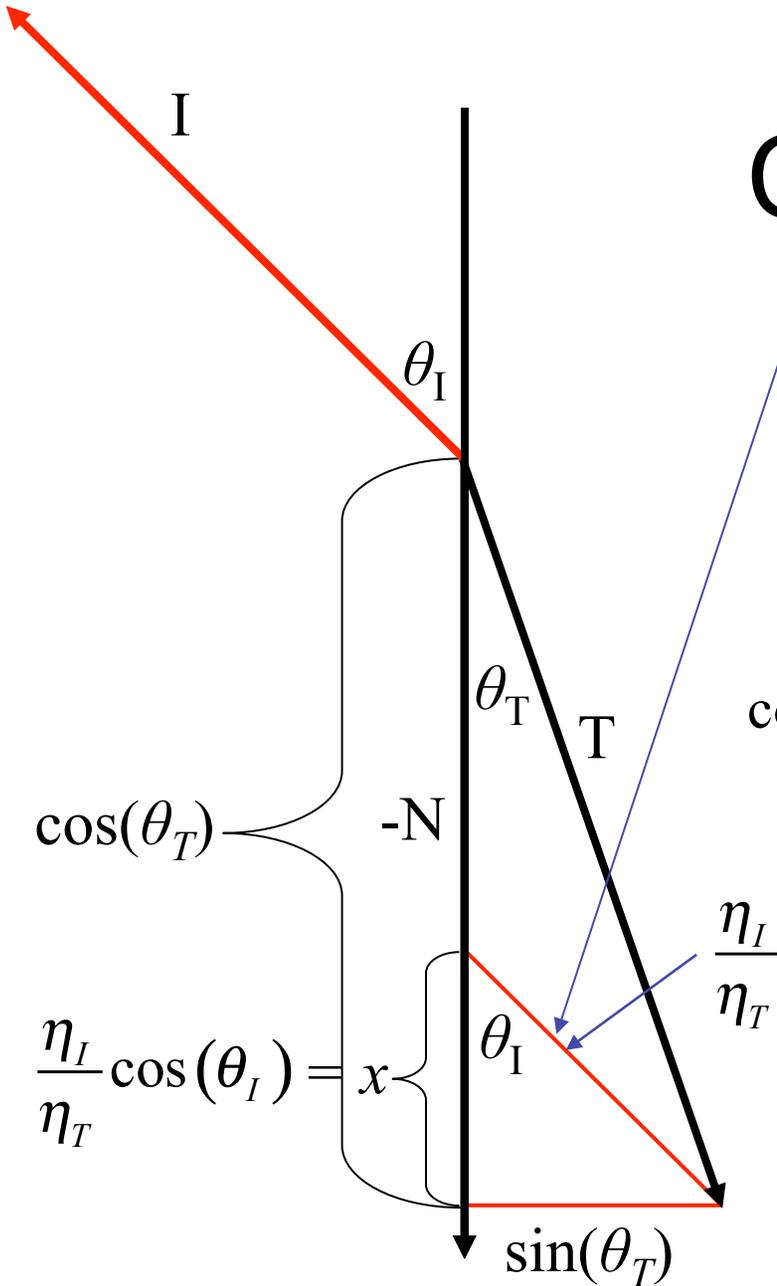
Parallel to I

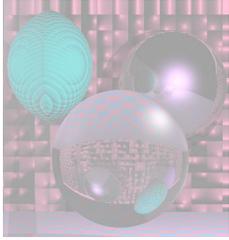
$$T = - \left( \cos(\theta_T) - \frac{\eta_I}{\eta_T} \cos(\theta_I) \right) N - \frac{\eta_I}{\eta_T} I$$

$$\cos(\theta_T) = \sqrt{1 - \sin^2(\theta_T)} = \sqrt{1 - \left( \frac{\eta_I}{\eta_T} \right)^2 \sin^2(\theta_I)}$$

$$= \sqrt{1 - \left( \frac{\eta_I}{\eta_T} \right)^2 (1 - \cos^2(\theta_I))}$$

$$= \sqrt{1 - \left( \frac{\eta_I}{\eta_T} \right)^2 (1 - (N \cdot I)^2)}$$





# Total Internal Reflection

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$$\cos(\theta_T) = \sqrt{1 - \left(\frac{\eta_I}{\eta_T}\right)^2 (1 - (N \cdot I)^2)}$$

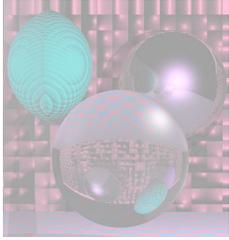
When is  $\cos(\theta_T)$  defined?

$$\text{When } 1 - \left(\frac{\eta_I}{\eta_T}\right)^2 (1 - (N \cdot I)^2) \geq 0.$$

If  $\eta_I > \eta_T$  and  $N \cdot I$  is close to 0,  $\cos(\theta_T)$  may not be defined.

