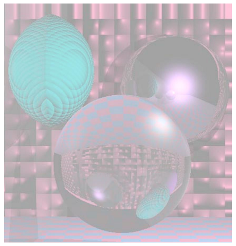


# CS 4300
# Computer Graphics

Prof. Harriet Fell

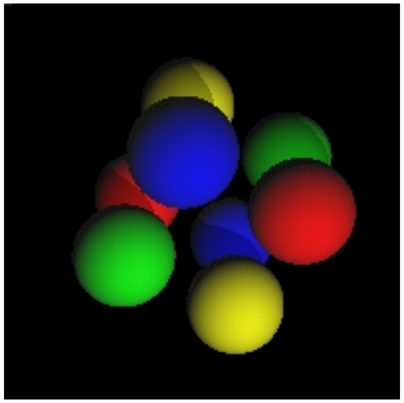
Fall 2012

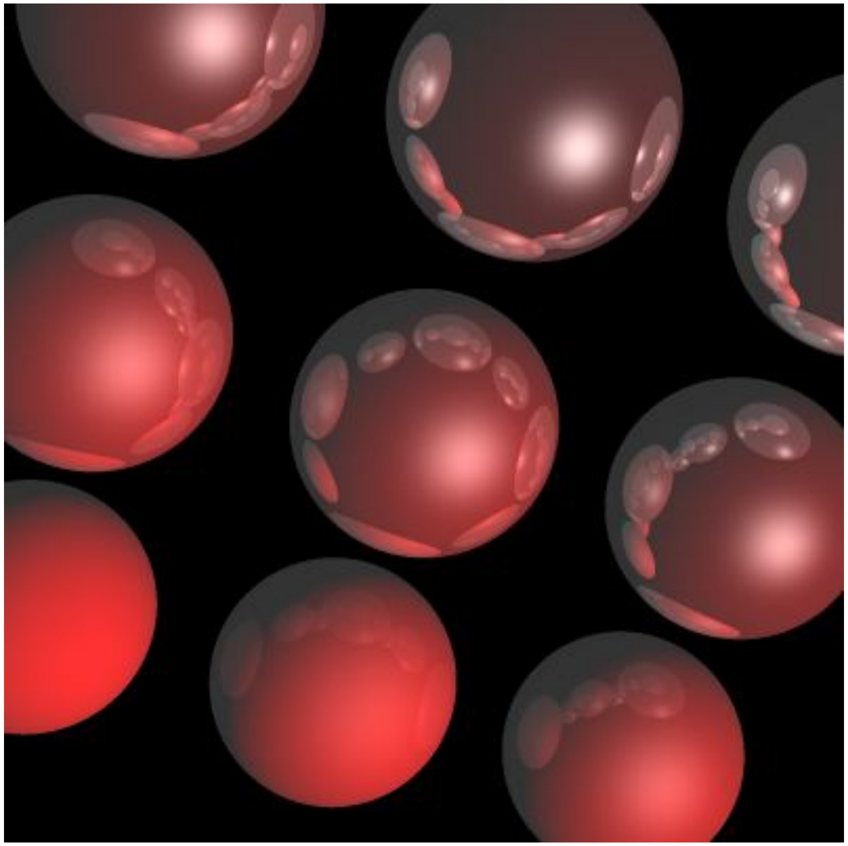
Lecture 24 – October 31, 2012

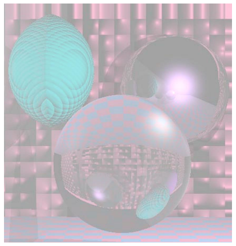
# Today's Topics

- Ray Casting

# Ray Tracing
# a World of Spheres



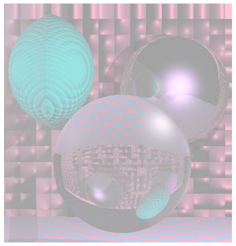




# What is a Sphere

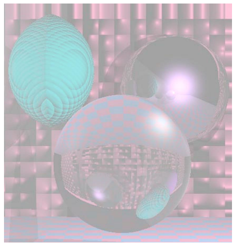
```
Vector3D   center;    // 3 doubles
double     radius;
double     R, G, B;   // for RGB colors between 0 and 1
double     kd;        // diffuse coeficient
double     ks;        // specular coeficient
(double    ka;        // ambient light coefficient)
```

```
-.01  .01   500 800 // transform theta phi mu distance
1 // antialias
1 // numlights
100 500 800 // Lx, Ly, Lz
9 // numspheres
//cx   cy cz radius R  G  B ka kd ks specExp kgr kt pic
-100 -100  0 40    .9  0  0 .2 .9 .0    4      0   0  0
-100    0  0 40    .9  0  0 .2 .8 .1    8     .1   0  0
-100  100  0 40    .9  0  0 .2 .7 .2   12     .2   0  0
   0 -100  0 40    .9  0  0 .2 .6 .3   16     .3   0  0
   0    0  0 40    .9  0  0 .2 .5 .4   20     .4   0  0
   0  100  0 40    .9  0  0 .2 .4 .5   24     .5   0  0
 100 -100  0 40    .9  0  0 .2 .3 .6   28     .6   0  0
 100    0  0 40    .9  0  0 .2 .2 .7   32     .7   0  0
 100  100  0 40    .9  0  0 .2 .1 .8   36     .8   0  0
```



# World of Spheres

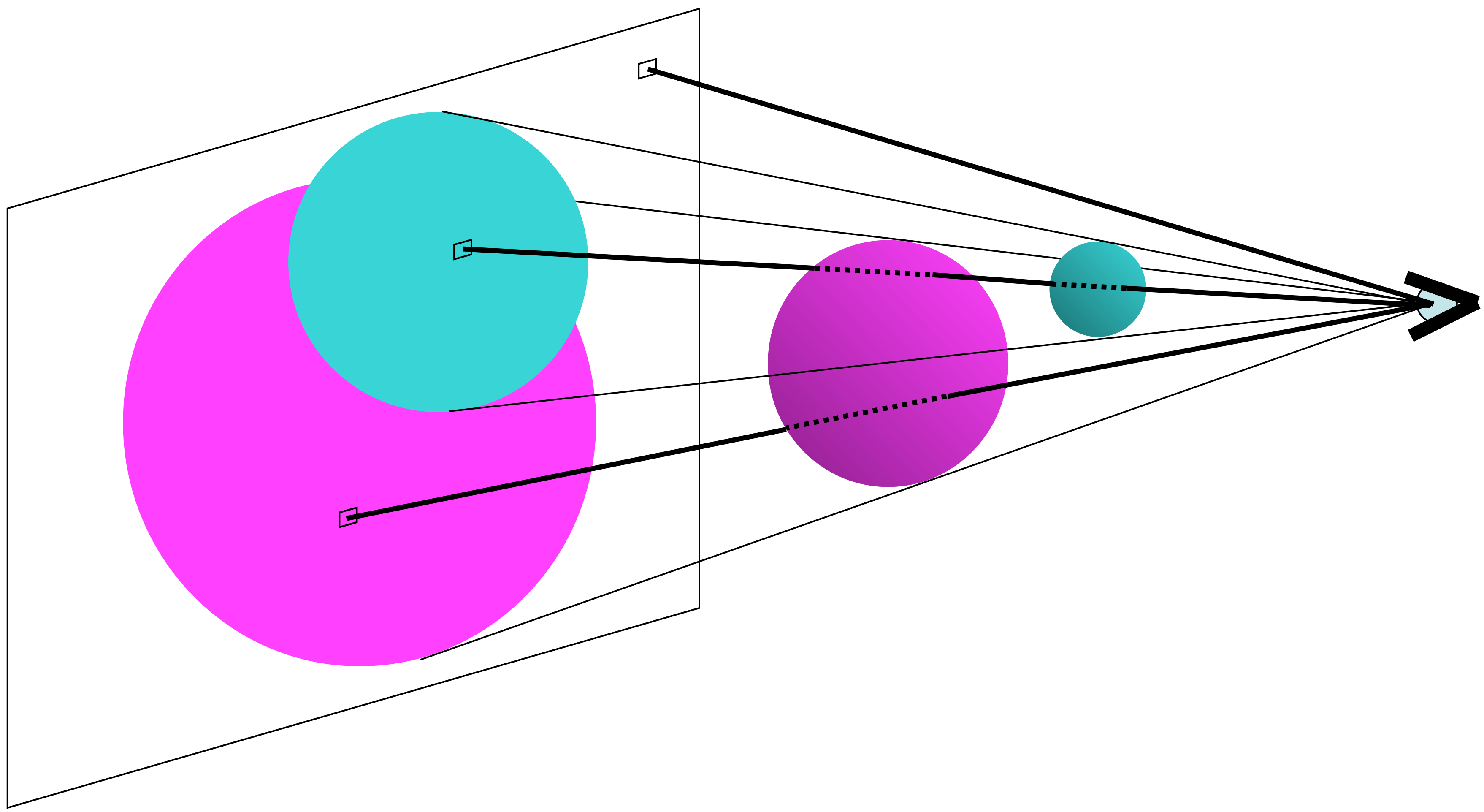
```
Vector3D VP;                 // the viewpoint
int numLights;
Vector3D theLights[5];       // up to 5 white lights
double ka;                   // ambient light coefficient
int numSpheres;
Sphere theSpheres[20];       // 20 sphere max

int ppmT[3];                 // ppm texture files
View sceneView;              // transform data
double distance;             // view plane to VP
bool antialias;              // if true antialias
```

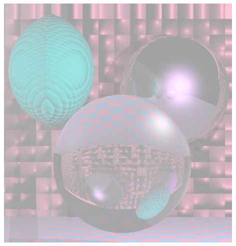


# Simple Ray Tracing for Detecting Visible Surfaces

```
select window on viewplane and center of projection
for (each scanline in image) {
   for (each pixel in the scanline) {
         determine ray from center of projection
                  through pixel;
         for (each object in scene) {
                  if (object is intersected and
                      is closest considered thus far)
                           record intersection and object name;
         }
         set pixel's color to that of closest object intersected;
   }
}
```

# Ray Trace 1
# Finding Visible Surfaces

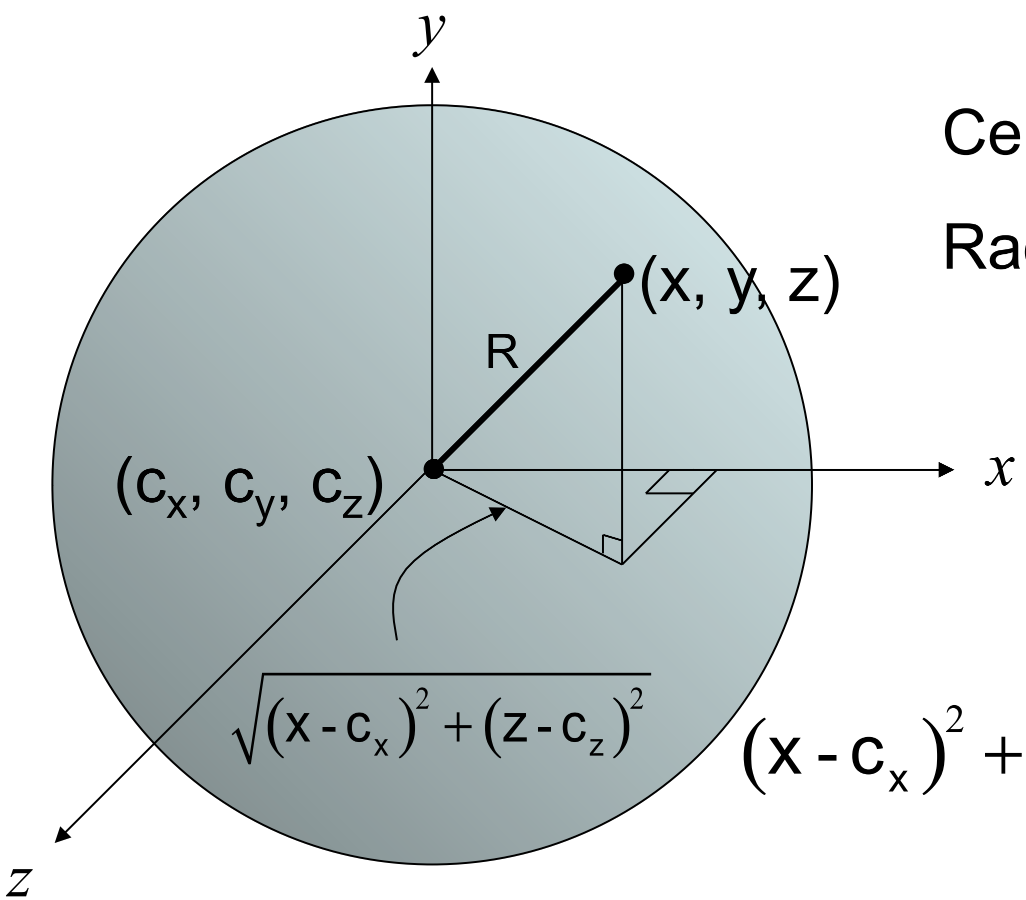




# Ray-Sphere Intersection

- Given
  - Sphere
    - Center $(c_x, c_y, c_z)$
    - Radius, $R$
  - Ray from $P_0$ to $P_1$
    - $P_0 = (x_0, y_0, z_0)$ and $P_1 = (x_1, y_1, z_1)$
  - View Point
    - $(V_x, V_y, V_z)$
- Project to window from $(0,0,0)$ to $(w,h,0)$

