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Handbook of Temporal Reasoning in Artificial Intelligence
Edited by M. Fisher, D. Gabbay and L. Vila

Chapter 1

Formal Theories of Time and Temporal Incidence

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The design of intelligent agents acting in a changing environment must be based on some form of temporal reasoning system. Such a system should, in turn, be founded on a formal theory of time. Theories of time are based on some primitive time units (instants, intervals, etc.) and determine both the expressiveness of the language and the completeness of the reasoning system.

The time theory of a temporal reasoning system is closely connected with the so-called theory of *temporal incidence*, meaning the set of domain-independent properties for the truth-value of temporal propositions throughout time. Classically, for a given domain, we distinguish between two classes of temporal propositions: changing domain properties (or *fluents*) and events whose occurrence may cause change on fluents.

Formal theories of time and temporal incidence involve some controversial issues such as (i) the expression of *instantaneous events* and *fluents* that hold instantaneously, (ii) the *dividing instant problem* and (iii) the formalization of the properties for *non-instantaneous holding of fluents*.

This chapter surveys the most relevant theories of time proposed in Artificial Intelligence according to various representational issues including the ones above. Also, the chapter presents a brief overview of *temporal incidence theories* and proposes a theory of temporal incidence defined upon a theory of instants and periods whose key insight is the distinction between continuous and discrete fluents.

1.1 Introduction

An intelligent agent interacting in a changing environment must be able to reason about these changes as well as the events and actions causing them, the effects it may have in the rest of the environment and the time when all these things happen or cease happening. Therefore, the design of intelligent agents acting in a changing environment must be based, among other components, on some form of temporal reasoning system. If we want this system to be well-founded and its properties formally studied it must be based upon a formal theory of time. Time theories are based in some time primitive unit (instants, intervals, etc.) and determine both the expressiveness of the language as well as the completeness of the reasoning system.

As a matter of fact, time has been recognized as a fundamental notion in reasoning about changing domains and many frameworks for reasoning about change and action are built upon a temporal representation [McDermott, 1982; Allen, 1984; Kowalski and Sergot, 1986; Dean and McDermott, 1987; Williams, 1986; Shoham, 1987; Kuipers, 1988; Forbus, 1989; Galton, 1990; Schwalb *et al.*, 1994; Pinto, 1994; Miller and Shanahan, 1994; Koubarakis, 1994a; Iwasaki *et al.*, 1995; Fusaoka, 1996; Bacchus and Kabanza, 1996; Vila and Reichgelt, 1996]. In these frameworks, the domain at hand is formalized by expressing how propositions are true or false throughout time. Commonly there is a distinction between propositions describing the state of the world (*fluents*) and those representing occurrences that happen in the world and may change its state (*events*). Examples of fluents are "the light is on", "the ball is moving at speed v", and "the battery charge is increasing", and examples of events are "turn the light off", "kick the ball" and "the controller sent a signal to the relay".

A temporal representation has two basic components: (i) a theory of time, and (ii) a theory of temporal incidence. The *theory of time* defines the structure of the primitive time units (e.g. *transitivity* of the ordering relation over instants). The *theory of temporal incidence* defines the domain-independent properties for the truth-value of fluents and events throughout time (e.g. if a fluent is true during a period it must be true during the instants within that period).

Time and temporal incidence are issues that traditionally attracted the interest of areas such as philosophy [Whitehead, 1919; Russell, 1956; Hamblin, 1972; Kamp, 1979; Newton-Smith, 1980], *physics* and *linguistics* [Kenny, 1963; Vendler, 1967; Davidson, 1967; Jackendoff, 1976; Mourelatos, 1978; Dowty, 1979; Allen, 1984; Bach, 1986]. But, why are time and temporal incidence theories important for *automated temporal reasoning* ? They have impact on major properties of the system for answering queries with some temporal component such as "Was the light open when the controller sent the signal to the relay ? "At what times has been the light on and the door open simultaneously ?". Consider, for instance, *Constraint Logic Programming* [Jaffar and Maher, 1994] with temporal constraints [Hrycej, 1993; Brzoska, 1993; Früehwirth, 1996; Schwalb *et al.*, 1996]: the theory of time characterizes the *constraint domain* which determines the properties for the constraint solving procedure; the theory of temporal incidence has impact on the completeness of the overall proof procedure.

From an efficiency point of view, these theories enable inferring implicit, redundant or inconsistent temporal information that can be used to expedite the query answering search. For example, consider two tasks competing on a single resource. The theory of temporal incidence allows us to infer that the time periods during which these tasks utilize the common resource do not overlap. In turn it enables some temporal constraint propagation.

A lot of research in *Artificial Intelligence* has focussed on formalizing time and temporal incidence [van Benthem, 1983; Allen and Hayes, 1985; Ladkin, 1987; Shoham, 1987; Tsang, 1987a; Allen and Hayes, 1989; Galton, 1990; Lin, 1991; Vila, 1994]; however it turns out not to be a simple task. First because a theory of time must naturally reflect commonsense intuitions about time, and sometime these intuitions have to do with instantaneous phenomena while others more naturally concern durative phenomena. Second because they must be adequate to describe events that happen and change the values of fluents without contradicting those intuitions. For instance, determining the value of a fluent at the time it changes has been a controversial issue (the so-called *dividing instant problem*). Moreover, we may face different types of change: discrete, continuous, etc. Real world domains usually involve parameters whose change is modelled as continuous and others whose change is viewed as discrete. A system with both types of change is called a *hybrid system*, and its model a *hybrid model* [Iwasaki *et al.*, 1995]. For example, consider an electro-mechanical battery charger: re-charging a battery can be viewed as a continuous change whereas closing a relay would be better regarded as discrete*. When both types of change happen concurrently, describing what is true and false becomes more difficult.

This chapter presents these challenges more precisely, presents the most relevant theories of time proposed in Artificial Intelligence and discusses the successes and failures with respect to these representational issues. Also, the chapter presents a brief overview of temporal incidence theories and proposes a theory of *temporal incidence* defined upon a theory of instants and periods whose key insight is the distinction between continuous and discrete fluents.

1.2 Requirements and Problems

In this section we identify some problematic issues that must be addressed when formalizing time and temporal incidence:

Instantaneous Events A dynamic system often involves events that are naturally modelled as instantaneous. Some prototypical examples are "turn off the light", "shoot the gun", "start moving", "sign a contract". Representing such events can cause some problems, especially when *sequences* of them occur in presence of continuous change (Section 1.11 discusses this in detail).

Fluents that Hold at an Instant We often talk about the value of fluents at certain instants (e.g. "the temperature of patient X at 9:00 was high", "Was the light red when the car hit John?"). Also, modelling *continuous change* requires having fluents that may hold at isolated instants. A simple, representative example is the parameter *speed* (v) of a ball tossed upwards in what we call the *Tossed Ball Scenario* (TBS) (see Figure 1.1). The ball moves up during

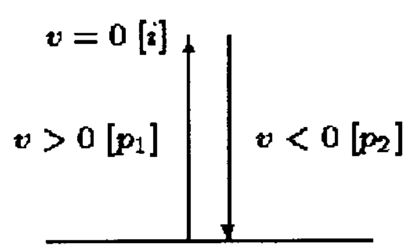

Figure 1.1: The Tossed Ball Scenario (TBS).

p_1 and down during p_2. By continuity of v, there must be a time piece "in between" p_1 and p_2 where the $v = 0$. Since the ball cannot be stopped in the aïr for a while, such a time piece can only be durationless. However, being able to talk about the truth value of fluents at durationless times may lead to the problem described next.

*Hybrid models are important because many daily used electro-mechanical devices are suitably modelled as such.

The Dividing Instant Problem (DIP) Let us assume that time is composed of both instants and periods, and we need to determine the truth-value of a fluent f (e.g. "the light is on") at an instant i, given that f is true on a period p_1 ending at i and is false at a period p_2 beginning at i (see Figure 1.2) [Hamblin, 1972; van Benthem, 1983; Allen, 1984; Galton, 1990]. The problem is a matter of logical consistency: if periods are closed then f

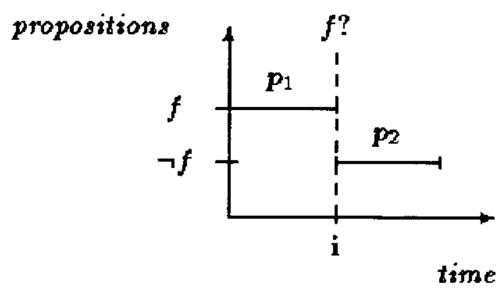

Figure 1.2: The Dividing Instant Problem.

and $\neg f$ are both true at i which is inconsistent; if they are open we might have a "truth gap" at i; finally, the choices open/closed and closed/open are judged to be artificial [Allen, 1984; Galton, 1990].