Then there is no transmitting ray and we have

*total internal reflection.*



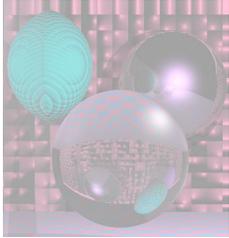
# Index of Refraction

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The speed of all electromagnetic radiation in vacuum is the same, approximately  $3 \times 10^8$  meters per second, and is denoted by  $c$ . Therefore, if  $v$  is the phase velocity of radiation of a specific frequency in a specific material, the refractive index is given by

$$\eta = \frac{c}{v}$$

[http://en.wikipedia.org/wiki/Refractive\\_index](http://en.wikipedia.org/wiki/Refractive_index)

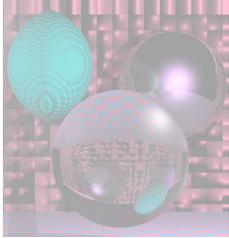


# Indices of Refraction

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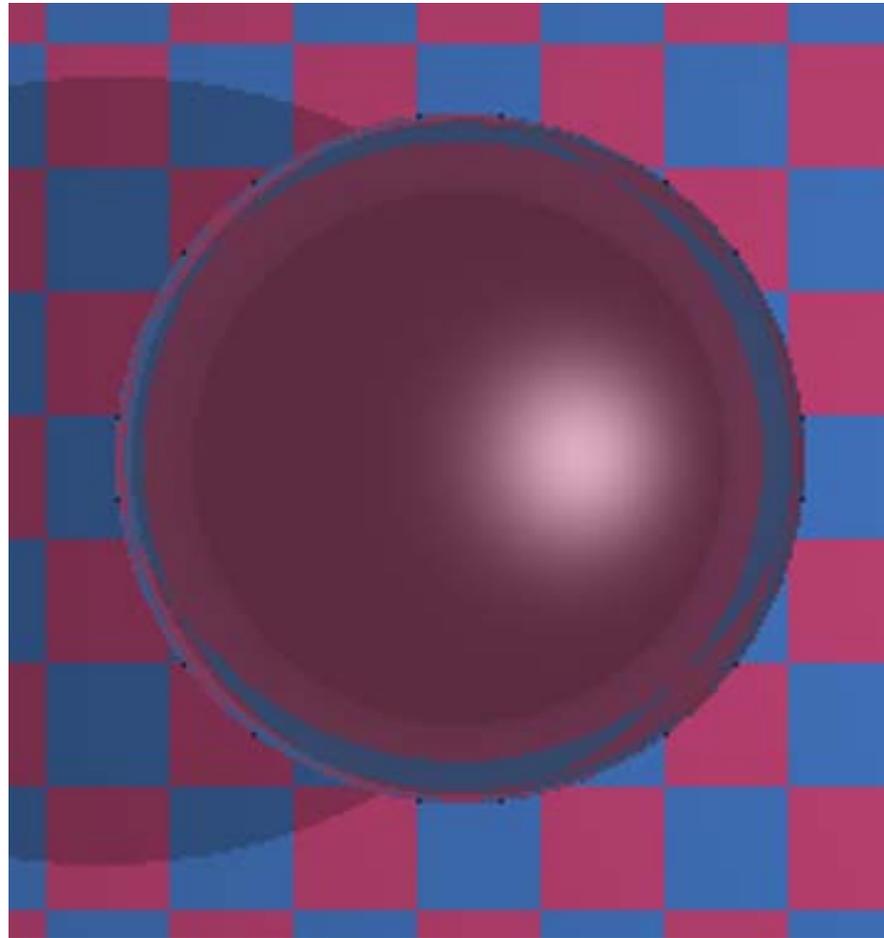
<b>Material</b>	<b><math>\eta</math> at <math>\lambda=589.3</math> nm</b>
vacuum	1 (exactly)
helium	1.000036
air at STP	1.0002926
water ice	1.31
liquid water (20°C)	1.333
ethanol	1.36
glycerine	1.4729
rock salt	1.516
glass (typical)	1.5 to 1.9
cubic zirconia	2.15 to 2.18
diamond	2.419