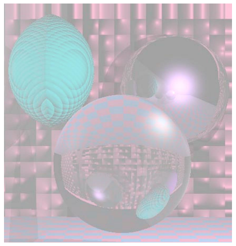
# Sphere Equation




Center $C = (c_x, c_y, c_z)$

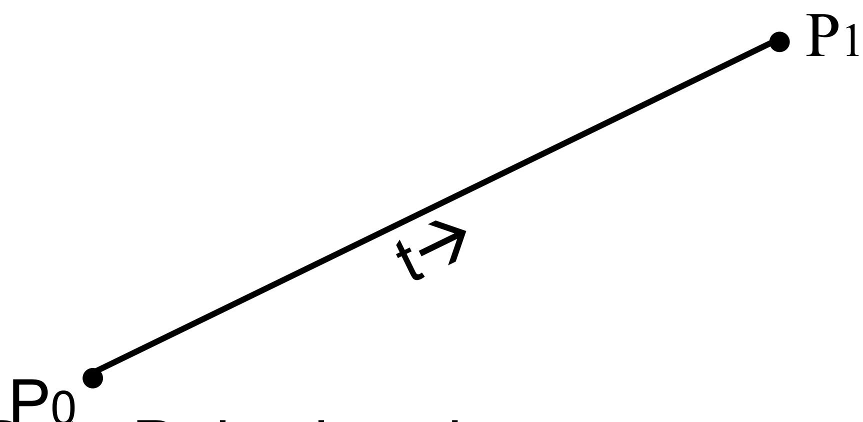
Radius R

$$(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = R^2$$



# Ray Equation

$P_0 = (x_0, y_0, z_0)$ and $P_1 = (x_1, y_1, z_1)$




The ray from $P_0$ to $P_1$ is given by:

$$P(t) = (1 - t)P_0 + tP_1 \qquad 0 <= t <= 1$$

$$= P_0 + t(P_1 - P_0)$$

# Intersection Equation

$P(t) = P_0 + t(P_1 - P_0)$ $\quad 0 <= t <= 1$

is really three equations

$x(t) = x_0 + t(x_1 - x_0)$

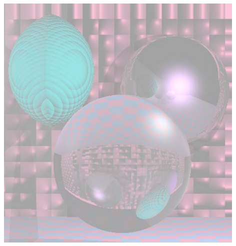
$y(t) = y_0 + t(y_1 - y_0)$

$z(t) = z_0 + t(z_1 - z_0)$ $\quad 0 <= t <= 1$

Substitute x(t), y(t), and z(t) for x, y, z, respectively in

$$(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = R^2$$

$$((x_0 + t(x_1 - x_0)) - c_x)^2 + ((y_0 + t(y_1 - y_0)_1) - c_y)^2 + ((z_0 + t(z_1 - z_0)) - c_z)^2 = R^2$$



# Solving the Intersection Equation

$$\left(\left(x_0 + t\left(x_1 - x_0\right)\right) - c_x\right)^2 + \left(\left(y_0 + t\left(y_1 - y_0\right)_1\right) - c_y\right)^2 + \left(\left(z_0 + t\left(z_1 - z_0\right)\right) - c_z\right)^2 = R^2$$

is a quadratic equation in variable t.

For a fixed pixel, VP, and sphere,

$x_0$, $y_0$, $z_0$, $x_1$, $y_1$, $z_1$, $c_x$, $c_y$, $c_z$, and R

are all constants.

We solve for t using the quadratic formula.

# The Quadratic Coefficients

$$\left(\left(x_0 + t(x_1 - x_0)\right) - c_x\right)^2 + \left(\left(y_0 + t(y_1 - y_0)_1\right) - c_y\right)^2 + \left(\left(z_0 + t(z_1 - z_0)\right) - c_z\right)^2 = R^2$$

Set $d_x = x_1 - x_0$

$d_y = y_1 - y_0$

$d_z = z_1 - z_0$

Now find the the coefficients:

$$At^2 + Bt + C = 0$$

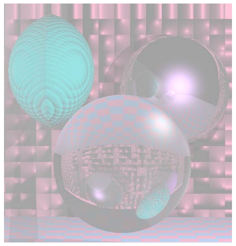
# Computing Coefficients

$$((x_0 + t(x_1 - x_0)) - c_x)^2 + ((y_0 + t(y_1 - y_0)) - c_y)^2 + ((z_0 + t(z_1 - z_0)) - c_z)^2 = R^2$$

$$((x_0 + td_x) - c_x)^2 + ((y_0 + td_y) - c_y)^2 + (((z_0 + td_z) - c_z)^2 = R^2$$

$$(x_0 + td_x)^2 - 2c_x(x_0 + td_x) + c_x{}^2 +$$
$$(y_0 + td_y)^2 - 2c_y(y_0 + td_y) + c_y{}^2 +$$
$$(z_0 + td_z)^2 - 2c_z(z_0 + td_z) + c_z{}^2 - R^2 = 0$$

$$x_0{}^2 + 2x_0 td_x + t^2 d_x{}^2 - 2c_x x_0 - 2c_x td_x + c_x{}^2 +$$
$$y_0{}^2 + 2y_0 td_y + t^2 d_y{}^2 - 2c_y y_0 - 2c_y td_y + c_y{}^2 +$$
$$z_0{}^2 + 2z_0 td_z + t^2 d_z{}^2 - 2c_z z_0 - 2c_z td_z + c_z{}^2 - R^2 = 0$$



# The Coefficients

$$x_0^2 + 2x_0 t d_x + t^2 d_x^2 - 2c_x x_0 - 2c_x t d_x + c_x^2 +$$
$$y_0^2 + 2y_0 t d_y + t^2 d_y^2 - 2c_y y_0 - 2c_y t d_y + c_y^2 +$$
$$z_0^2 + 2z_0 t d_z + t^2 d_z^2 - 2c_z z_0 - 2c_z t d_z + c_z^2 - R^2 = 0$$

$$A = d_x^2 + d_y^2 + d_z^2$$

$$B = 2d_x(x_0 - c_x) + 2d_y(y_0 - c_y) + 2d_z(z_0 - c_z)$$

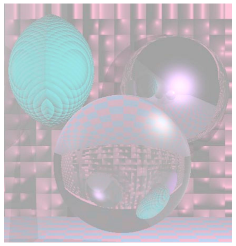
$$C = c_x^2 + c_y^2 + c_z^2 + x_0^2 + y_0^2 + z_0^2 + -2(c_x x_0 + c_y y_0 + c_z z_0) - R^2$$

# Solving the Equation

$$At^2 + Bt + C = 0$$

$$\text{discriminant} = D(A,B,C) = B^2 - 4AC$$

$$D(A,B,C)\begin{cases} < 0 & \text{no intersection} \\ = 0 & \text{ray is tangent to the sphere} \\ > 0 & \text{ray intersects sphere in two points} \end{cases}$$



The intersection nearest $P_0$ is given by:

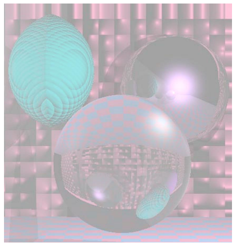
$$t = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

To find the coordinates of the intersection point:

$$x = x_0 + td_x$$

$$y = y_0 + td_y$$

$$z = z_0 + td_z$$



# First Lighting Model

- Ambient light is a global constant.

  Ambient Light = $k_a (A_R, A_G, A_B)$

  $k_a$ is in the "World of Spheres"

  $0 \le k_a \le 1$

  $(A_R, A_G, A_B)$ = average of the light sources

  $(A_R, A_G, A_B) = (1, 1, 1)$ for white light

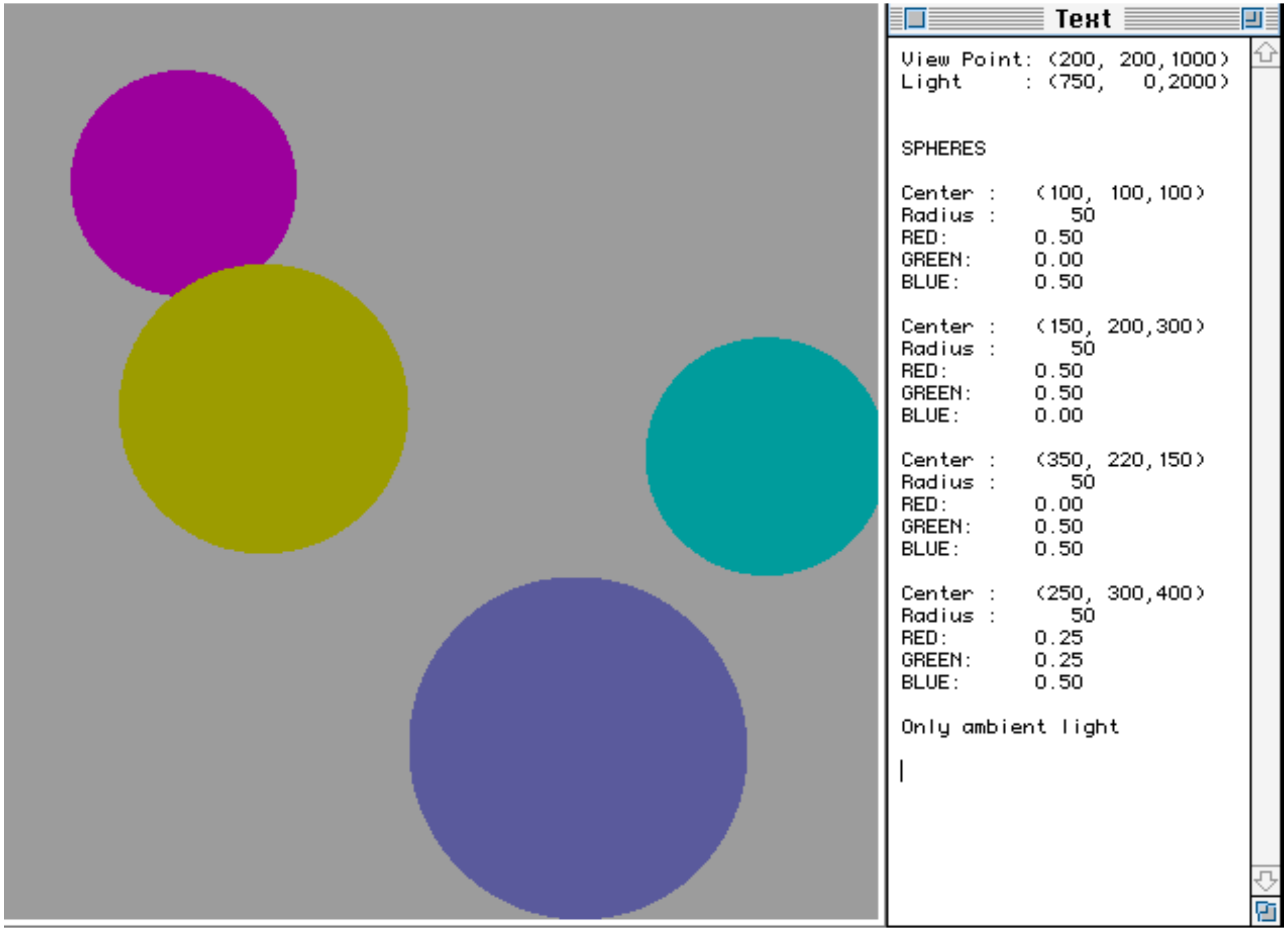
- Color of object $S = (S_R, S_G, S_B)$
- Visible Color of an object S with only ambient light

  $C_S = k_a (A_R S_R, A_G S_G, A_B S_B)$

- For white light

  $C_S = k_a (S_R, S_G, S_B)$

# Visible Surfaces
# Ambient Light



# Second Lighting Model

- Point source light $L = (L_R, L_G, L_B)$ at $(L_x, L_y, L_z)$
- Ambient light is also present.
- Color at **point p** on an object S with ambient & diffuse reflection