Non-Instantaneous Holding of Fluents Formalizing the properties of temporal incidence for non-instantaneous fluents can be problematic. There are two major classes of them. The first class are properties about the holding of a fluent at related times. Instances of this class are:

- *Homogeneity*: If a fluent is true on a piece of time it must hold on any sub-time [Allen, 1984; Galton, 1990].
- *Concatenability**: If a fluent is true on two consecutive pieces of time it must be true on the piece of time obtained by concatenating them. Notice that there may be different views for the meaning of *consecutive*.

The second class are properties relating the holding of contradictory fluents at related times. It includes:

- *Non-holding*: If a fluent is not true on a piece of time, there must be a sub-time where its negation holds [Allen, 1984; Galton, 1990].
- *Disjointness*: Any two periods such that a fluent is true on one and its negation is true on the other, must be disjoint. There may be different views for the meaning of *disjoint* too.

*As opposed to homogeneity, very little attention has been paid to concatenability. Nevertheless, it is a semantical issue that also may have some computational benefits since it allows for a compact representation of a fluent holding on numerous, overlapping periods.

Non-Atomic Fluents The holding of negation, conjunction and disjunction of atomic fluents can be a non-trivial issue [Shoham, 1987; Galton, 1990].

After presenting our theories of time and temporal incidence we shall revisit all these issues. Next, we'll discuss theories of time. Starting from different intuitions, several theories of time have been proposed*. They can be classified into three classes according to whether the primitive time units are instants, periods or events†.

1.3 Instant-based Theories

Instants are defined as a durationless pieces of time. An alternative, more precise definition identifies instants to pieces of time whose begin and end are not distinct. In *physics* it has been standard practice to model time as an unbounded continuum of instants [Newton, 1936] structured as the real numbers set. Instant-based theories have been used in several AI systems [McCarthy and Hayes, 1969; Bruce, 1972; Kahn and Gorry, 1977; McDermott, 1982; Shoham, 1987].

An instant-based theory is defined on a structure $\langle \mathcal{I}, \prec \rangle$ with a number of properties:

- *Ordering.* The *minimum* properties for instants ordering are those of a *partially ordered set* (POSET):

  IRREF $\neg(i \prec i)$
  ASYM $i \prec i' \Rightarrow \neg(i' \prec i)$
  TRANS $i \prec i' \wedge i' \prec i'' \Rightarrow i \prec i''$

  Additionally, one may want to impose some *"linearity"* of time. If it is imposed only towards the past

  Left-LIN $i' \prec i \wedge i'' \prec i \Rightarrow i' \prec i'' \vee i' = i'' \vee i'' \prec i'$

  we obtain a branching time structure towards the future [McDermott, 1982] that might be appropriate to model various possible futures. Otherwise, we can impose linearity on both directions

  LIN $i \prec i' \vee i = i' \vee i' \prec i$

  forcing instants to be arranged in a single line.

- *Boundedness.* The instants ordered structure may have beginning and end or, conversely, have no end towards past and future. The later is caputured by the following axiom:

  SUCC $\forall i \exists i' \, (i' \prec i') \quad \forall i \exists i' \, (i \prec i)$

  The idea of unbounded time corresponds to a more general view whereas bounded time can be more appropriate in some particular contexts.

- *Dense/Discrete.* Denseness forces to have an instant between any two instants

  DENS $\forall i, i' \, (i \prec i' \Rightarrow \exists i'' \, (i \prec i'' \prec i'))$

*Some intuitions as well as pointers to some relevant works are from [Lin, 1991].

†Note that the word event is overloaded: in the context of time theories, *event* denotes any temporal proposition, thus it includes both events and fluents as defined in the introduction section.

This property may be important to model continuous change. A consequence is that any stretch of time can be decomposed into sub-times which can have interest in planning where tasks are usually decomposed into subtasks. Another consequence is that one cannot refer to the previous and next instants. Discreteness is enforced by the following axiom:

$$\textbf{DISC} \quad \forall i, i' \ (i < i' \Rightarrow \exists i'' \ (i \prec i'' \land \neg \exists i''' \ i \prec i''' \prec i''))$$
$$\forall i, i' \ (i < i' \Rightarrow \exists i'' \ (i'' \prec i' \land \neg \exists i''' \ i'' \prec i''' \prec i'))$$

As it happens, each finite strict partial order is also discrete.

The above principles are sufficient to achieve a certain level of completeness. Two theories are known to be *syntactically complete* [van Benthem, 1983]: the *unbounded dense linear* theory axiomatized by

IRREF, TRANS, LIN, SUCC, DENS

and the *unbounded discrete linear* theory axiomatized by

IRREF, TRANS, LIN, SUCC, DENS

The alternatives to instant-based theories are theories built upon primitive units more related to our experience than instants. One alternative are period-based theories since "periods are usually associated with events that take time". A further step in that direction is directly developing theories based on events.

1.4 Period-based Theories

Period-based theories interested researchers in philosophy, linguistics and AI [Walker, 1948; Hamblin, 1972; Newton-Smith, 1980; Dowty, 1979; Allen, 1984; Allen and Hayes, 1989]. For instance, Allen proposes a theory exclusively based on periods and the 13 relations between pairs of them shown in Figure 1.3. This theory has been analysed and reformulated

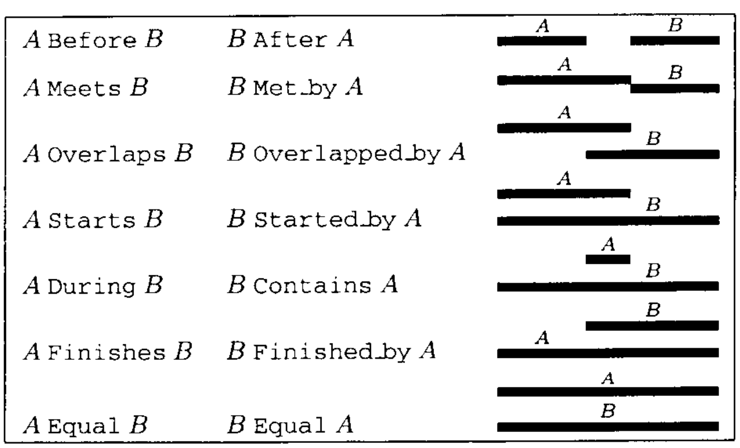

Figure 1.3: The 13 relations between temporal intervals.

in terms of the sole relation `Meets` by Allen & Hayes and Ladkin.

1.4.1 Allen's Period Theory

Allen [Allen, 1983] takes an initial structure $\langle \mathcal{P}, \mathbf{AR} \rangle$ where $\mathbf{AR}$ denotes the set of the 13 primitive period relations that correspond to every possible simple qualitative relationship that may exist between two intervals (see Figure 1.3). The behaviour or $\mathbf{AR}$ is informally specified by the following axiom schemas [Allen, 1984]:

1. Given any period, there exists another period related to it by each relationship in $\mathbf{AR}$.
2. The relationships in $\mathbf{AR}$ are mutually exclusive.
3. The relationships have a transitive behaviour, e.g. if p_1 `Before` p_2 and p_2 `Meets` p_3 then p_1 `Before` p_3.

We shall henceforth refer to this theory as $\mathcal{A}$. We propose the following formalization:

$\mathbf{A_1}$ $\forall P \in \mathcal{P}, R \in \mathbf{AR}\ \exists P'(R(P, P'))$
$\mathbf{A_2}$ $\forall P, P' \in \mathcal{P}, R \in \mathbf{AR}\ \forall R' \in \mathbf{AR} - R\ (R(P, P') \Rightarrow \neg R'(P, P'))$
$\mathbf{A_3}$ Allen's transitive table [Allen, 1983]

1.4.2 Allen & Hayes's Period Theory

Allen's theory is re-defined in terms of the `Meets` relation in [Allen and Hayes, 1985]* ($\|$ denotes `Meets` and $\oplus$ the or-exclusive logical connective):

$\mathbf{AH_1}$ $\forall p, q, r, s\ (p\|q \wedge p\|s \wedge r\|q \Rightarrow r\|s)$
$\mathbf{AH_2}$ $\forall p, q, r, s\ (p\|q \wedge r\|s \Rightarrow \exists t\ (p\|t\|s) \oplus p\|s \oplus \exists t\ (r\|t\|q))$
$\mathbf{AH_3}$ $\forall p\ \exists q, r\ q\|p\|r$
$\mathbf{AH_4}$ $\forall p, q, r, s\ (p\|q\|s \wedge p\|r\|s \Rightarrow q = r)$
$\mathbf{AH_5}$ $\forall p, q\ (p\|q \Rightarrow \exists r, s, t\ (r\|p\|q\|s \wedge r\|t\|s))$