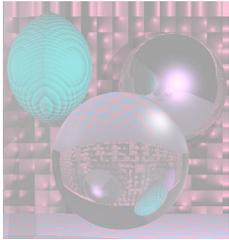
[http://en.wikipedia.org/wiki/List\\_of\\_indices\\_of\\_refraction](http://en.wikipedia.org/wiki/List_of_indices_of_refraction)



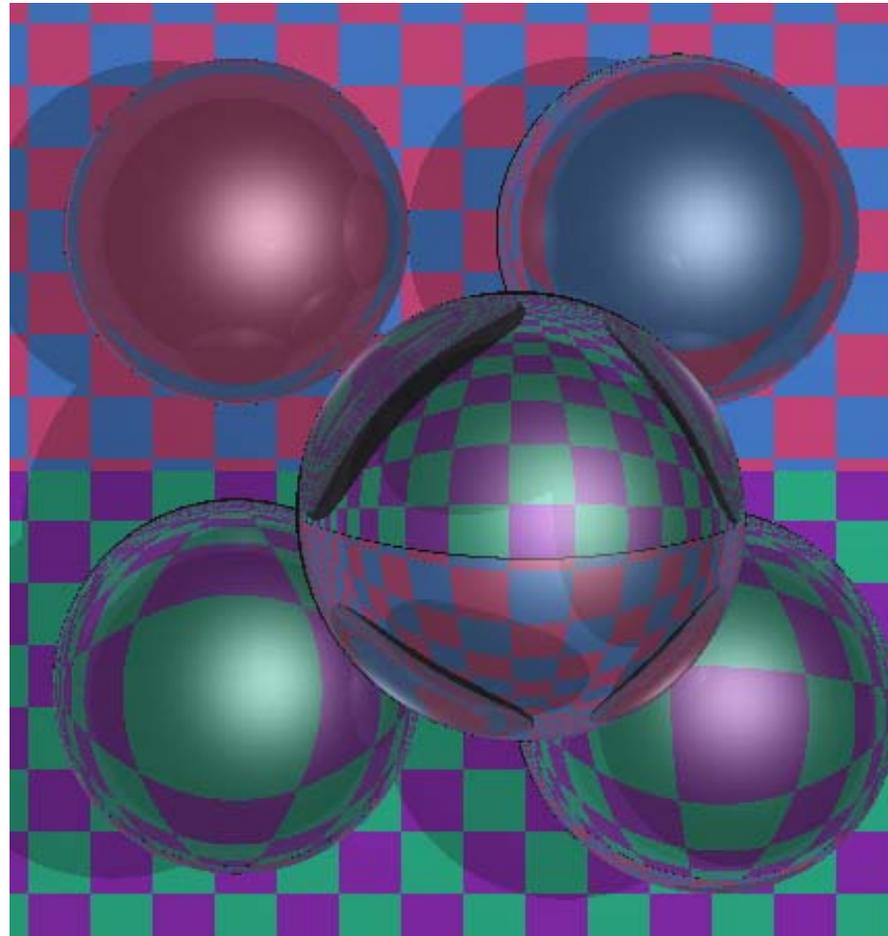
# One Glass Sphere

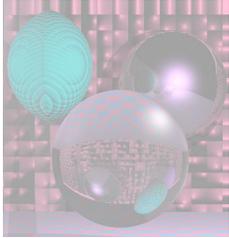
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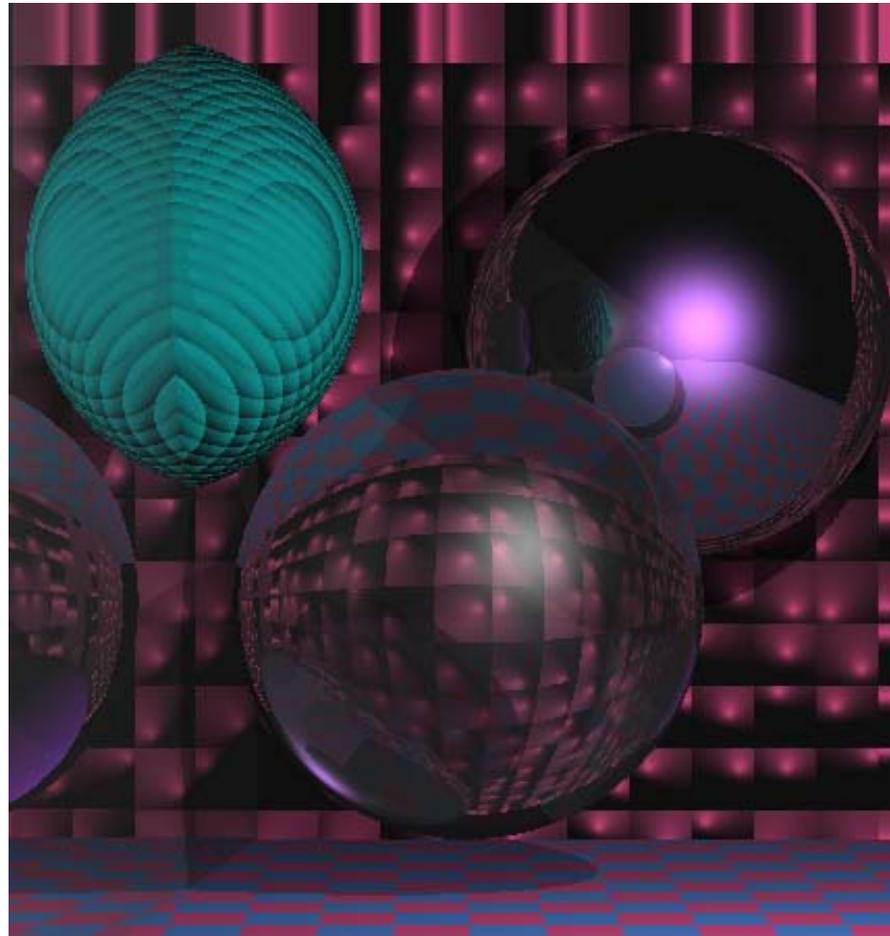
# Five Glass Balls