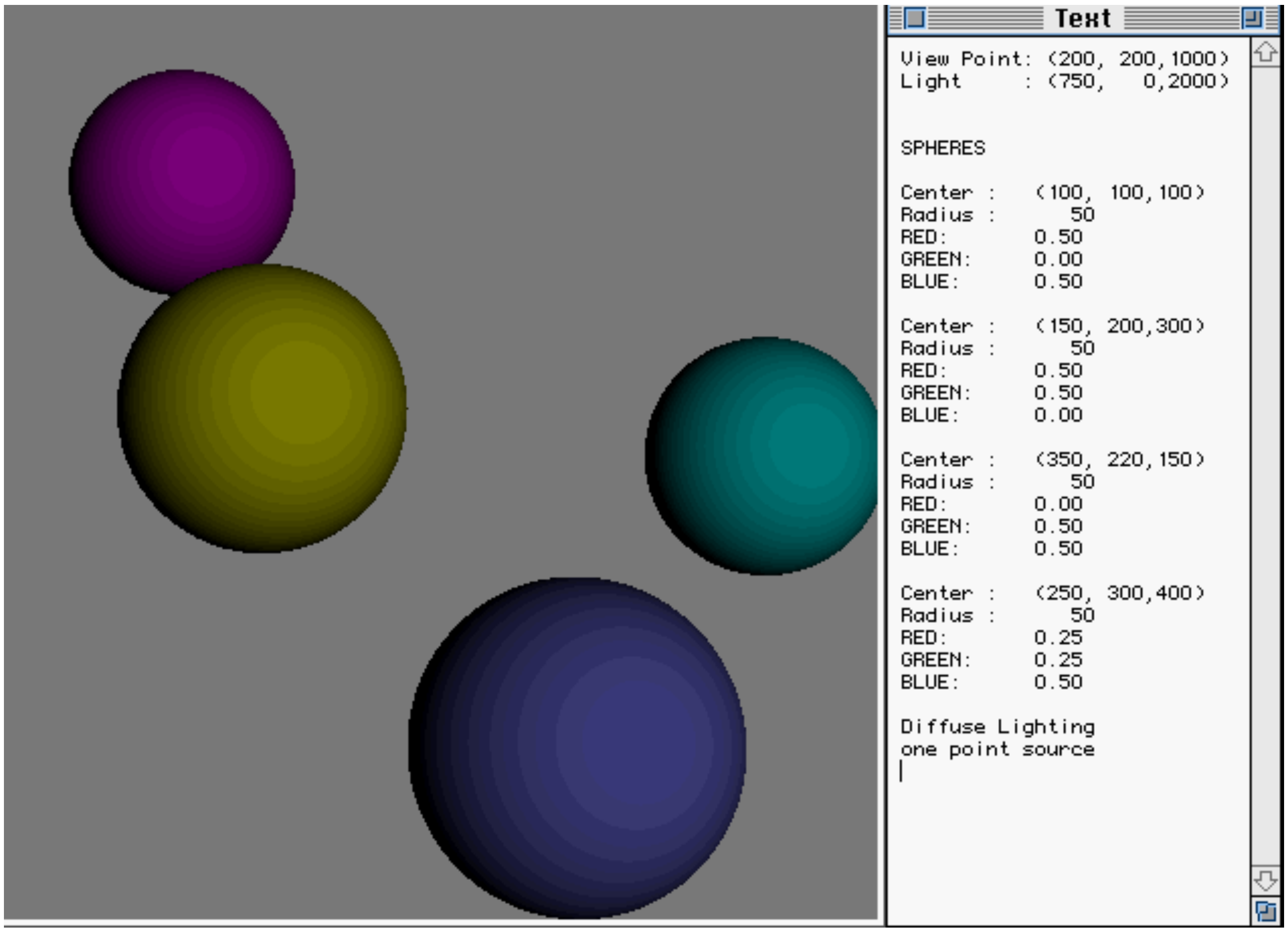
$$C_p = k_a (A_R S_R, A_G S_G, A_B S_B) + k_d k_p (L_R S_R, L_G S_G, L_B S_B)$$

- For white light, L = (1, 1, 1)

$$C_p = k_a (S_R, S_G, S_B) + k_d k_p (S_R, S_G, S_B)$$

- $k_p$ depends on the **point p** on the object and $(L_x, L_y, L_z)$
- $k_d$ depends on the object (sphere)
- $k_a$ is global
- $k_a + k_d \leq 1$

# Diffuse Light

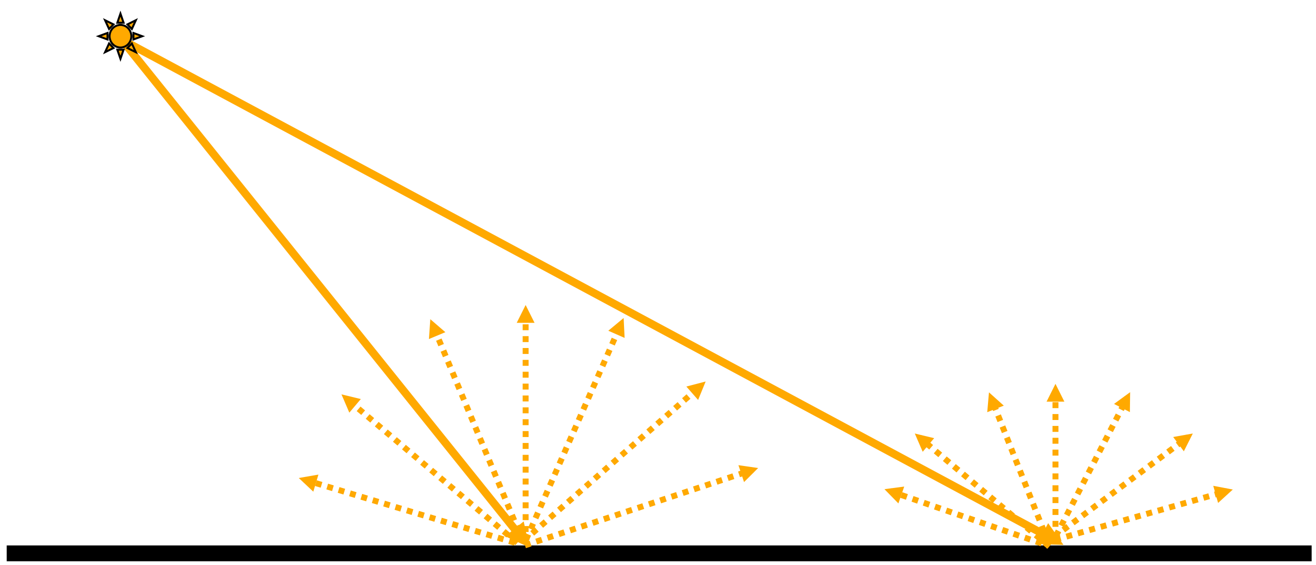


# Lambertian Reflection Model
# Diffuse Shading

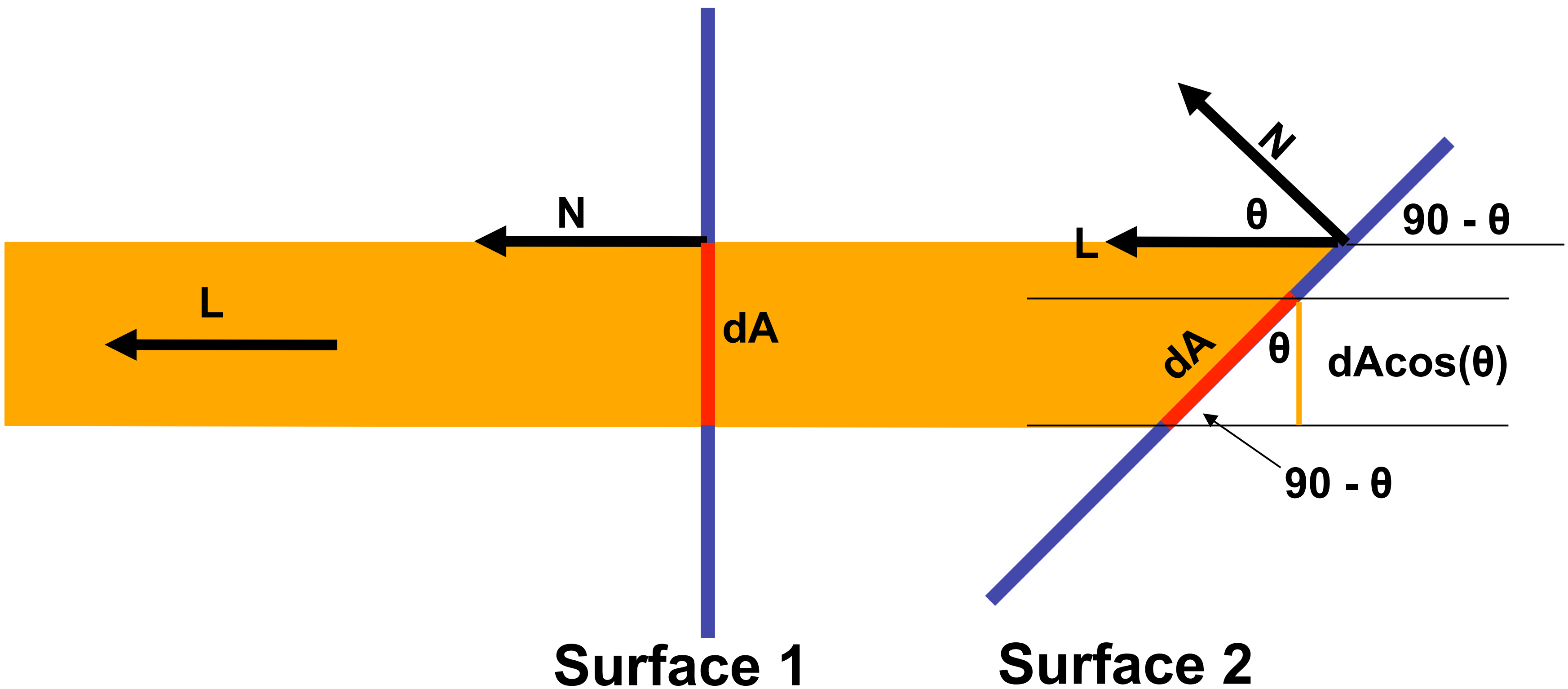
- For matte (non-shiny) objects
- Examples
  - Matte paper, newsprint
  - Unpolished wood
  - Unpolished stones
- Color at a point on a matte object does not change with viewpoint.

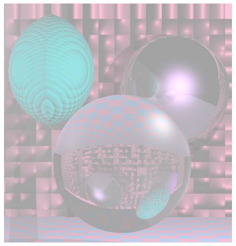
# Physics of Lambertian Reflection

- Incoming light is partially absorbed and partially transmitted equally in all directions