We call it $\mathcal{AH}$. It has been also by Ladkin [Ladkin, 1987] who (i) claims that axiom $\mathbf{AH_5}$ is redundant (it is, in fact, not true [Galton, 1996a]), (ii) relates $\mathcal{AH}$ to other period-based theories of time, (iii) completely characterizes its models, and (iv) proposes a completion to obtain an axiomatization of the theory of rational intervals which is proved to be *countably categorical* [van Benthem, 1983] and, therefore, complete. The completion is obtained by adding the following *denseness* axiom $\mathbf{N_1}$:

$$\mathbf{N_1}\quad \forall p, q, r, s\ \exists x, y\ \text{Pointless}(p, q, x, y) \wedge \text{Pointless}(x, y, r, s)$$

where pointless is defined as follows:

Definition 1.4.1. ("pointless") *Given the intervals p, q, r, s, Pointless(p, q, r, s) iff*

$$\exists u, v, w\ [p, q] \sim_{\mathcal{I}} [u, v] \wedge [r, s] \sim_{\mathcal{I}} [v, w]$$

and $\sim_{\mathcal{I}}$ denotes the period relation of "having the same meeting point".

*Notice that this re-definition is not equivalent to Allen's initial axiomatics. Both are analyzed below in deeper detail.

1.4.3 Relation between $\mathcal{A}$ and $\mathcal{AH}$

Surprisingly, $\mathcal{A}$ and $\mathcal{AH}$ are not that similar. On the one hand $\mathcal{A}$ accepts models which do not fit in $\mathcal{AH}$.

Lemma 1.4.1. $\mathcal{A} \not\subset \mathcal{AH}$

Proof. We may find several counter-examples of models of $\mathcal{A}$ which are not models of $\mathcal{AH}$.

- Counter-example 1 (axiom $\mathbf{AH_4}$): Let us take M as the set of non-empty open intervals on Q plus a second "copy" of an arbitrary interval, let us take for instance (1,2)', which happens to be different from its "original" (1,2) though it keeps every relation that (1,2) has with any other interval. Thus, M is a model of $\mathcal{A}$ but it is not a model of $\mathcal{AH}$ since it does not satisfy $\mathbf{AH_4}$.
- Counter-example 2 (axiom $\mathbf{AH_2}$): Let us take M as two copies of the set of non-empty open intervals on Q, being the intervals in one copy not related by any relationship the intervals of the other copy. M is a model of $\mathcal{A}$ but linear ordering is not satisfied since any interval in one copy is not ordered with respect to any interval of the other one.

□

On the other hand, $\mathcal{A}$ is stronger than $\mathcal{AH}$ since it imposes *denseness* Axiom $\mathbf{A_1}$ in $\mathcal{A}$ guarantees decomposability which in period-based theories corresponds to denseness. Contrarily, $\mathcal{AH}$ has been designed to be weak enough to embrace both discrete and dense models.

Lemma 1.4.2. $\mathcal{AH} \not\subset \mathcal{A}$

Proof. As a counter-example take any discrete model –for instance the intervals formed over the set of integer numbers. It is a model of $\mathcal{AH}$ [Allen and Hayes, 1985] but is ruled out by $\mathbf{A_1}$. For instance, there is no period which `Overlaps` the period [1,2]. □

1.4.4 Revised $\mathcal{A}$

A more accurate look at $\mathcal{A}$ reveals that nothing accounts for the intuition that "periods are all contained in a single time dimension"*. Our refinement of $\mathcal{A}$ is based on adding an axiom schema with such a role:

Definition 1.4.2. ($\mathcal{A}'$) *$\mathcal{A}'$'axiomatization is $\mathcal{A}$'s plus the following additional axiom schema:*

$$\mathbf{A'_1} \quad \forall P, P' \, \exists R \in \mathbf{AR} \; R(P, P')$$

This refinement produces a rather remarkable change in the accepted models. We demonstrate it by analyzing how this theory relates to Allen & Hayes's theory. We use the definitions of period relations in terms of the single relation `Meets` given in [Allen and Hayes, 1985].

Lemma 1.4.3. $\mathcal{A}' \subseteq \mathcal{AH}$

*This is an idea which Allen seems to be in sympathy with since he explicitly refuses alternative structures like McDermott's *branching time* construction.

Proof.

AH_1. Suppose $\neg r\|s$.

Then, by $\mathbf{A'_1}$, there must be a period relation R such that $R \neq \|$ and rRs. It is easy to check that whatever relation we take, it stands in contradiction with the period transitivity table and/or with axiom A_2. For example let us assume that (r `Before` s), which is equivalent to (s `After` r). Using the period constraints in the premise and our assumption, we have (p `Meets` s `After` r `Meets` q). By successively applying Allen's transition table we get

(p `After Met_by Overlaps Overlapped_by During Contains Start Started_by Finishes Finished_by Equal` q)

which is inconsistent with $p\|q$ in the premise.

AH_2. From the hypothesis (a) $p\|q \wedge r\|s$ we want to show that one of the following alternatives exclusively hold: (b) $p\|s$, (c) p `Before` s, (d) r `Before` q –according to the definition of `Before`. By A'_1 we know that one of the 13 period relations must hold between p and s. Since (b) and (c) are mutually exclusive (by A_2) we only need to prove the following three statements:

1. (a) $\wedge \neg(p\|s) \wedge \neg(p$ `Before` $s) \Rightarrow r$ `Before` q
2. (a) $\wedge\, p\|s \Rightarrow \neg(r$ `Before` $q)$
3. (a) $\wedge\, p$ `Before` $s \Rightarrow \neg(r$ `Before` $q)$

1. Let us assume $\neg(p\|s)$, $\neg(p$ `Before` $s)$ and $\neg(r$ `Before` $q)$. By $\mathbf{A'_1}$ and $\mathbf{A_2}$ one of the other 12 period relations holds. If $r\|q$ then from $r\|q \wedge r\|s$ it follows –by applying the transitivity table– that $q(= \vee$ `Starts` $\vee$ `Started_by`$)s)$, which combined with $p\|q$ by transitivity and conjunction gives $p\|s$ which stands in contradiction with our starting assumption. For any other case it holds that $\exists t'\ (p\|t'\|s)$ (this can be checked by revising the definition of the remaining 12 relations in terms of $\|$) that is equivalent to p `Before` s which is contradictory with the hypothesis $\neg(p$ `Before` $s)$. **2** and **3** are shown by assuming r `Before` q and standing in contradiction by consecutively applying the transitivity table a number of times.

AH_3. is trivially derived from $\mathbf{A_1}$.

AH_4. (By $\mathbf{A_1}$) q and r must be related by some period relation, namely R. Suppose that R is not $=$. Then using the definitions of each period relation one stands in contradiction with one from $\{p\|q, q\|s, p\|r, r\|s\}$.

□

Lemma 1.4.4. $\mathcal{AH} \vdash \mathbf{A'_1}, \mathbf{A_2}, \mathbf{A_3}$

Proof. $\mathbf{A'_1}$ and $\mathbf{A_2}$. For any two periods I, J we have (by $\mathbf{AH_3}$) that $\exists a, b, a', b'\ a\|I\|b$ and $a'\|J\|b'$. By $\mathbf{AH_2}$ we have

1. $\exists t_1\ a\|t_1\|J \oplus a\|J \oplus \exists t'_1\ a'\|t'_1\|I$
2. $\exists t_2\ a\|t_2\|b' \oplus a\|b' \oplus \exists t'_1\ J\|t'_2\|I$

3. $\exists t_3\ I \| t_3 \| J \oplus I \| J \oplus \exists t'_3\ a' \| t'_3 \| b$
4. $\exists t_4\ I \| t_4 \| b' \oplus I \| b' \oplus \exists t'_4\ J \| t'_4 \| b$

We may combine these different mutually exclusive choices. We obtain 3^4 *a priori* possible alternatives, but not every combination is feasible. We must use the remaining $\mathcal{AH}$ axioms to discard disallowed possibilities. For instance, let us take the first choice in (1): $\exists t_1\ a \| t_1 \| J$. (By $\mathbf{AH_5}$) there exists t_2 and b such that $t_2 = t_1 + J$ and $a \| t_2 \| b'$ which is the first choice in (2) and, due to the $\oplus$, the sole one allowed. Thus, we have (1.2) $a \| t_1 \| J \| b'$. This is compatible with every choice in (3) each of which in turn is compatible with some in (4). Finally we get (1.2) with each of the following relations:

$$\begin{array}{l} I \| t_3 \| J \\ I \| J \\ a' \| t'_3 \| b \wedge I \| t_4 \| b' \\ a' \| t'_3 \| b \wedge I \| b' \\ a' \| t'_3 \| b \wedge J \| t'_4 \| b \end{array}$$

By exhaustively applying this process we obtain an exclusive disjunctive formula where each disjunctive element exactly matches the definition in terms of the single relation `Meets` of one of the 13 period relations. Thus, we prove the mutual exclusivity of period relations ($\mathbf{A_2}$). Their existence ($\mathbf{A'_1}$) is also guaranteed since every auxiliary period has been introduced either through $\mathbf{AH_3}$ or $\mathbf{AH_2}$ and period addition used at some stages is supported by $\mathbf{AH_5}$.

