Northeastern University
CS 4180/5180 - Reinforcement Learning and Decision Making Fall 2019, Chris Amato

## Self Test

Name: $\qquad$

| Problem | Points |
| :--- | ---: |
| 1. BAYES' RULE | $/ 0$ |
| 2. Probability Distributions | $/ 0$ |
| 3. DISCRETE ExPECTATION | $/ 0$ |
| 4. ExPECTATION PROPERTIES | $/ 0$ |
| 5. DERIVATIVES | $/ 0$ |
| 6. MATRICES/LINEAR EQUATIONS | $/ 0$ |
| 7. MATRICES | $/ 0$ |
| Total | $/ 0$ |

## Instructions

- This assignment will not be graded for correctness
- Use this as an opportunity to self-assess your math background and self-study as appropriate.


## (0 pts.) 1. Bayes' Rule

The Weatherly app predicts rain tomorrow. In recent years, it has rained only 73 days each year. When it actually rains, the Weatherly app correctly forecasts rain $70 \%$ of the time. When it doesn't rain, the app incorrectly forecasts rain $30 \%$ of the time. What is the probability that it will rain tomorrow?

Hint: $P(H \mid D)=\frac{P(H) P(D \mid H)}{P(D)}$

## (0 pts.) 2. Probability Distributions

Given the following probability density function (PDF) of a random variable $x \ldots$

$$
p(x)= \begin{cases}4 x & 0 \leq x \leq \frac{1}{2} \\ -4 x+4 & \frac{1}{2} \leq x \leq 1\end{cases}
$$

What is the equation and graph of the corresponding cumulative density function (CDF)?

## (0 pts.) 3. Discrete Expectation

Calculate the expected value of $X, E[X]$, where $X$ is a random variable representing the outcome of a roll of a trick die. Use the sample space $x \in\{1,2,3,4,5,6\}$ (i.e. six-sided die) and let

$$
P(X=x)= \begin{cases}\frac{1}{2} & x=1 \\ \frac{1}{10} & x \neq 1\end{cases}
$$

## (0 pts.) 4. Expectation Properties

Use the properties of expectation to show that we can rewrite the variance of a random variable $X \ldots$
$\operatorname{Var}[X]=E\left[(X-\mu)^{2}\right]$
as ...
$\operatorname{Var}[X]=E\left[X^{2}\right]-(E[X])^{2}$

## (0 pts.) 5. Derivatives

Calculate the following derivatives
$e^{x^{3}+5 x^{2}}$
$3^{x} \log (x)$

## (0 pts.) 6. Matrices/Linear Equations

Consider the following system of equations .....
$2 x_{1}+x_{2}+x_{3}=3$
$4 x_{1}+2 x_{3}=10$
$2 x_{1}+2 x_{2}=-2$
a. Write the system as a matrix equation of the form $A x=b$.
b. Write the solution of the system as a column $s$ and verify by matrix multiplication that $A s=b$.
c. Write $b$ as a linear combination of the columns in $A$.

## (0 pts.) 7. Matrices

Consider the following matrix ...

$$
A=\left(\begin{array}{lll}
1 & 2 & 3 \\
1 & 4 & 3 \\
1 & 3 & 4
\end{array}\right)
$$

a. What is the determinant, $\operatorname{det}(A)$ or $|A|$, of the matrix?
b. Is the matrix invertible?
c. What is the rank of the matrix?