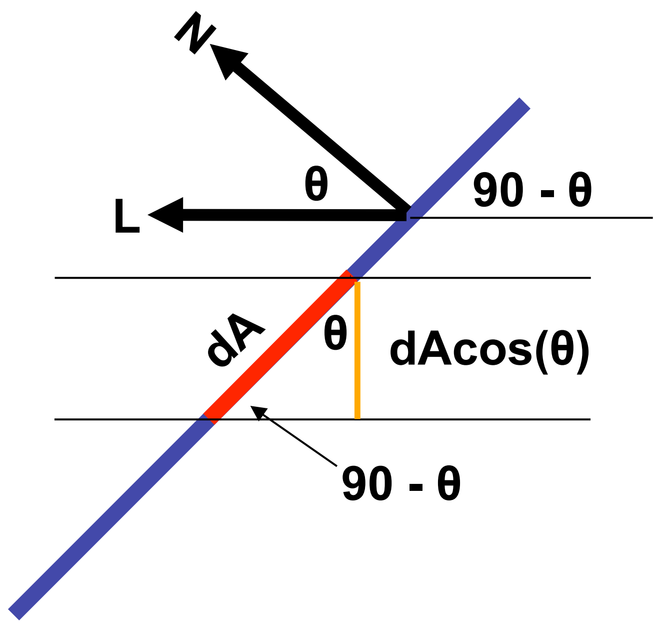


# Geometry of Lambert's Law



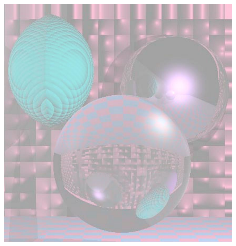


# $\cos(\boldsymbol{\theta})=N \cdot L$



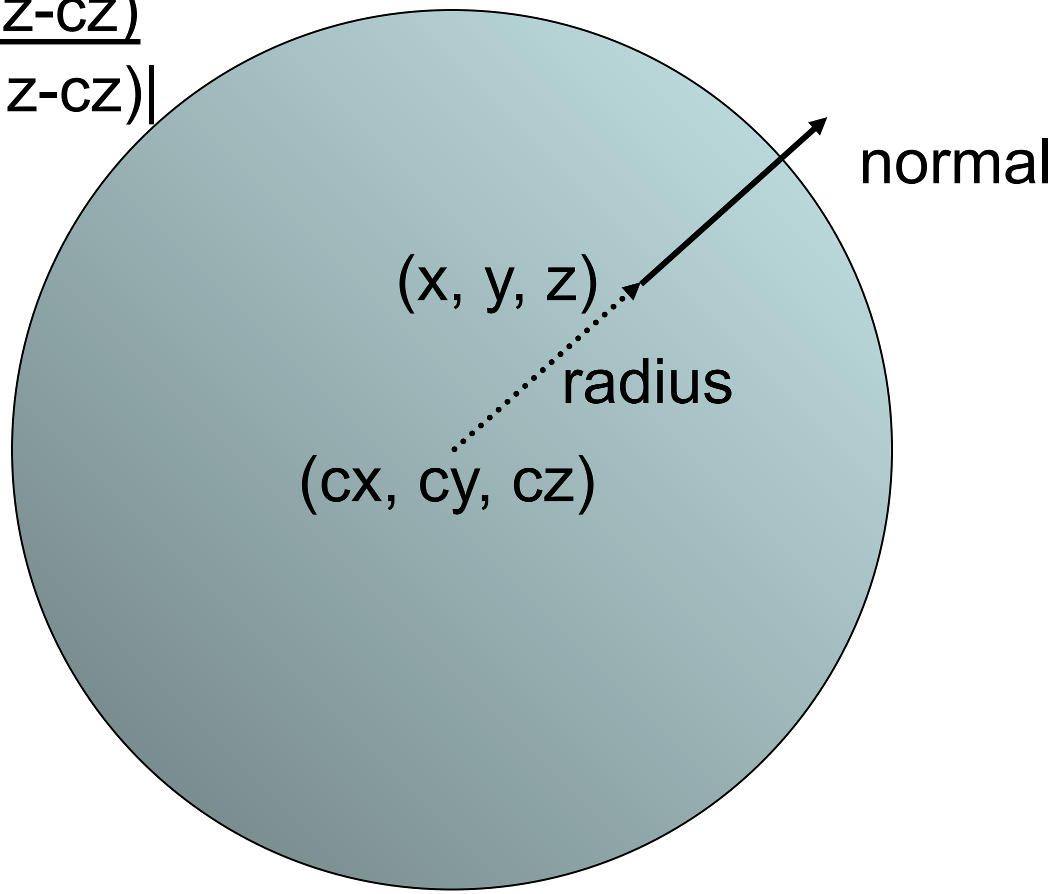

**Surface 2**

$$Cp= ka\ (SR, SG, SB) + kd\ \mathbf{N \cdot L}\ (SR, SG, SB)$$

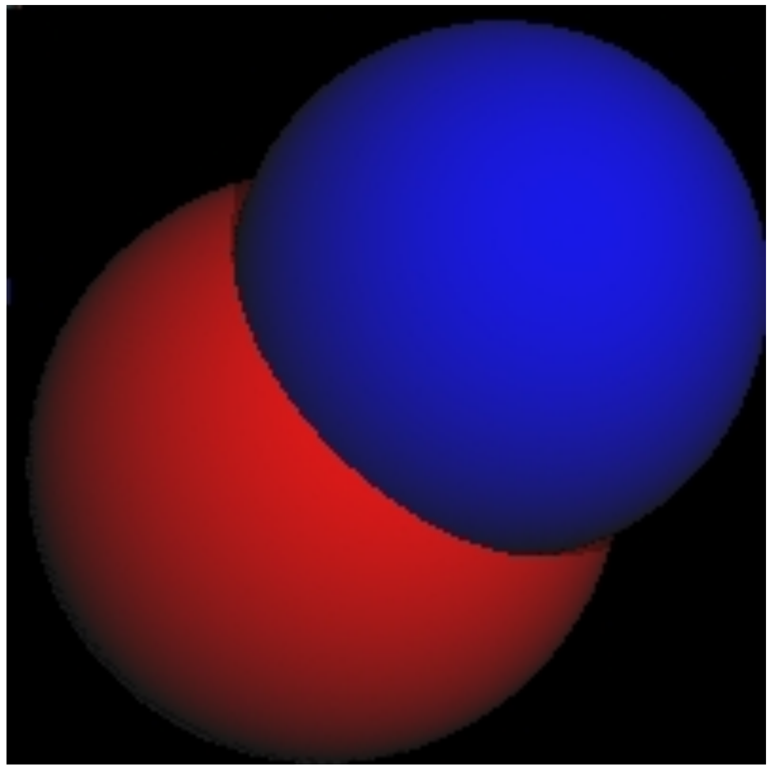


# Finding N

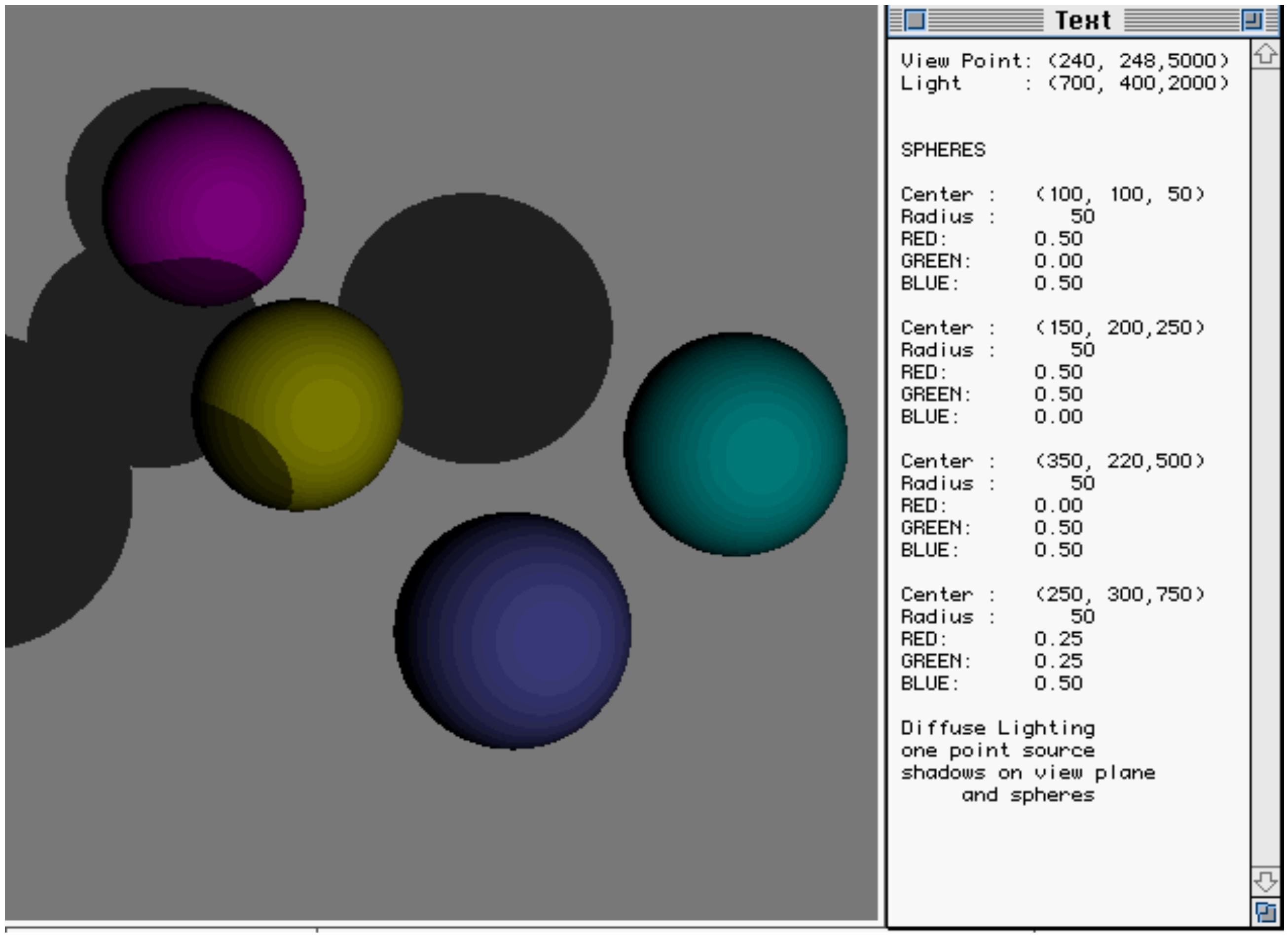
$$\mathbf{N} = \frac{(x-cx,\ y-cy,\ z-cz)}{|(x-cx,\ y-cy,\ z-cz)|}$$