$\mathbf{A_3}$ is easy though a bit tedious to prove by using the transitivity table [Allen and Hayes, 1985]. □

Regarding $\mathbf{A_1}$, the relationships `Meets` and `Before` (and their inverses) can be easily derived from one and two applications respectively of axiom $\mathbf{AH_3}$ towards the future (towards the past), but this is not the case for the remaining ones. To derive them we would require the *denseness* axiom $\mathbf{N_1}$:

Lemma 1.4.5. $\mathcal{AH}$, $\mathbf{N_1} \vdash \mathbf{A_1}$

Proof. By using the denseness axiom one may always find the appropriate endpoints which define the period P' that satisfies $R(P, P')$. □

Theorem 1.4.1. $Th(\mathcal{AH}, \mathbf{N_1}) \equiv \mathcal{A}'$

Proof. Given lemmas 1.4.3, 1.4.4 and 1.4.5, it suffices to prove that $\mathcal{A}' \vdash \mathbf{N_1}$ which is straight forward by applying $\mathbf{A_1}$ with the relationship `Started_by` on the period bounded by the initial points. □

1.4.5 Extending $\mathcal{A}$ with Instants

Allen's theory can be properly extended with instants by implementing the idea of instants as period the meeting points. Now the ontology is made of non-empty sets of instants and periods in a structure such as $\langle \mathcal{I}, \mathcal{P}, \mathtt{Limits}, \mathbf{AR} \rangle$. where `Limits` is a instant-period relation. $\mathcal{A}'$ axioms are extended with the following:

$$\begin{array}{ll} \mathbf{IM_1} & \forall P, P'\ (P \| P' \Rightarrow \exists i\ (\mathtt{Limits}(i, P) \wedge \mathtt{Limits}(i, P'))) \\ \mathbf{IM_2} & \forall i \exists P, P'\ (P \| P' \wedge \mathtt{Limits}(i, P) \wedge \mathtt{Limits}(i, P')) \end{array}$$

1.5 Events

Event-based theories are motivated by the following intuition:

> "time is no more than the totality of temporal relations between the events and processes which constitute the history of our world. Then defining time is a question about the actual relations between these events and processes."

This approach mostly interested philosophers such as [Russell, 1956; Whitehead, 1919; Kamp, 1979] and a few AI people [Tsang, 1987a].

Time is defined as the structure $\langle E, \prec, O\rangle$ where E is a non-empty set of events, $\prec$ is a precedence relation and O is a overlapping relation. The axioms of the theory, called $\mathcal{E}$, are as follows [Kamp, 1979]:

$\mathbf{E}_1$	$e \prec e' \Rightarrow \neg(e' \prec e)$	**NO SYM**($\prec$)
$\mathbf{E}_2$	$e \prec e' \wedge e' \prec e'' \Rightarrow e \prec e''$	**TRANS**($\prec$)
$\mathbf{E}_3$	$eOe' \Rightarrow e'Oe$	**SYM**(O)
$\mathbf{E}_4$	eOe	**REFL**(O)
$\mathbf{E}_5$	$e \prec e' \Rightarrow \neg(eOe')$	**SEP**
$\mathbf{E}_6$	$e \prec e' \wedge e'Oe'' \wedge e'' \prec e''' \Rightarrow e \prec e'''$	**TRANS**($\prec, O$)
$\mathbf{E}_7$	$e \prec e' \vee eOe' \vee e' \prec e$	**LIN**

$\mathbf{E}_1$ and $\mathbf{E}_2$ state partial order for $\prec$. O is reflexive and symmetric ($\mathbf{E}_3$ and $\mathbf{E}_4$) but not transitive. $\mathbf{E}_5$ to $\mathbf{E}_7$ state relations between $\prec$ and O: they are mutually exclusive, exhibit a sort of transitivity and establish a linear ordering over events. The properties of $\mathcal{E}$ have extensively studied in [Kamp, 1979] and later in [Tsang, 1987a; Lin, 1991].

Since period-based theories are based on the intuition that periods are the stretches of time occupied by events, the properties of periods and events are very similar. However, event-based theories are conceptually different in the sense that events are not "pure time entities" but the occurrences themselves. In other words, a clear dissociation between occurrences and their times of occurrence is not established.

1.5.1 Relation between $\mathcal{E}$ and $\mathcal{AH}$

Since events happen on periods, period-based theories and event-based are very similar. The main distinction is that two events that happen at the same are not necessarily the same events. If we take Allen's relations as the reference, $e \prec e'$ is equivalent to e `Before` $\vee$ `Meets` e' and O is equivalent to $\neg(e \prec e') \wedge \neg(e' \prec e)$. According to Tsang [Tsang, 1987b], $\mathcal{E}$ can be extended to obtain a theory equivalent to $\mathcal{AH}$ by adding the following axioms (where ! and $\cap$ denote period intersection and union respectively):

$\mathbf{E}_8$	$\exists e'\ e' \prec e \wedge \neg(\exists x\ (e' \prec x \wedge x \prec e)$
$\mathbf{E}_9$	$e \prec e' \wedge e' \prec e'' \Rightarrow e \prec e''$
$\mathbf{E}_{10}$	$eOe' \Rightarrow e!e'$
$\mathbf{E}_{11}$	$\exists e''\ e'' = e \cup e$

$\mathbf{E}_8$ and $\mathbf{E}_9$ are needed to guarantee period meeting unboundedness ($\mathbf{AH}_e$ axiom), $\mathbf{E}_{10}$ is needed to derive axiom $\mathbf{AH}_1$, and $\mathbf{E}_{11}$ is needed to derive $\mathbf{AH}_2$ and $\mathbf{AH}_5$. Given the theory $\mathbf{E}_1 \div \mathbf{E}_{11}$, called $\mathcal{E}^*$ by Tsang, we have:

Theorem 1.5.1. $\mathcal{E} \equiv \mathcal{AH}$

1.6 Analysing the Time Theories

Let us now analyze these alternatives from the philosophical, notational, computational technical viewpoints:

Philosophical Event-based theories clearly are the most attractive from a philosophical point of view as they are directly based on perceived phenomena. However, as noticed by [Lin, 1991], are instant and period -based theories that have gained wide acceptance in AI:

> "The reason for this seems to be that people are accustomed to think that time is an "independent" entity where events take place."

Period-based theories are appealing since they make this distinction while they preserve the starting intuition that our direct experience is with events that take time. In fact, period-based theories capture the most relevant relations between events.

Several philosophical arguments have been put forward against instants such as "Our direct experience is with phenomena that take time", "It takes to many instants to make up a durable experience ?" [Hamblin, 1972; Kamp, 1979], "the point-based, continuous model ... they start with is too rich" [Allen and Hayes, 1989], "... (instant-based models) permit the description of phenomenally impossible states of affairs" [Hamblin, 1972]. One argument in favor of instants must be mentioned though. As with durative events, we seem to have mental experience with instantaneous phenomena as well. It is reflected by many references in natural language expressions (see the examples given in Section 1.2 such as "the time I started moving" or "the temperature of the patient at 9:00"). The claim is not about the existence of instantaneous phenomena but about the fact that we are accustomed to think about the notion of instantaneous.

Notational vs. Computational Two types of expressions must be considered: *temporal assertions* and *temporal relations*. We next discuss both, being the second the one that has some computational consequences.

Expressing temporal assertions: If our ontology is provided with instants, expressing instantaneous events and instantaneous holding of fluents is straight forward. This becomes a difficult issue in period-based theories because instants cannot be represented as very short periods (as proposed in [Allen, 1984]) because a short period does not *divide* a period into two meeting periods. For example, if i in Figure 1.2 is modelled as a short period then p_1 does not meet p_2. The same applies to expressing instantaneous fluents (like in the TBS). The proposal of modelling instants as *indivisible* periods, called *moments* [Allen and Hayes, 1985], does not work either for the same reason. The option of representing instants as zero duration periods is problematic too because Allen's transitive table needs to be transformed into a much weaker table, otherwise the distinction between the period relations that are not `Before`, `Equal` and `After` becomes meaningless [Schwalb, 1996].