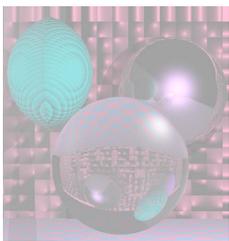




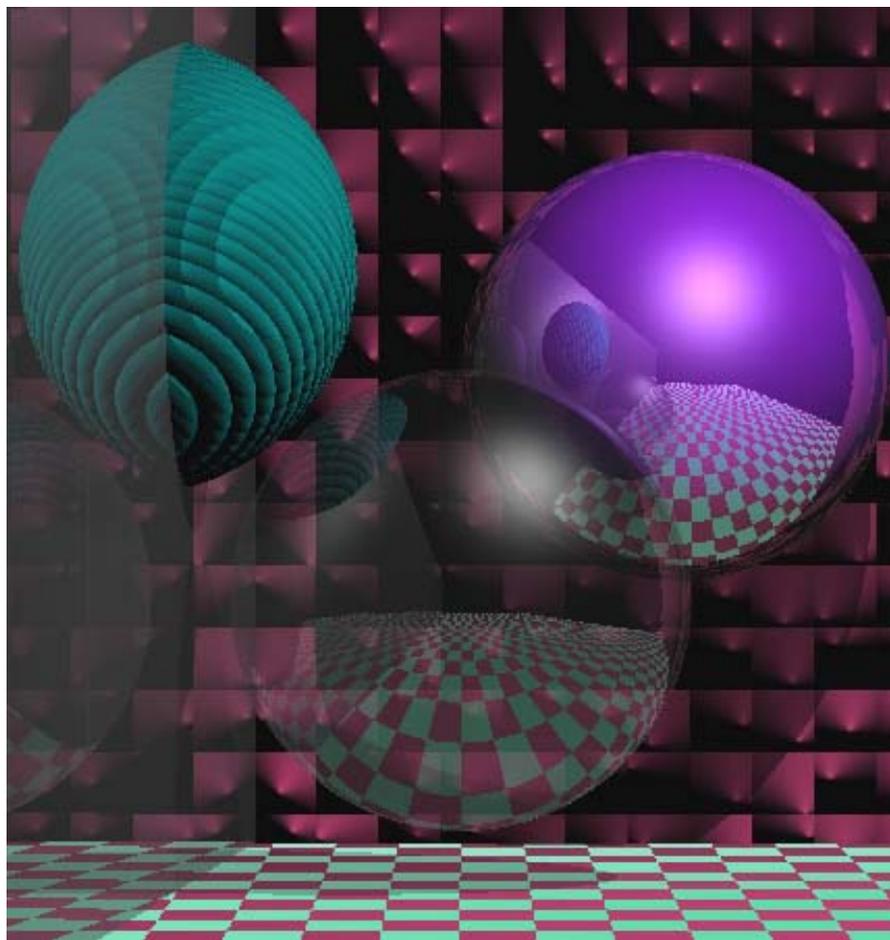
# A Familiar Scene

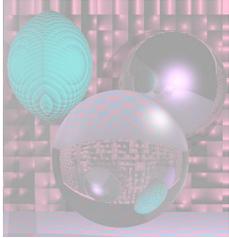