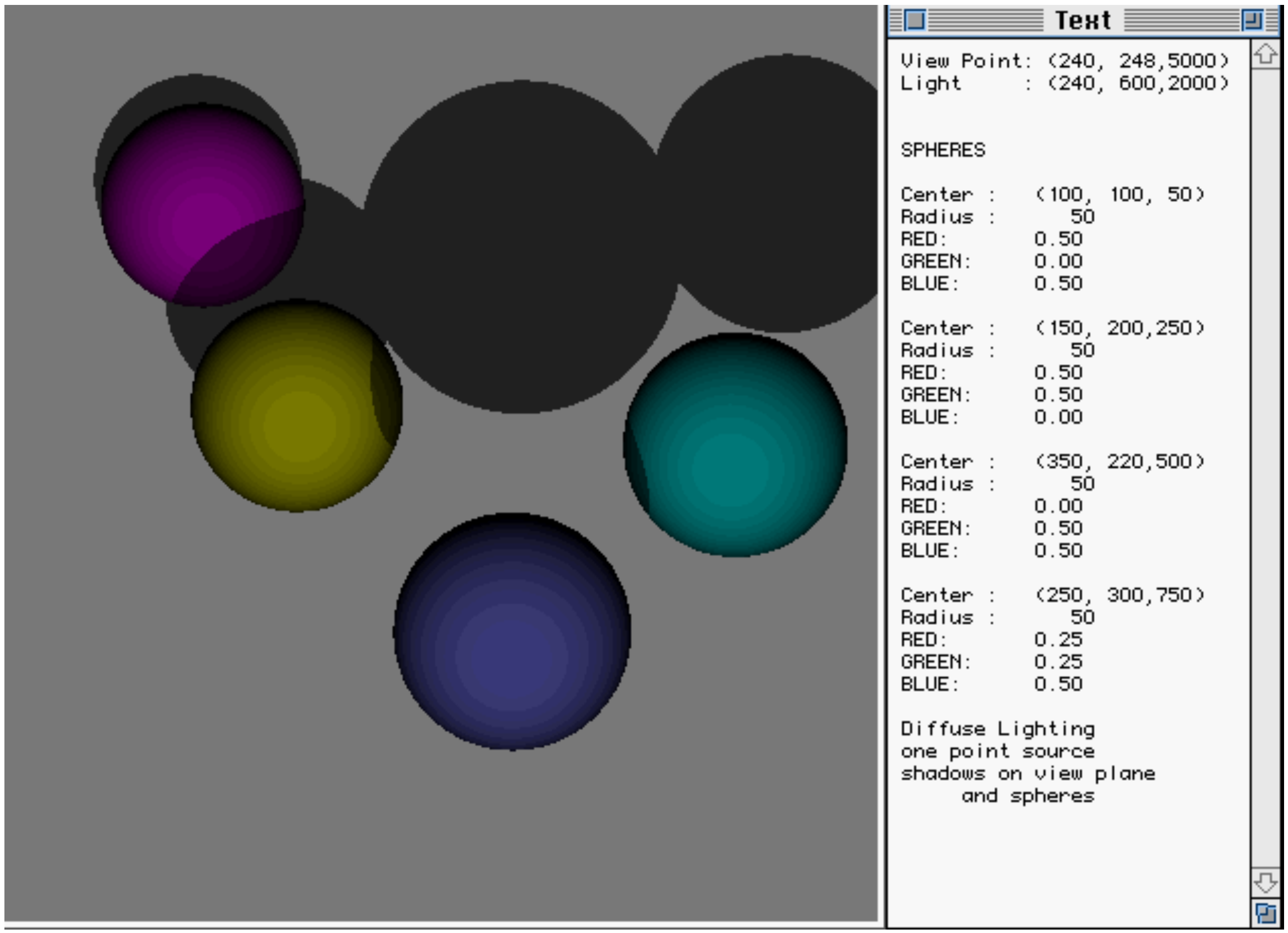
# Diffuse Light 2



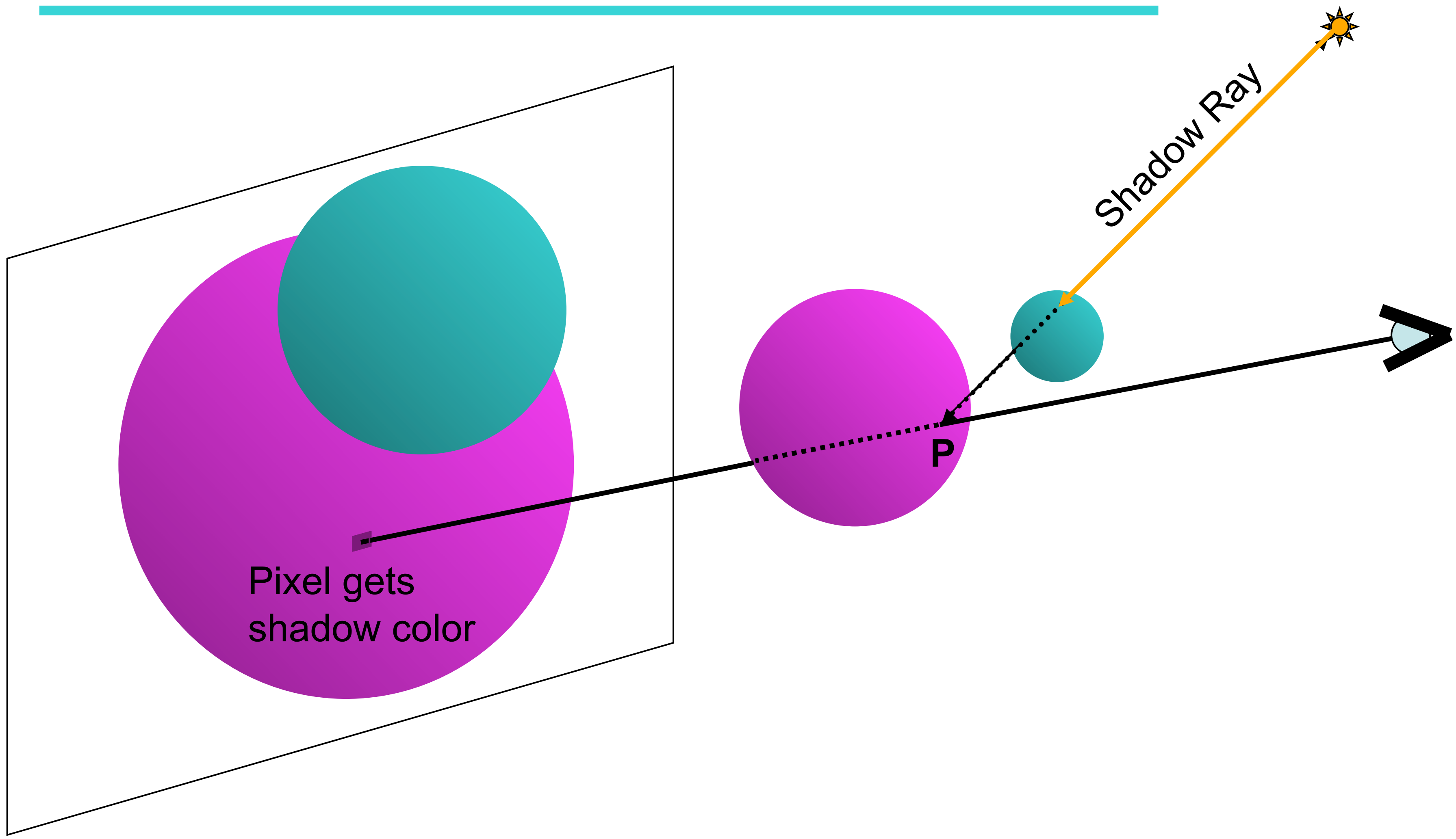
# Shadows on Spheres

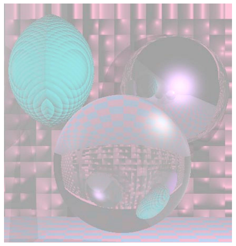


# More Shadows



# Finding Shadows





# Shadow Color

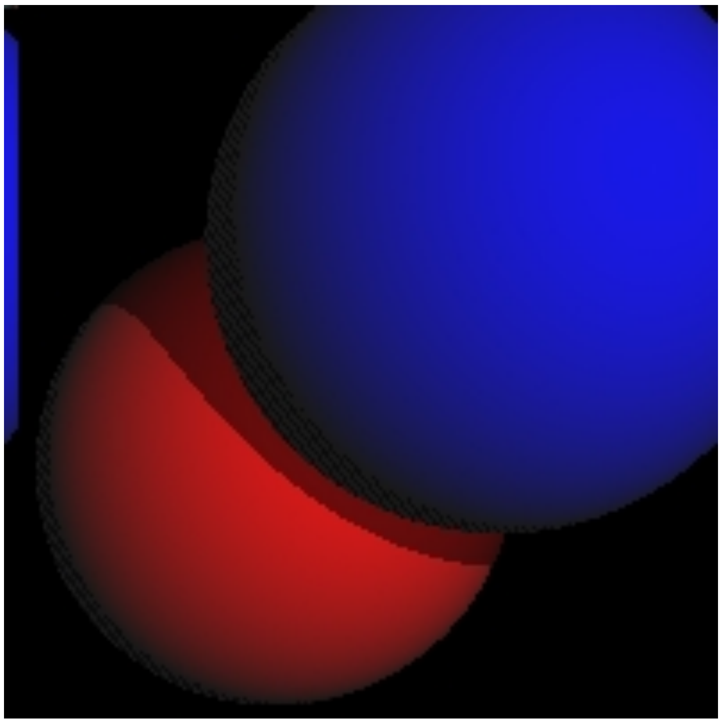
- Given

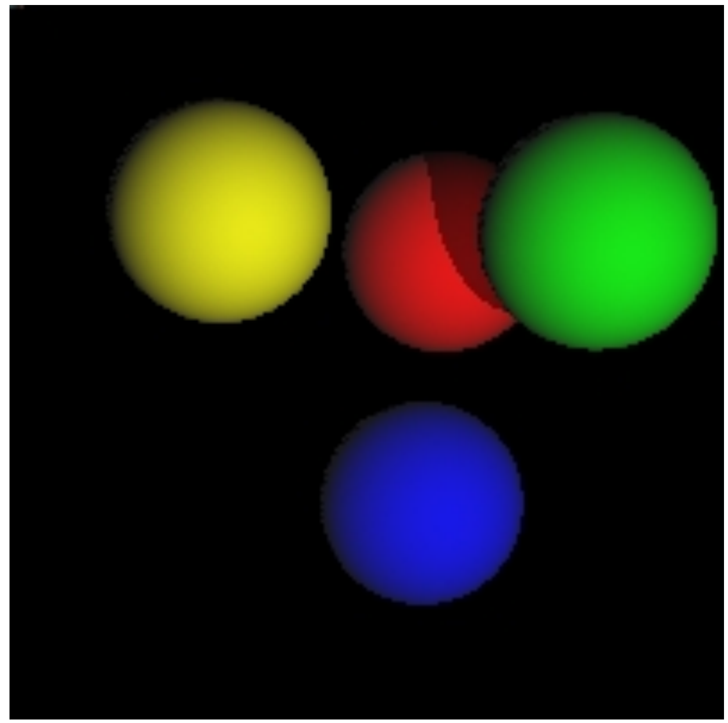
  Ray from P (point on sphere S) to L (light)

  $P = P_0 = (x_0, y_0, z_0)$ and $L = P_1 = (x_1, y_1, z_1)$

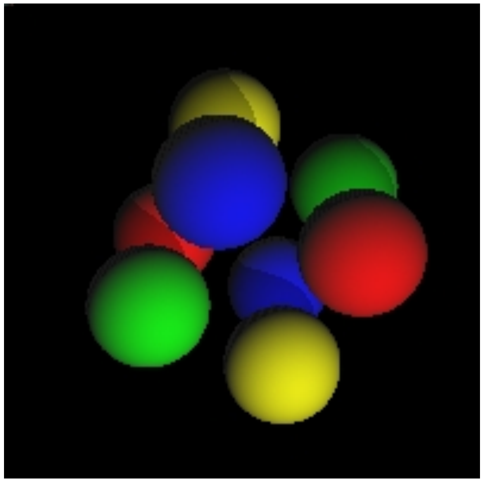
- Find out whether the ray intersects any other object (sphere).
  - If it does, P is in shadow.
  - Use only ambient light for pixel.