A sounder, more sophisticated technique is defining instants as sets of periods. Mathematicians proposed a number of set-theoretic constructions of points from intervals such as (i) an instant is identified with the maximal set of intervals that have a non-empty intersection [Whitehead, 1919] (called *nests*), or (ii) an instant is defined as the equivalence class of pairs of meeting intervals that meet "at the same place"*. Now, the points are (i) whether

* Attributed to Bolzano.

there is any simpler, more natural alternative to this class of instants, and (ii) whether these instants can be used to talk about the occurrence of events and holding of fluents. We discuss both in forthcoming sections.

Expressing temporal relations: Any temporal relation between two periods can be specified in terms of instant relations between the endpoints of the periods. In particular, every basic period relation in Figure 1.3 can be specified by a conjunction of binary instant relations. Furthermore, some temporal relations are more efficiently expressed in terms of relations between instant than between periods. For example

$$p_1 \texttt{Before Meets Overlaps Finished\_by Contains}\, p_2 \equiv \texttt{begin}(p_1) \prec \texttt{begin}(p_2)$$

However, the instant-based notation may be less efficient because of several reasons. First, a binary relation may become higher arity relation ($n \geq 2$) when stated in terms of instants. For example, given the periods p_1 and p_2,

$$p_1 \texttt{ Before After } p_2 \equiv \texttt{end}(p_1) \prec \texttt{begin}(p_2) \ \vee \ \texttt{end}(p_2) \prec \texttt{begin}(p_1)$$

Second, instant-based expressions are sometimes more cumbersome. Third, as the number of events grows, the number of conjunctive combinations grows exponentially (as noticed by Tsang [Tsang, 1987a]). For example,

$$p_1 \texttt{ Overlaps Finishes } p_2 \quad \wedge \quad p_2 \texttt{ Meets Overlaps } p_3$$

is represented as the disjunction of each element of the cartesian set expressed in terms of instant relations, namely

$$\begin{array}{lll}
\texttt{begin}(p_1) \prec \texttt{begin}(p_2) \wedge \texttt{end}(p_1) \prec \texttt{end}(p_2) & \wedge & \texttt{end}(p_2) = \texttt{begin}(p_3) \\
\vee & & \\
\texttt{begin}(p_1) \prec \texttt{begin}(p_2) \wedge \texttt{end}(p_1) \prec \texttt{end}(p_2) & \wedge & \texttt{begin}(p_2) \prec \texttt{begin}(p_3) \wedge \texttt{end}(p_2) \prec \texttt{end}(p_3) \\
\vee & & \\
\texttt{begin}(p_2) \prec \texttt{begin}(p_1) \wedge \texttt{end}(p_1) = \texttt{end}(p_2) & \wedge & \texttt{end}(p_2) = \texttt{begin}(p_3) \\
\vee & & \\
\texttt{begin}(p_2) \prec \texttt{begin}(p_1) \wedge \texttt{end}(p_1) = \texttt{end}(p_2) & \wedge & \texttt{begin}(p_2) \prec \texttt{begin}(p_3) \wedge \texttt{end}(p_2) \prec \texttt{end}(p_3)
\end{array}$$

expressed in terms of instant relations.

Having a compact, low order expression of temporal relations is not only a notational issue but it also has some impact on the computational cost of reasoning with them.

Technical Instant-based axiomatizations apparently allow for a better understanding and control of the properties we want for time. There is no general agreement on this point. The interested reader may compare the theories provided in the appendices.

1.7 Instants and Periods

According to the above analysis, it seems that an interesting approach would be a theory of time based on both instants and periods. It would enjoy the following advantages:

- *Natural expression*: Instants are used to express instantaneous events and fluents, and periods to express the durable ones.

- *Efficient notation and computation*: Either instant or period relations, according to what is more efficient, can be used to express the temporal relations at hand.

To define such a theory two alternatives have been explored: (i) starting with a concerted instant-period ontology, or (ii) defining instants from periods. Semantical arguments, such as the DIP, led a number of researchers to follow the second alternative. In Section 1.6 we have seen that simple techniques of representing an instant as a period do not work, and only more complex mathematical constructions do. The interest of deriving instants from periods is unclear since (as noted by Allen & Hayes [Allen and Hayes, 1989]) "we may end up in the same place" as if we start with an instants structure, whereas both the axioms and the instants construction are clearly less intuitive. Indeed we show (Section 1.7.2) that Allen & Hayes's theory [Allen and Hayes, 1985; Allen and Hayes, 1989] together with "derived instants" admits the same models than our instant-period theory of time.

Therefore, if we want both instants and periods its preferable to take the route of starting with both as ontological primitives of our time model. To our knowledge, the only proposal in this direction is Galton's theory of time which we discuss below and elsewhere in this collection.

1.7.1 Galton's Instants and Periods Theory

In Galton's theory [Galton, 1990] neither periods are a set-theoretic construction from instants nor vice versa. Both have the same ontological status as a primitive. The underlying intuition is

> "...there being an instant at the point where two periods meet".

Time is defined as the structure $\langle I, P, \texttt{Within}, \texttt{Limits}, \text{Allen's relations} \rangle$ where I and P are non-empty sets of instants and periods respectively, `Within` and `Limits` are instant-period relations with the obvious meaning. In addition to the 13 Allen's period relations, the period relation `In` is defined as the disjunction of `During`, `Starts` and `Finishes`. The set of axioms (we call it $\mathcal{G}$) is as follows (i denotes an instant and p, q, r periods):

$$\begin{array}{ll} \mathbf{G_1} & \forall p \exists i\ \texttt{Within}(i,p) \\ \mathbf{G_2} & \texttt{Within}(i,p) \wedge \texttt{In}(p,q) \Rightarrow \texttt{Within}(i,q) \\ \mathbf{G_3} & \texttt{Within}(i,p) \wedge \texttt{Within}(i,q) \Rightarrow \exists r\ (\texttt{In}(r,p) \wedge \texttt{In}(r,q)) \\ \mathbf{G_4} & \texttt{Within}(i,p) \wedge \texttt{Limits}(i,q) \Rightarrow \exists r\ (\texttt{In}(r,p) \wedge \texttt{In}(r,q)) \end{array}$$

We assume that by "...together with the various relations between intervals ..." ([Galton, 1990], p166) Galton means that $\mathcal{A}$ axioms are also included. Neither that nor a characterization of the models of the theory is formally given by Galton. $\mathcal{G}$ avoids DIP-like arguments and turns out to be useful for Galton to prove the relations between his temporal occurrence predicates (such as HOLDS_{on}, HOLDS_{in}, HOLDS_{at}), however they exhibit two major shortcomings. The first regards the relations between instants. They are not sufficient for the needs of a practical reasoner. For instance, the relation `Limits` is not sufficient for distinguishing between the begin and the end of a period. Notice that there is no account for any ordering over instants. Although it is a very basic notion, it is neither explicitly stated nor induced by the period axioms as we discuss next. The second shortcoming regards the connection between instants and periods. Although it is not easy to figure out what are the intended models of $\mathcal{G}$, a careful analysis reveals that the theory is not strong enough to properly connect instants and periods. It is easy to identify examples of counter-intuitive, accepted models:

Example 1.7.1. *Let us take a basic model M composed of an infinite set of periods and Allen's relations satisfying $\mathcal{I}_{\mathcal{A}}$ axioms plus an infinite set of instants which make M satisfy I_1 -for example INT(Q) as periods and Q as instants:*

- *Example model 1: M plus a single instant $i \notin Q$ which limits a certain period P in M and only that one. In particular it does not limit any of those periods that meet or are met by P.*
- *Example model 2: M plus a single instant $i \notin Q$ which limits a certain period P in M and is not within any period. In particular it is not within any of those periods that overlap P.*
- *Example model 3: M plus a single instant $i \notin Q$ which limits a certain period P in M and also is within P.*

The obvious undesirable consequence of $\mathcal{G}$ weakness is that some queries will not receive the expected intuitive answers. In the first example, given the assertions $\texttt{Within}(i, p)$ and $\texttt{Meets}(p, p')$, it is not possible to derive an answer for the query $\texttt{Limits}(i, p')$.

1.7.2 $\mathcal{IP}$

$\mathcal{IP}$ [Vila and Schwalb, 1996] has two sorts of symbols, instants ($\mathcal{I}$) and periods ($\mathcal{P}$) which are formed by two infinite disjoint sets of symbols, and three primitive binary relation symbols $\prec: \mathcal{I} \times \mathcal{I}$ and $\texttt{begin}, \texttt{end} : \mathcal{I} \times \mathcal{P}$.