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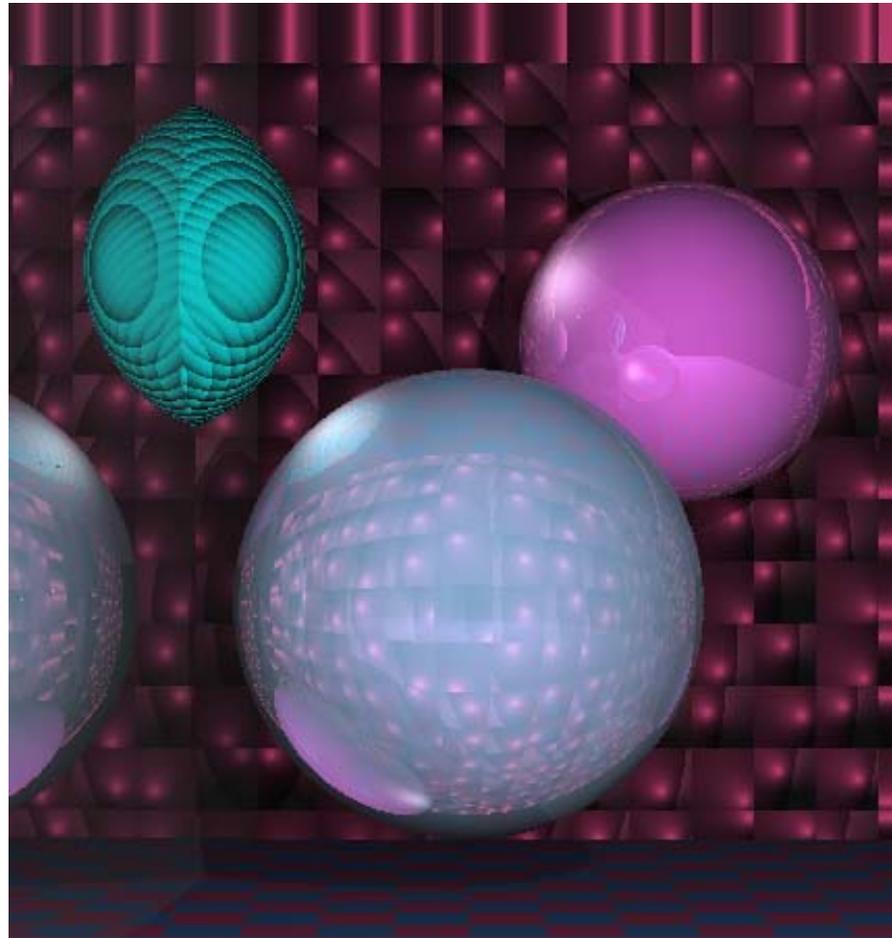


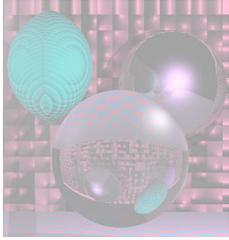
# Bubble





# Milky Sphere





# Lens - Carl Andrews 1999