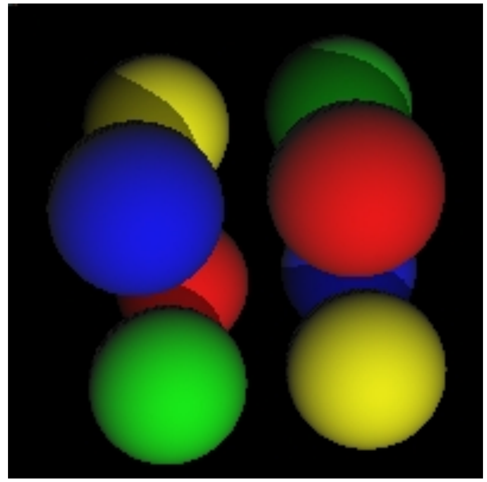
# Shape of Shadows

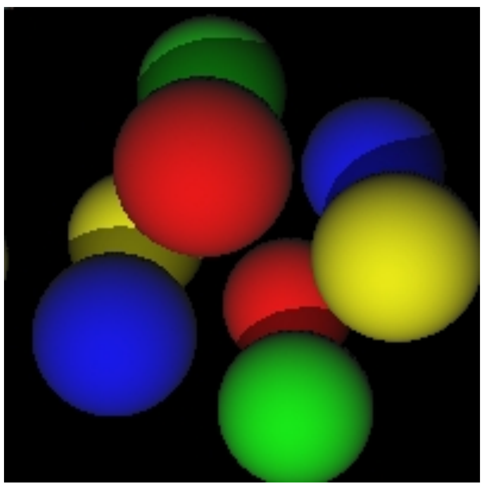


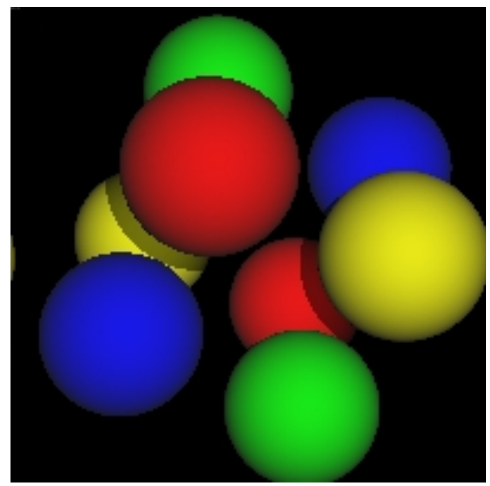


# Different Views

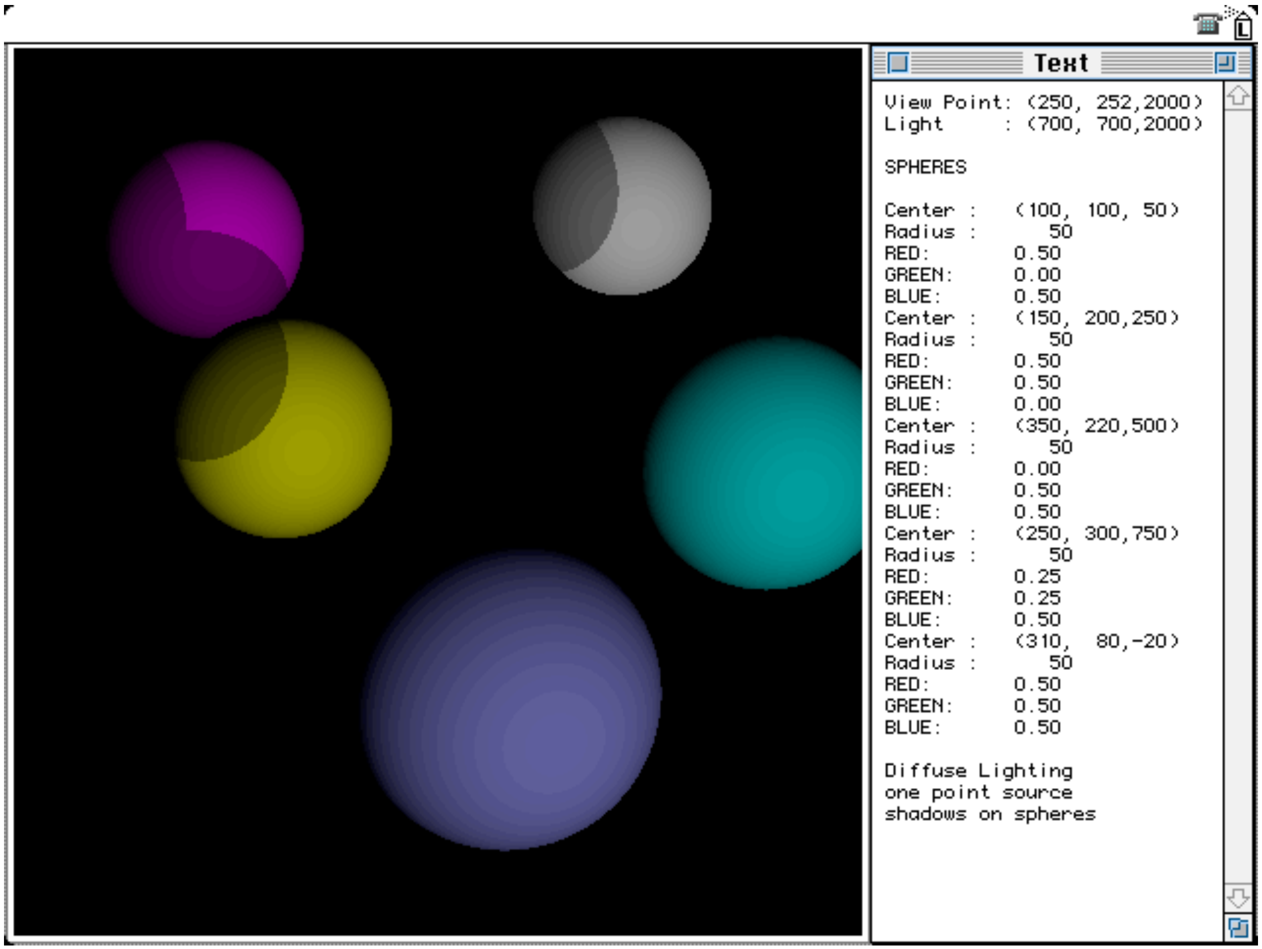




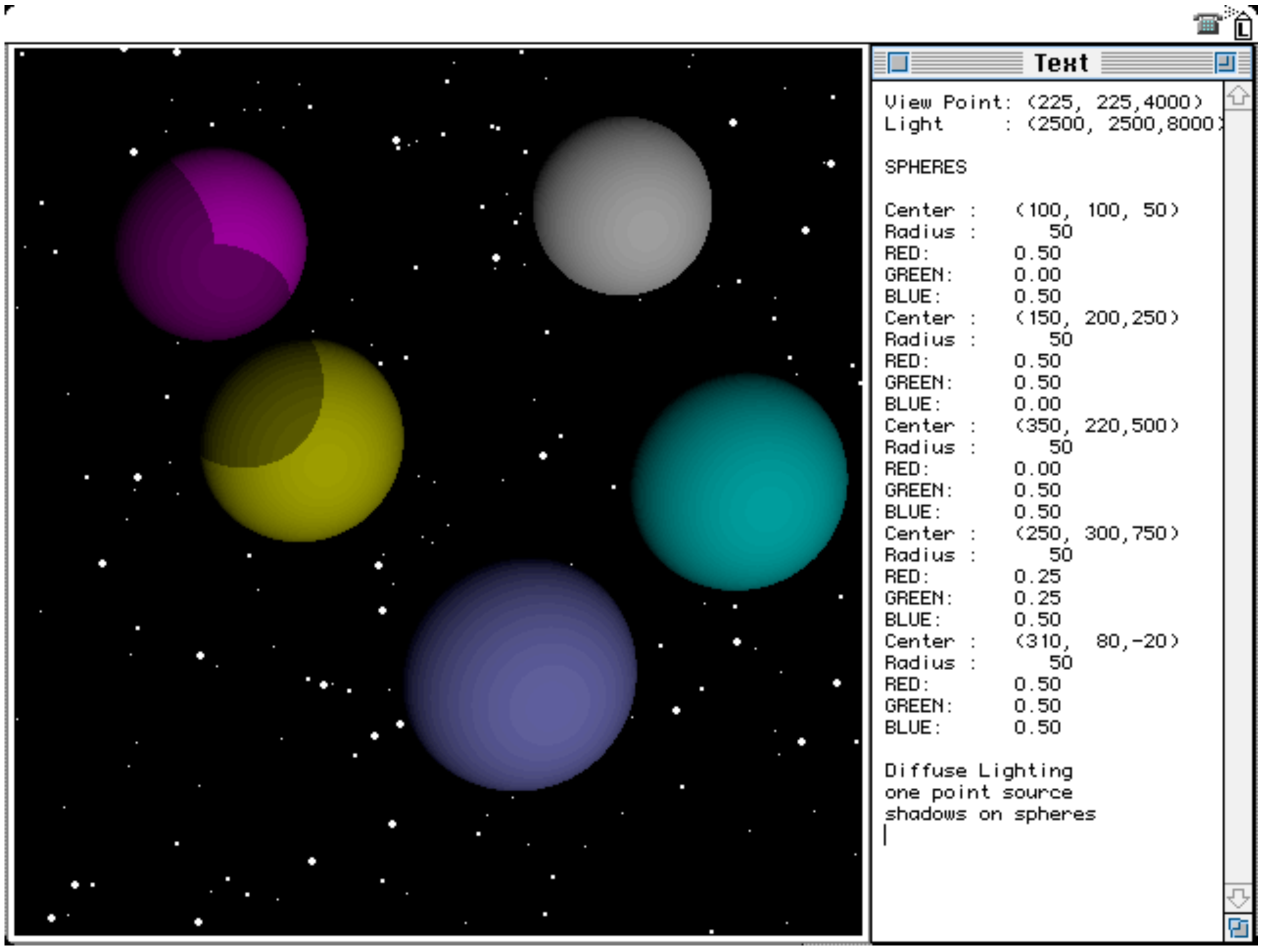




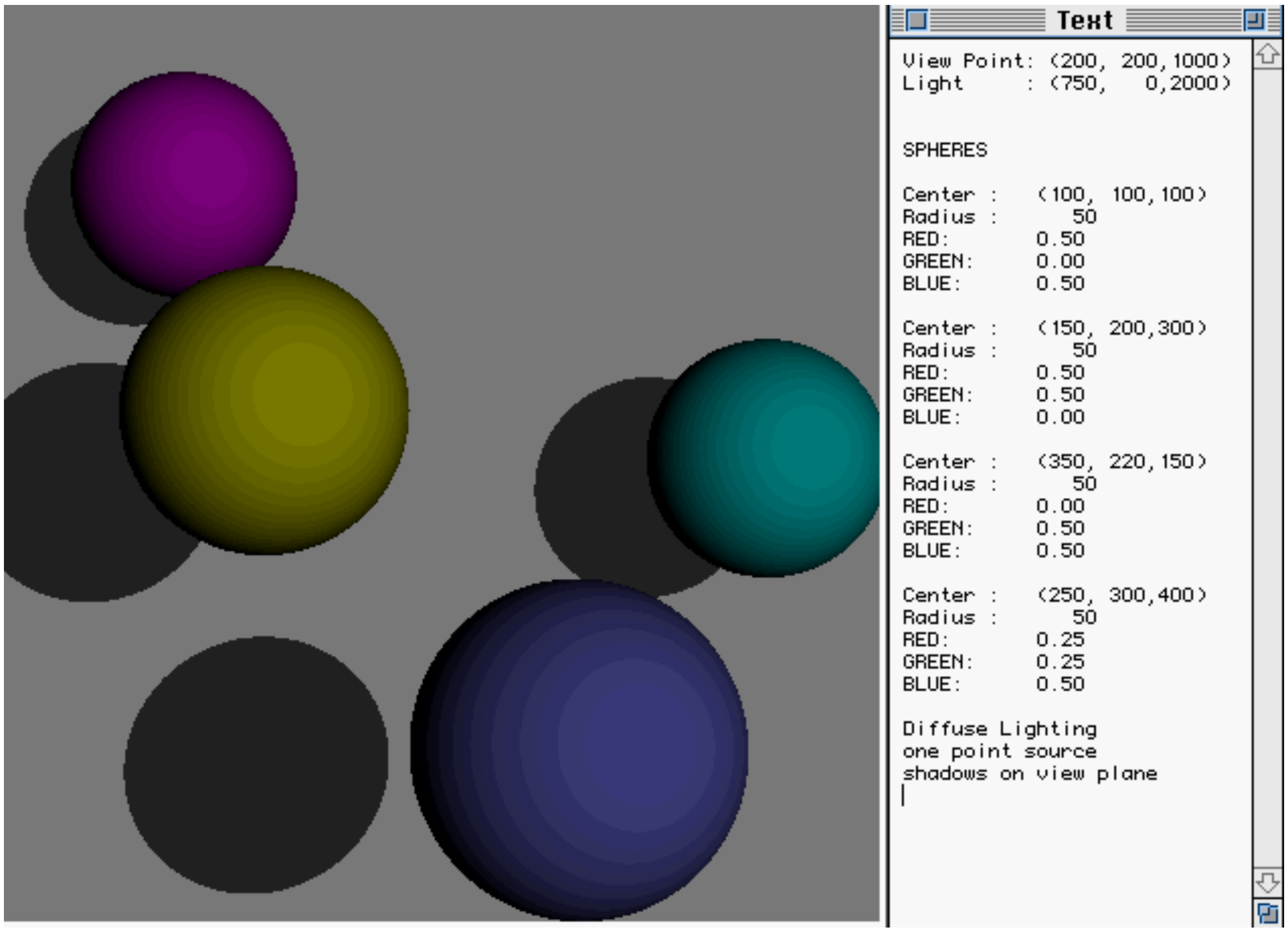
# Planets


Text
View Point: (250, 252,2000)
Light : (700, 700,2000)
SPHERES
Center : (100, 100, 50)
Radius : 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50
Center : (150, 200,250)
Radius : 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00
Center : (350, 220,500)
Radius : 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50
Center : (250, 300,750)
Radius : 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50
Center : (310, 80,-20)
Radius : 50
RED: 0.50
GREEN: 0.50
BLUE: 0.50
Diffuse Lighting
one point source
shadows on spheres