The first-order axiomatization of $\mathcal{IP}$ theory is as follows:

$\mathbf{IP_1}$	$\neg(i \prec i)$
$\mathbf{IP_2}$	$i \prec i' \Rightarrow \neg(i' \prec i)$
$\mathbf{IP_3}$	$i \prec i' \wedge i' \prec i'' \Rightarrow i \prec i''$
$\mathbf{IP_4}$	$i \prec i' \vee i \prec i' \vee i = i'$
$\mathbf{IP_{5.1}}$	$\exists i' \, (i' \prec i)$
$\mathbf{IP_{5.2}}$	$\exists i' \, (i \prec i')$
$\mathbf{IP_6}$	$\texttt{begin}(i,p) \wedge \texttt{end}(i',p) \Rightarrow i \prec i'$
$\mathbf{IP_{7.1}}$	$\exists i \, \texttt{begin}(i,p)$
$\mathbf{IP_{7.2}}$	$\exists i \, \texttt{end}(i,p)$
$\mathbf{IP_{8.1}}$	$\texttt{begin}(i,p) \wedge \texttt{begin}(i',p) \Rightarrow i = i'$
$\mathbf{IP_{8.2}}$	$\texttt{end}(i,p) \wedge \texttt{end}(i',p) \Rightarrow i = i'$
$\mathbf{IP_9}$	$i \prec i' \Rightarrow \exists p \, (\texttt{begin}(i,p) \wedge \texttt{end}(i',p))$
$\mathbf{IP_{10}}$	$\texttt{begin}(i,p) \wedge \texttt{end}(i',p) \wedge$ $\wedge \, \texttt{begin}(i,p') \wedge \texttt{end}(i',p') \Rightarrow p = p'$

$\mathbf{IP_1}$ – $-\mathbf{IP_4}$ are the conditions for $\prec$ to be a *strict linear order* — namely irreflexive, asymmetric, transitive and linear– relation over the instants*. $\mathbf{IP_5}$ imposes unboundedness on this ordered set. $\mathbf{IP_6}$ orders the extremes of a period. This axiom rules out durationless

*Notice that $\mathbf{IP_1}$ is actually redundant since it can be derived from $\mathbf{IP_2}$. We include it for clarity.

periods which are not necessary since we have instants as a primitive. The pairs of axioms $\mathbf{IP}_{7..}$ and $\mathbf{IP}_{8..}$ formalize the intuition that the beginning and end instants of a period always *exist* and are *unique* respectively. Conversely, axioms $\mathbf{IP}_9$ and $\mathbf{IP}_{10}$ close the connection between instants and periods by ensuring the *existence* and *uniqueness* of a period for a given ordered pair of instants.

Next we characterize the models of $\mathcal{IP}$ and determine its relationships with other theories.

The Models

The models are defined over $\mathcal{IP}$-structures.

Definition 1.7.1. **($\mathcal{IP}$-structure)** *An $\mathcal{IP}$-structure is a tuple $\langle \mathcal{I}_d, \mathcal{P}_d, <_d, \mathtt{begin}_d, \mathtt{end}_d \rangle$ where $\mathcal{I}_d$ and $\mathcal{P}_d$ are sets of instants and periods respectively, $<_d$ is a binary relation on $\mathcal{I}_d$ and $\mathtt{begin}_d, \mathtt{end}_d$ are binary relations on $\mathcal{I}_d, \mathcal{P}_d$.*

Periods are merely viewed as ordered pairs of instants.

Definition 1.7.2. (pairs) *Given a set $\mathcal{S}$ over which an ordering relation $<$ is defined, we note by $\mathtt{pairs}(\mathcal{S})$ the set of $<$-ordered pairs of distinct elements of $\mathcal{S}$: $\mathtt{pairs}(\mathcal{S}) = \{(x,y) \mid x, y \in \mathcal{S} \wedge x < y\}$. Over a set of pairs we define the following relations: (i) first$(x, (y,z)) \stackrel{def}{=} x = y$ and (ii) second$(x, (y,z)) \stackrel{def}{=} x = z$.*

Now we show — similar to Ladkin [Ladkin, 1987] — that the elements and the pairs of an unbounded linear order $\mathcal{S}$ form a model for $\mathcal{IP}$.

Theorem 1.7.1. **(a model)** *Given an infinite set $\mathcal{S}$ and an unbounded strict linear order $<$ on it, the $\mathcal{IP}$-structure $\langle \mathcal{S}, \mathtt{pairs}(\mathcal{S}), <, first, second\rangle$ forms a model of $\mathcal{IP}$.*

Proof. (sketch) It is easy to prove that every axiom of $\mathcal{IP}$ is satisfied if we interpret the instants on the set $\mathcal{S}$, the periods on the set $\mathtt{pairs}(\mathcal{S})$, the ordering as $<$, and begin and end relations as first and second respectively. □

Indeed these are the only models of $\mathcal{IP}$.

Theorem 1.7.2. **(the models)** *Any model $M = \langle \mathcal{I}_d, \mathcal{P}_d, <_d, \mathtt{begin}_d, \mathtt{end}_d \rangle$ of $\mathcal{IP}$ is* isomorphic *to the structure $\langle \mathcal{I}_d, \mathtt{pairs}(\mathcal{I}_d), <_d, first, second\rangle$ where pairs, first and second are defined as above.*

Corollary 1. *Every model of $\mathcal{IP}$ is completely characterized by an infinite set $\mathcal{S}$ and an unbounded strict linear order $<$ on it (not necessarily dense).*

Note that $\mathcal{IP}$ accepts both dense and discrete models of time.

Dense $\mathcal{IP}$

The sub-theory that embraces dense models only, we call it $\mathcal{IP}_{dense}$, is axiomatized by adding the *denseness* axiom over instants:

$$\mathbf{IP}_{11} \quad i \prec i' \Rightarrow \exists i'' \, (i \prec i'' \prec i')$$

Theorem 1.7.3. (dense models) *The models of* $\mathcal{IP}_{dense}$ *are characterized by the set of elements and the set of ordered pairs of distinct elements of an unbounded,* dense, *strict linearly ordered set. Moreover* $\mathcal{IP}_{dense}$ *is a* complete *axiomatization for the theory of rationals and rational intervals, namely* $Th(Q, INT(Q))$.

Relation between $\mathcal{IP}$ and $\mathcal{AH}$

To compare our theory with Allen & Hayes theory (called $\mathcal{AH}$) we use the same technique as Ladkin [Ladkin, 1987]. Instants are derived from periods by first defining the notion of pair of meeting periods, second applying the equivalence relation "having the same meeting point" and, finally, associating an instant to each class. Let us call the resulting theory $\mathcal{I}_{\mathcal{AH}_{\sim\mathcal{I}}}$. Its class of models is the same as $\mathcal{IP}$.

Theorem 1.7.4. $\mathcal{IP} \equiv \mathcal{I}_{\mathcal{AH}_{\sim\mathcal{I}}}$

Theorem 1.7.5. $\mathcal{IP}_{dense} \equiv Th(\mathcal{AH}, N_1)$

Relation between $\mathcal{IP}$ and $\mathcal{G}$

As we discuss in Section 1.6, the instants in $\mathcal{G}$ do not correspond with the places where periods meet. For instance, nothing forces the instant that exists within a period by axiom $\mathbf{G_1}$ to be a place where two periods meet. They do not correspond to the idea of period endpoints either, which is our approach. As a matter of fact, $\mathcal{G}$ is weaker than $\mathcal{IP}_{dense}$.

Theorem 1.7.6. $\mathcal{IP}_{dense} \subset \mathcal{G}$

Proof. (sketch) $\mathcal{G}$ axioms are derived from $\mathbf{IP_{10}}$, linearity, extremes ordering, existence of both instants and periods and density. □

The reason of $\mathcal{G}$ weakness is the loose connection between instants and periods. There is no direct relation between $\mathcal{IP}$ and $\mathcal{G}$: $\mathcal{IP}$ accepts discrete models, whereas $\mathcal{G}$ imposes a sort of denseness by axiom $\mathbf{G_1}$. Characterize the models of $\mathcal{G}$ and its relation with $\mathcal{IP}$ is an open issue.

1.8 Temporal Incidence

For the sake of showing how a time theory is put at work, we complement this survey of theories of time with a section on temporal incidence, we propose a specific temporal incidence theory that works very well with an instant-period time theory, and, finally, we show on an example how the representational issues above a outlined th a classical example from naive physics.

Classical temporal logics in AI [McCarthy and Hayes, 1969; McDermott, 1982; Allen, 1984; Shoham, 1987; Haugh, 1987; Galton, 1991] mostly agree upon the temporal incidence properties that distinguish *fluents* from *events*. Fluents hold homogeneously whereas events event occurrences are anti-homogeneous. Instant-based approaches allow (i) a direct expression of instantaneous events and fluents, and (ii) an easy specification of temporal incidence properties. In McDermott's framework, for example, homogeneity of fluents is specified by

$$\text{Throughout}(T_1, T_2, F) \Leftrightarrow \forall\, T_1 \leq T \leq T_2\ \text{True}(T, F)$$

These advantages are more obvious in Shoham's work where a much richer categorization of proposition types is defined.

In period-based theories temporal incidence specification is more difficult. For example, Allen's axiom for fluent homogeneity is as follows:

$$\textbf{H.2} \quad \textsc{Holds}(F, I) \Leftrightarrow \forall\, I' \in I\; \exists I'' \in I'\; \textsc{Holds}(F, I'')$$

It is not only a cumbersome axiom, but in fact allows some non-intended models*. Moreover Galton [Galton, 1990] proves that axiom **H.2** conflicts with axiom **H.4** which specifies the *holding of negated fluents*. It is not clear how it can be avoided.

Also, period-based theories have problems to express the holding of a fluent at an instant, either because instants cannot be directly referred or because of the DIP. Allen & Hayes's reply to this issue as follows ([Allen and Hayes, 1989], Section 4):

> "We avoid it by resolutely refusing to allow fluents to hold at points".

They propose the following alternative: "One could define a notion of a fluent X being true at a point p by saying that X is true at p just when there is some interval I containing p during which X is true". It is easy to see that it does not work for modelling continuous fluents (consider the $v = 0$ fluent in the TBS). In Section 1.10 we determine the conditions for the DIP to be a problem and we propose a simple approach that satisfies them.

Let us now address the issue of modelling continuous change. There is a general agreement upon the importance of this issue for common-sense reasoning. In spite of it, no previous work formally accounts for the essential temporal incidence differences between holding of discrete and continuous fluents. Galton's work [Galton, 1990] is the only attempt in that direction, up to our knowledge. Fluents are diversified into instantaneous/durable and states of position/states of motion:

> "A *state of position* can hold at isolated instants; if it holds during a period it holds at its limits (e.g. a quantity taking a particular value). ... A *state of motion* cannot hold at isolated instants (e.g. a body being at rest)."

Galton's approach presents two problems:

1. The utility of Galton's new classes of fluents is not clear since more than one class is needed to model a single continuously changing parameter. Let us illustrate it with the TBS. Consider the fluent $f = (v \neq 0)$. It cannot be modelled as a state of position because f holds on both p_1 and p_2 which must contain the limiting instant i where $\neg f$ holds ($v = 0$). A state of motion cannot be used either because it cannot hold at isolated instants: we are not allowed to say that $\neg f$ is true at i.

2. While states of position are *concatenable*, states of motion are not always. It is rather counter-intuitive: it seems that states of position should not be concatenable since the parameter they represent may have a different value at the meeting point. Since it is not the case for states of motion it seems that they should be concatenable. Next we follow this intuition.

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*An instance, due to Shoham, is a model in which time has the structure of real numbers and a property holds only over all its subintervals whose endpoints are rational.

1.9 $\mathcal{CD}$

$\mathcal{CD}$ the theory of temporal incidence initially proposed in [Vila and Schwalb, 1996], is based on the following ideas:

1. *We allow fluents to hold at points.* It allows modelling continuously changing fluents and makes the resulting theory much simpler to define. We discuss why it in fact does not originate any problem.

2. *We distinguish between continuous and discrete fluents.* We diversify fluents according to whether the change on the parameter they model is *continuous* or *discrete*.

To present our approach we assume the standard temporal reified first-order language with equality (as in [McDermott, 1982; Allen, 1984; Galton, 1990]) as underlying language. We propose a temporal representation with the following features:

- *Time theory*: We take $\mathcal{IP}_{dense}$. We define the instant-to-period relations (such as `Within`) and period-to-period relations (such as `Meets`) upon $\prec$, `begin` and `end`.

- *Reified propositions*: Reified propositions are classified into *continuous fluents*, *discrete fluents** and *events*.

- *Temporal Incidence Predicates (TIPs).* We introduce a different TIP for each combination of temporal proposition and temporal primitive (similar to [Kowalski and Sergot, 1986; Galton, 1990]):

$\text{HOLDS}^{\sim}_{on}(f,p)$	$\stackrel{def}{=}$	The continuous fluent f holds throughout the period p
$\text{HOLDS}^{\neg}_{on}(f,p)$	$\stackrel{def}{=}$	The discrete fluent f holds throughout the period p
$\text{HOLDS}^{\sim}_{at}(f,i)$	$\stackrel{def}{=}$	The continuous fluent f holds at the instant i
$\text{HOLDS}^{\neg}_{at}(f,i)$	$\stackrel{def}{=}$	The discrete fluent f holds at the instant i
$\text{OCCURS}_{on}(e,p)$	$\stackrel{def}{=}$	The event e occurs on the period p
$\text{OCCURS}_{at}(e,i)$	$\stackrel{def}{=}$	The event e occurs at the instant i

Terminology. Henceforth we use the following notational shorthands. We use `begin` and `end` in functional form (e.g. $i = \texttt{begin}(p)$). HOLDS_{on} stands for both $\text{HOLDS}^{\sim}_{on}$ and $\text{HOLDS}^{\neg}_{on}$, and HOLDS_{at} for $\text{HOLDS}^{\sim}_{at}$ and $\text{HOLDS}^{\neg}_{at}$.

$\mathcal{CD}$ axioms are as follows. Since instants and periods are both primitives, we are not forced to accept any assumption on the relation between the holding of a fluent on a period and its holding at the period endpoints. A fluent holds during a period iff it holds at its *inner* instants:

$$\textbf{CD}_1 \quad \text{HOLDS}_{on}(f,p) \Leftrightarrow (\texttt{Within}(i,p) \Rightarrow \text{HOLDS}_{at}(f,i))$$

From it, nothing can be derived about the holding of f at $\texttt{begin}(p)$ and $\texttt{end}(p)$.

Continuous Fluents A continuous fluent may hold both during a period and at a particular instant without any restriction. This is not the case for discrete ones.

*We use the equality relation to express a fluent representing a parameter taking a certain value. E.g. the speed of a ball being positive on p is expressed as $\text{HOLDS}(\text{speed} = +, p)$. We omit necessary axioms imposing the exclusivity among the different values of a parameter.

Discrete Fluents The genuine property of discrete fluents is that *they cannot hold at an isolated instant*:

$$\mathbf{CD_2} \quad \text{HOLDS}_{\overset{\neg}{at}}(f,i) \Rightarrow \exists p\,(\text{HOLDS}_{\overset{\neg}{on}}(f,p) \wedge (\texttt{Within}(i,p) \vee \texttt{begin}(i,p) \vee \texttt{end}(i,p)))$$

Our distinction between continuous and discrete events is different from Galton's distinction between states of position and states of motion. Identifying it as a key property in modelling changing domains is a contribution of this chapter.

Non-Instantaneous Events The intuition behind events (both instantaneous and durable) is that of an *accomplishment* that may have relevant consequences over the state of the world. Unlike preceding approaches, *our theory does not include any axiom governing the occurrence of events that take time.* It reflects the intuition that whether two *accomplishments* may happen concurrently depends on the abstraction degree of the analysis. For example, the event "I programmed the program p1" can not occur over two periods that are not disjoint. It is not the case, however, if the event under consideration is merely "programming a program". Therefore, no domain-independent axiom can be stated as part of a general theory of temporal incidence.

Non-Atomic Fluents Our theory directly addresses the issue of the holding of non-atomic fluents with the following axioms:

Negation:	$\mathbf{CD_3}$	$\text{HOLDS}_{at}(\neg f,i) \Leftrightarrow \neg\text{HOLDS}_{at}(f,i)$
Conjunction:	$\mathbf{CD_4}$	$\text{HOLDS}_{at}(f \wedge f',i) \Leftrightarrow \text{HOLDS}_{at}(f,i) \wedge \text{HOLDS}_{at}(f',i)$
Disjunction:	$\mathbf{CD_5}$	$\text{HOLDS}_{at}(f \vee f',i) \Leftrightarrow \text{HOLDS}_{at}(f,i) \vee \text{HOLDS}_{at}(f',i)$

Deriving the properties of non-instantaneous holding of non-atomic fluents from these axioms is straight forward.

1.10 Revisiting the Issues

Let us see now how the problems presented in Section 1.2 are addressed using $\mathcal{IP}$ together with $\mathcal{CD}$.