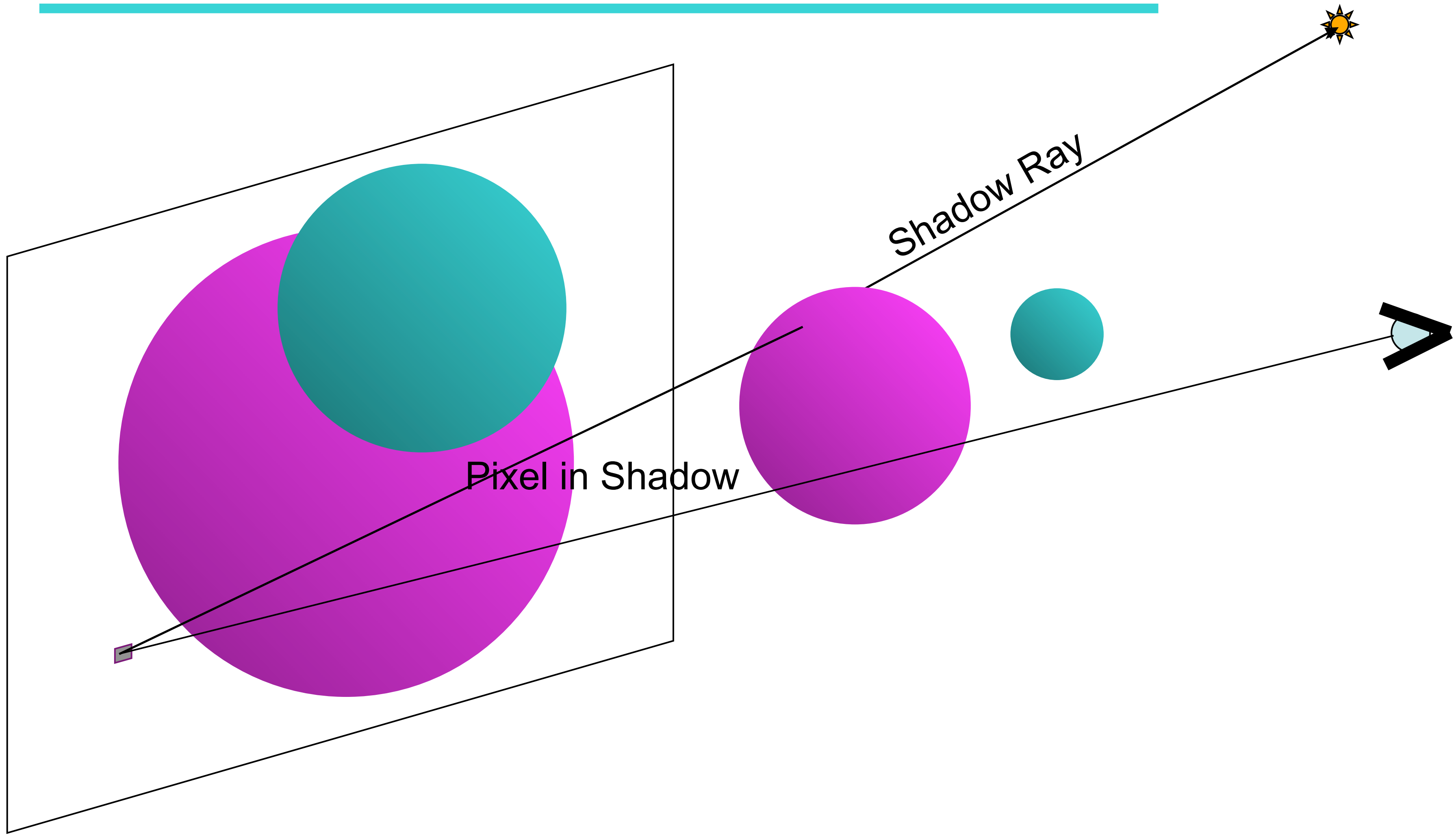
# Starry Skies



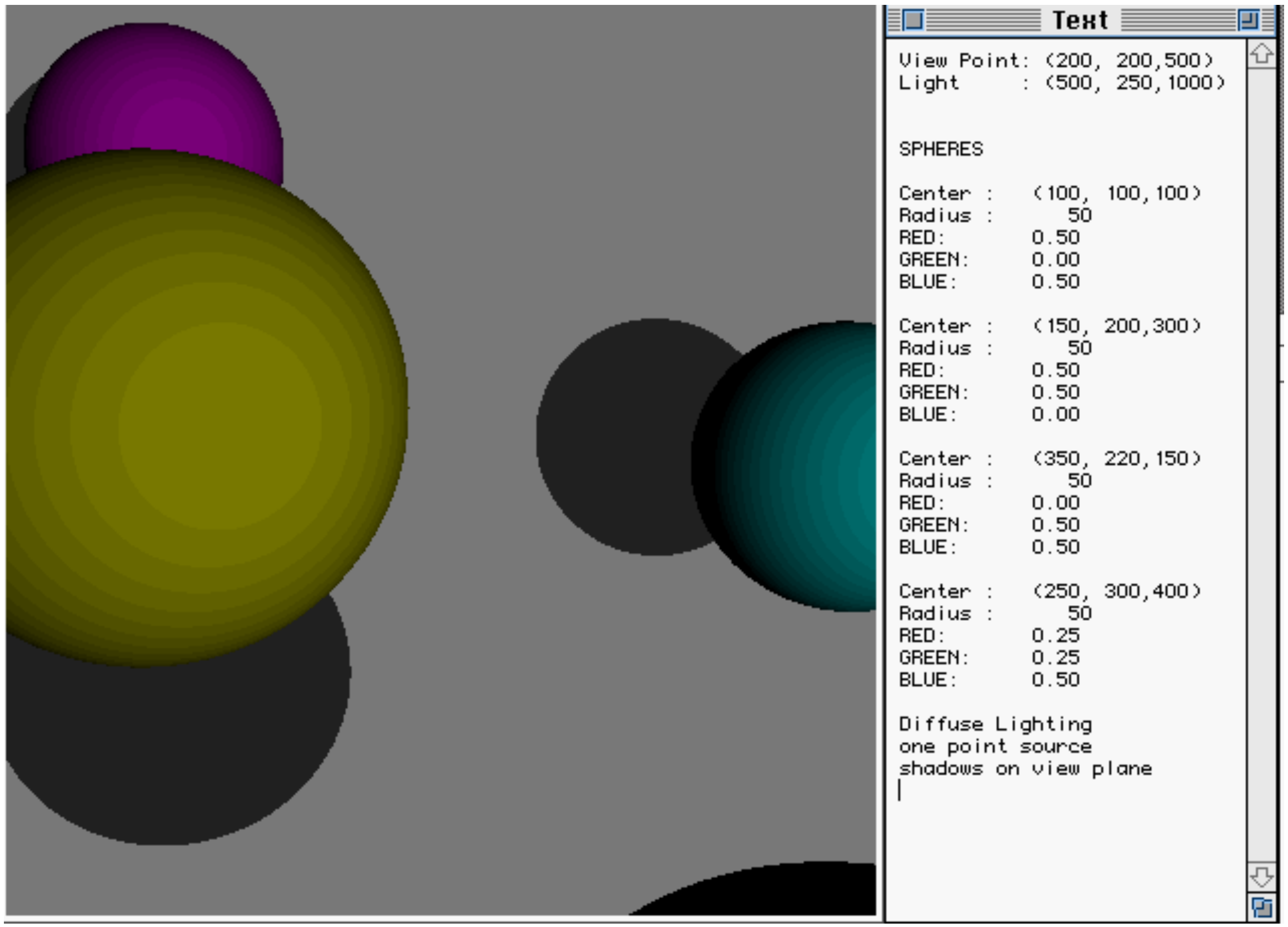
# Shadows on the Plane



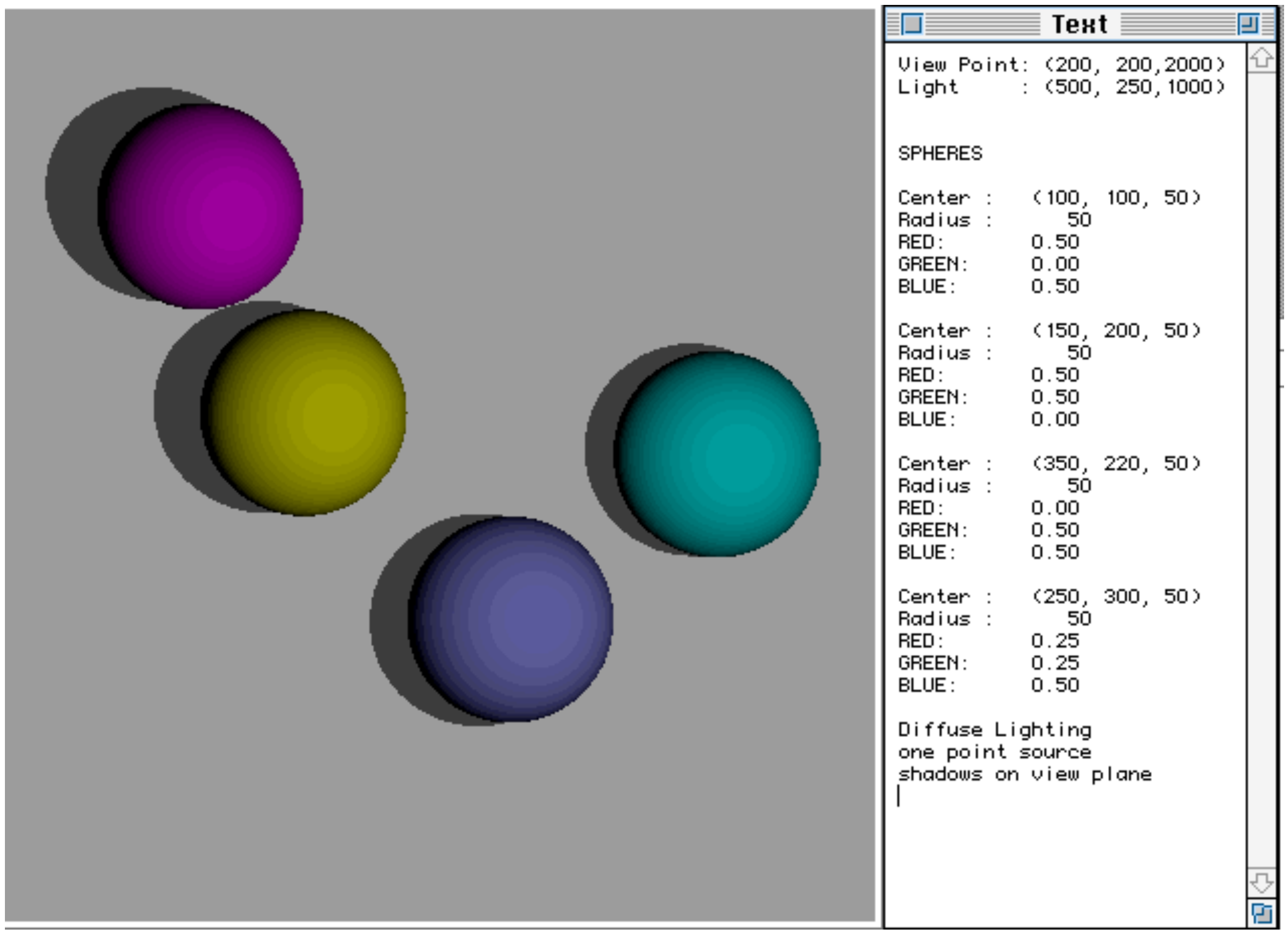
# Finding Shadows on the Back Plane



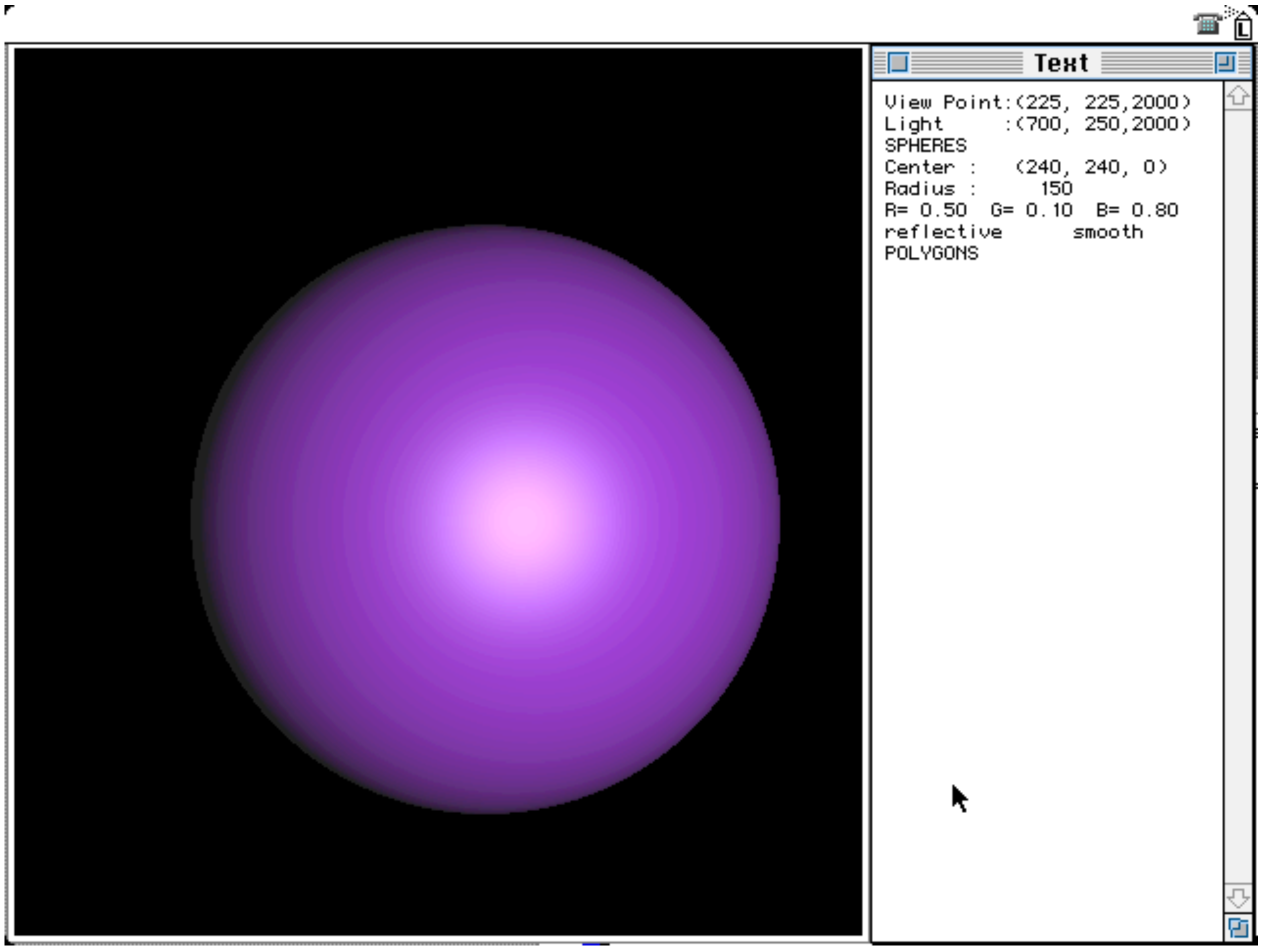
# Close up

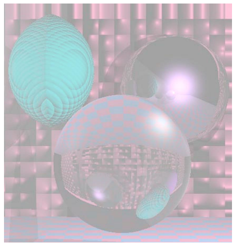

Text
View Point: (200, 200,500)
Light : (500, 250, 1000)
SPHERES
Center : (100, 100, 100)
Radius : 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50
Center : (150, 200, 300)
Radius : 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00
Center : (350, 220, 150)
Radius : 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50
Center : (250, 300, 400)
Radius : 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50
Diffuse Lighting
one point source
shadows on view plane

# On the Table


Text
View Point: (200, 200,2000)
Light : (500, 250,1000)
SPHERES
Center : (100, 100, 50)
Radius : 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50
Center : (150, 200, 50)
Radius : 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00
Center : (350, 220, 50)
Radius : 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50
Center : (250, 300, 50)
Radius : 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50
Diffuse Lighting
one point source
shadows on view plane

# Phong Highlight





# Phong Lighting Model

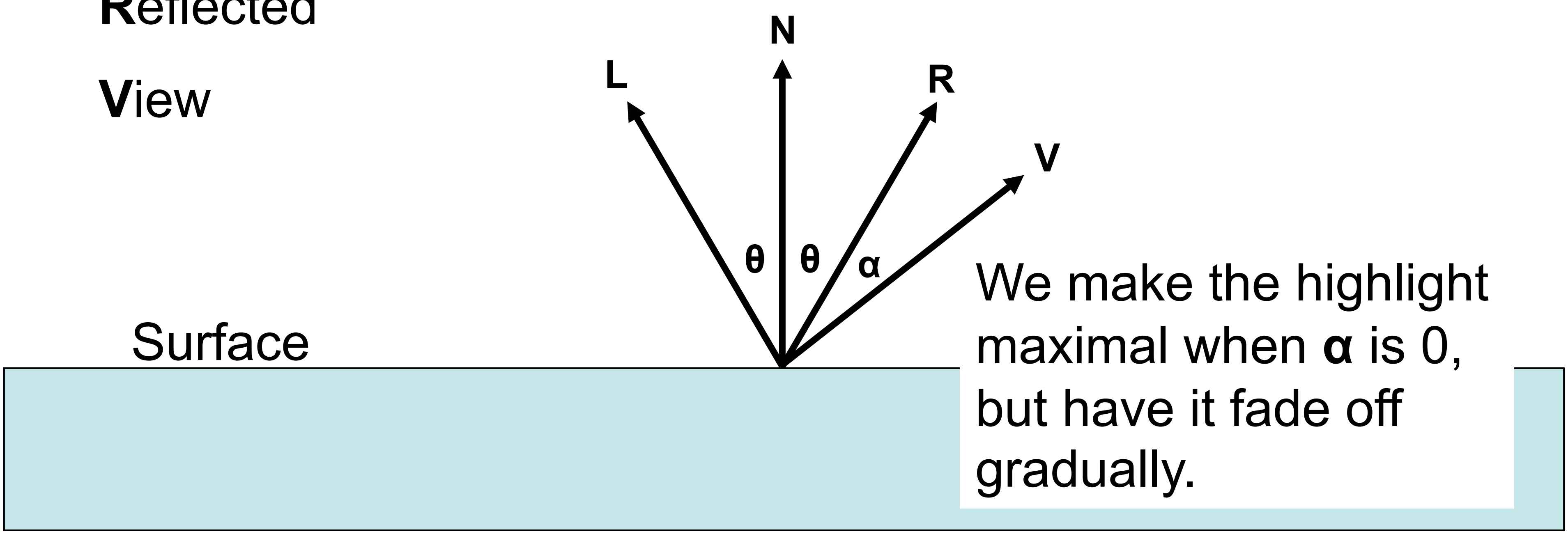
**L**ight

**N**ormal

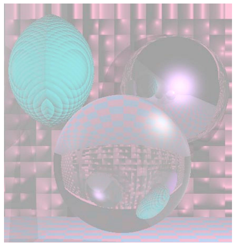
**R**eflected

**V**iew

The viewer only sees the light when **α** is 0.




We make the highlight maximal when **α** is 0, but have it fade off gradually.

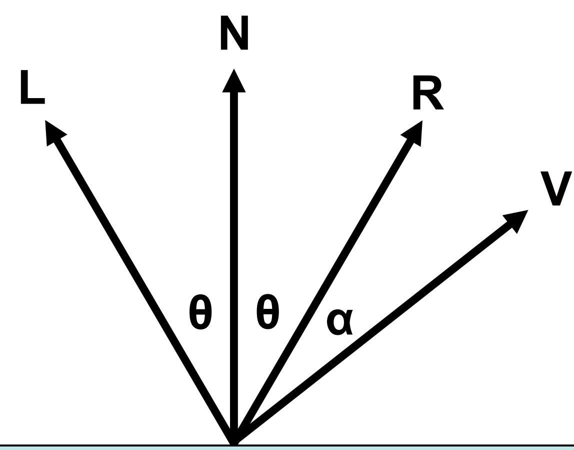


# Phong Lighting Model

$\cos(\theta) = \mathbf{R \bullet V}$

We use $\cos^n(\theta)$.

The higher n is, the faster the drop off.

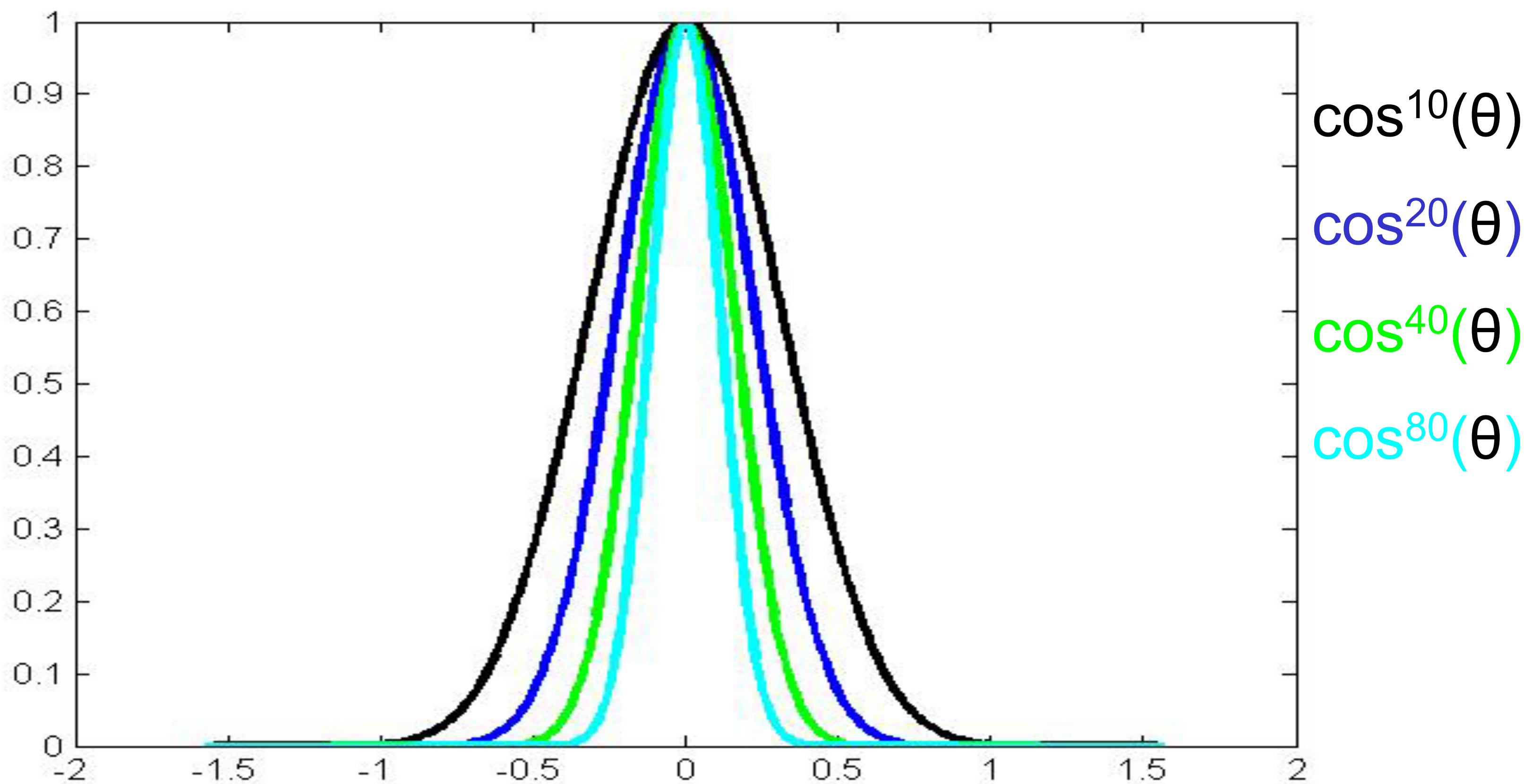



For white light

Surface

$$Cp = ka\ (SR, SG, SB) + kd\ \mathbf{N \bullet L}\ (SR, SG, SB) + ks\ (\mathbf{R \bullet V})^n (1, 1, 1)$$

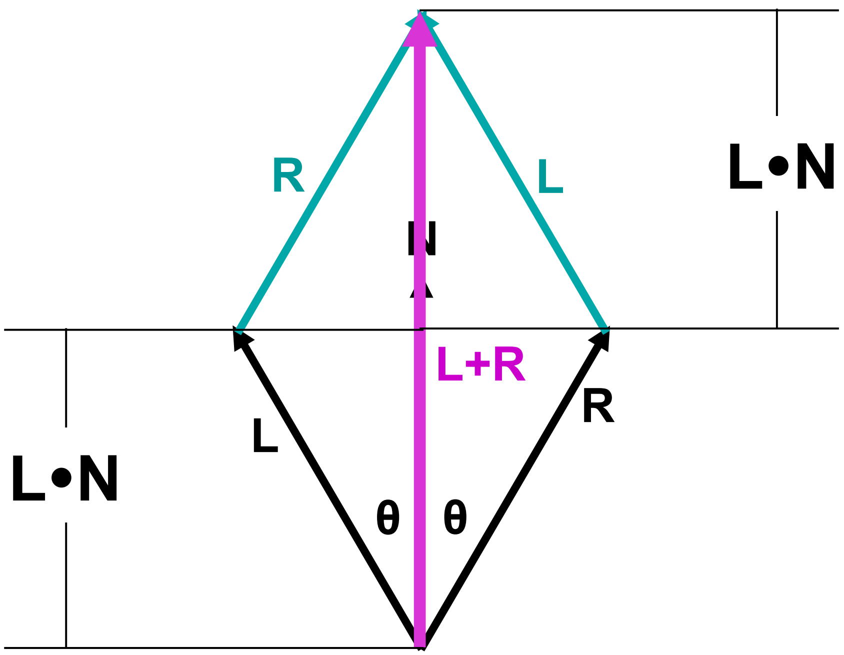
# Powers of cos(θ)

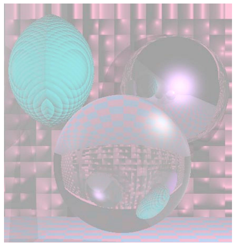


# Computing **R**

**L** + **R** = (2 **L•N**) **N**

**R** = (2 **L•N**) **N** - **L**





# The Halfway Vector

$$\mathbf{H} = \frac{\mathbf{L} + \mathbf{V}}{|\mathbf{L} + \mathbf{V}|}$$

Use **H•N**

instead of **R•V**.

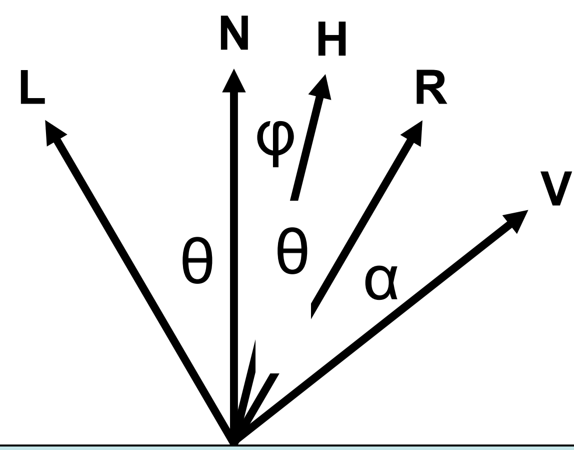
**H** is less expensive to compute than **R**.

From the picture

$\theta + \varphi = \theta - \varphi + \alpha$

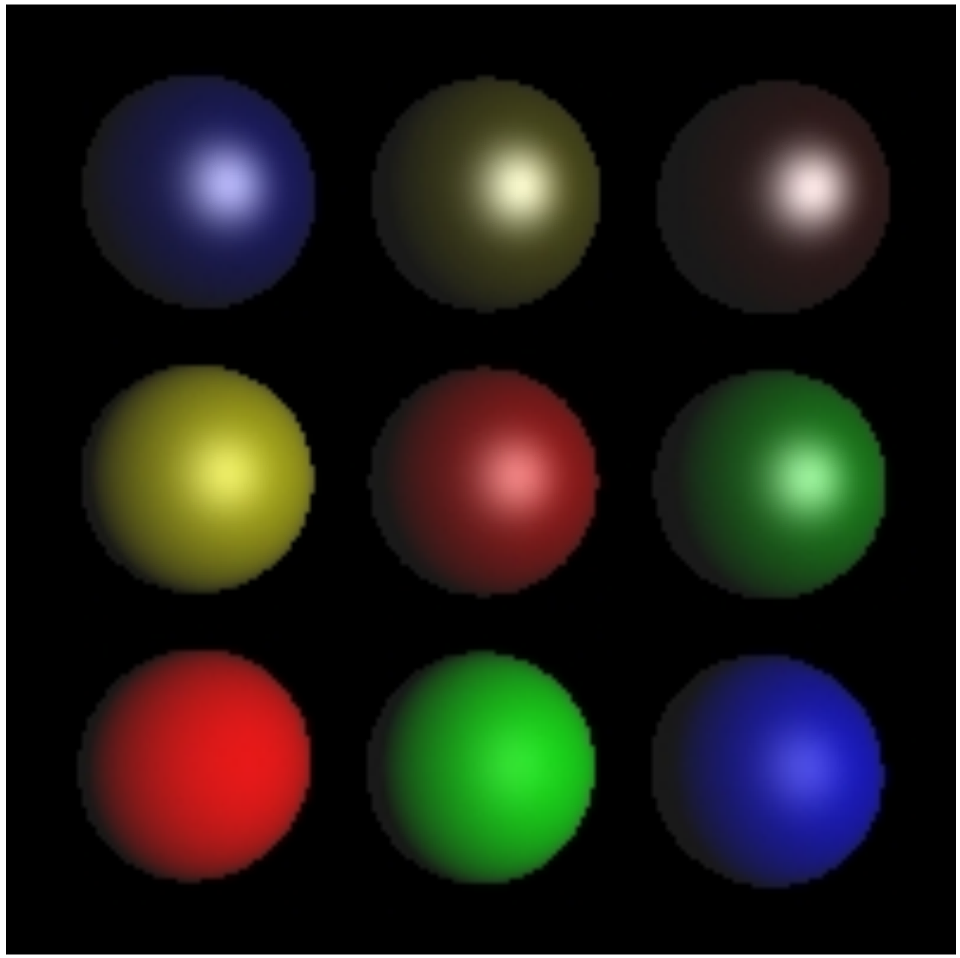
So $\varphi = \alpha/2$.

This is not generally true. Why?




Cp= ka (SR, SG, SB) + kd **N•L** (SR, SG, SB) + ks $(\mathbf{H} \bullet \mathbf{N})^n$ (1, 1, 1)

# Varied Phong Highlights



# Varying Reflectivity