Instantaneous Events Since we take instants as primitive, we can directly express instantaneous events using the predicate OCCURS_{at}. In the DIP scenario, for instance, we can write $\text{OCCURS}_{at}(\text{switchoff}, i)$. In Section 1.11 we discuss how to handle *sequences of instantaneous events*.

Instantaneous Holding We allow talking about a instantaneous holding of fluents by using HOLDS_{at} predicates. Axiom $\mathbf{CD_1}$ ensures that we are able to express the holding of contradictory fluents ending or beginning at a certain instant without conflict. Furthermore, we can express the holding of a continuous fluent at an isolated instant. The TBS scenario, for example, is merely represented as follows:

$$\begin{array}{ll} \text{HOLDS}_{\overset{\sim}{on}}(\text{speed} = +, p_1) & \\ \text{HOLDS}_{\overset{\sim}{at}}(\text{speed} = 0, i) & \texttt{end}(p_1) = i = \texttt{begin}(p_2) \\ \text{HOLDS}_{\overset{\sim}{on}}(\text{speed} = -, p_2) & \end{array}$$

The Dividing Instant Problem The DIP is not a problem for temporal incidence theories where the following two conditions hold:

1. The holding of a fluent over a period does not constrain its holding at the period's endpoints.

2. One can express that a fluent holds at an instant.

These conditions avoid logical contradiction and a truth gap at the dividing instant, respectively. In Figure 1.2 fluent f can be regarded as discrete and the DIP scenario can be formalized as follows:

$$\text{HOLDS}^{\neg}_{on}(\text{light} = \text{on}, p_1) \wedge \texttt{Meets}(p_1, p_2) \wedge \text{HOLDS}^{\neg}_{on}(\text{light} = \text{off}, p_2)$$

Given this information only, the query $\text{HOLDS}^{\neg}_{on}(\text{light} = \text{on}, \text{end}(p_1))$ merely gets no answer. The additional information required to answer it is domain-dependent. Some fluents hold on and at the end of a period (e.g. the fluent "being in contact with the floor" for a ball being lifted up), other fluents hold at the beginning and throughout the period (e.g. "not being in contact with the floor" for a ball that falls on the floor). In the light example, the most appropriate might be having three fluents light=on, light=off and light=changing, where the first and the second hold over open periods and the third holds at the dividing instant. Our approach avoids making any commitment about the holding at period's endpoints, whereas provides the means to safely specify what happens *at* the dividing instant. It requires an adequate theory of concatenability that we present below.

Non-Instantaneous Fluent Holding A nice feature of our proposal is that the above few axioms are enough to easily derive the fundamental properties of temporal incidence of fluents. For instance, *Allen's Homogeneity* $\text{HOLDS}^{\sim}_{on}(f,p) \Leftrightarrow \texttt{In}(p',p) \Rightarrow \text{HOLDS}^{\sim}_{on}(f,p')$ is easily derived from $\mathbf{CD}_1$. Before presenting the concatenability properties we define few basic notions. Given any two periods p, p' such that $\texttt{Meets}(p,p')$, $\texttt{meetpoint}(p,p') \stackrel{def}{=} \text{end}(p)$ and $\texttt{concat}(p,p') \stackrel{def}{=} p''$ s.t. $\texttt{begin}(p'') = \texttt{begin}(p) \wedge \texttt{end}(p'') = \texttt{end}(p')$. Also $\texttt{In} : \mathcal{P} \times \mathcal{P} \stackrel{def}{=} \texttt{Starts} \vee \texttt{During} \vee \texttt{Finishes}$, $\texttt{Disjoint}^{\neg}_{on} : \mathcal{P} \times \mathcal{P} \stackrel{def}{=} \texttt{Before} \vee \texttt{After}$ and $\texttt{Disjoint}^{\sim}_{on} : \mathcal{P} \times \mathcal{P} \stackrel{def}{=} \texttt{Before} \vee \texttt{Meets} \vee \texttt{Met\_by} \vee \texttt{After}$. The properties for concatenability are as follows:

Theorem 1.10.1. (Concatenability of discrete fluents)
If $\texttt{Meets}(p,p')$ *then*

$$\text{HOLDS}^{\neg}_{on}(f,p) \wedge \text{HOLDS}^{\neg}_{on}(f,p') \Leftrightarrow \text{HOLDS}^{\neg}_{on}(f, \texttt{concat}(p,p'))$$

Theorem 1.10.2. (Concatenability of continuous fluents)
If $\texttt{Meets}(p,p')$ *then*

$$\text{HOLDS}^{\sim}_{on}(f,p) \wedge \text{HOLDS}^{\sim}_{on}(f,p') \wedge \text{HOLDS}^{\sim}_{at}(f, \texttt{meetpoint}(p,p')) \Leftrightarrow$$
$$\Leftrightarrow \text{HOLDS}^{\sim}_{on}(f, \texttt{concat}(p,p'))$$

Concatenability can be regarded as a special case of *joinability*. Given two periods p and p', $\texttt{join}(p,p')$ is defined as a period p'' such that $\texttt{begin}(p'') = \texttt{min}(\texttt{begin}(p), \texttt{begin}(p'))$ and $\text{end}(p'') = \texttt{max}(\text{end}(p), \text{end}(p'))$, where $\texttt{min}$ and $\texttt{max}$ are defined according to $\prec$.

Theorem 1.10.3. (Joinability of discrete fluents)
If $\neg\mathtt{Disjoint}^{\neg}_{on}(p,p')$ *then*

$$\textsc{Holds}^{\neg}_{on}(f,p) \wedge \textsc{Holds}^{\neg}_{on}(f,p') \Leftrightarrow \textsc{Holds}^{\neg}_{on}(f,\mathtt{join}(p,p'))$$

Theorem 1.10.4. (Joinability of continuous fluents)
If $\neg\mathtt{Disjoint}^{\sim}_{on}(p,p')$, *or*
$\mathtt{Meets}(p,p') \wedge \textsc{Holds}_{at}(f,\mathtt{meetpoint}(p,p'))$, *or*
$\mathtt{Met\_by}(p,p') \wedge \textsc{Holds}_{at}(f,\mathtt{meetpoint}(p',p))$
then

$$\textsc{Holds}^{\sim}_{on}(f,p) \wedge \textsc{Holds}^{\sim}_{on}(f,p') \Leftrightarrow \textsc{Holds}^{\sim}_{on}(f,\mathtt{join}(p,p'))$$

There are also a number properties relating the holding of contradictory fluents at distinct, related times.

Theorem 1.10.5. (non-holding of discrete fluents)

$$\neg\textsc{Holds}^{\neg}_{on}(f,p) \Leftrightarrow \exists p'\ \mathtt{In}(p',p) \wedge \textsc{Holds}^{\neg}_{on}(\neg f,p')$$

Theorem 1.10.6. (non-holding of continuous fluents)

$$\neg\textsc{Holds}^{\sim}_{on}(f,p) \Leftrightarrow \exists i\ \mathtt{Within}(i,p) \wedge \textsc{Holds}^{\sim}_{at}(f,i)$$

Theorem 1.10.7. (disjointness)

$$\textsc{Holds}_{on}(f,p) \wedge \textsc{Holds}_{on}(\neg f,p') \Rightarrow \mathtt{Disjoint}^{\sim}_{on}(p,p')$$

At this point one may ask for how long can we go enumerating properties of temporal incidence. To answer this question, let us analyze the issue from a more general perspective. The above properties are particular cases of the following scheme (f is a fluent and $\overline{p}$ denotes the collection of periods $p_1,\ldots,p_n$) :

If $\textsc{Holds}(f,\overline{p})$ and $f \models f'$ then $\textsc{Holds}(f',\overline{p}')$
If $\textsc{Holds}(f,\overline{p})$ and $f \models \neg f'$ then $\textsc{Holds}(\neg f',\overline{p}')$

The scope of this chapter goes as far as showing that the most basic properties of this scheme are theorems of our theory.

1.11 Example: Modelling Hybrid Systems

In this section we illustrate the application of the proposed theory in qualitative modelling of physical systems. A (qualitative) model that includes both discrete and continuously changing parameters is called a *hybrid model*. Many physical systems, such as most electro-mechanical devices like photocopiers, cars, stereos, are suitably modelled as a hybrid model. Several approaches have been proposed to represent "qualitative" hybrid models [Nishida and Doshita, 1987; Forbus, 1989; Iwasaki and Low, 1992; Iwasaki *et al.*, 1995], however some semantical problems arise because of the different nature of discrete and continuous change. We shall see that an adequate theory of time and temporal incidence is fundamental to overcome them.

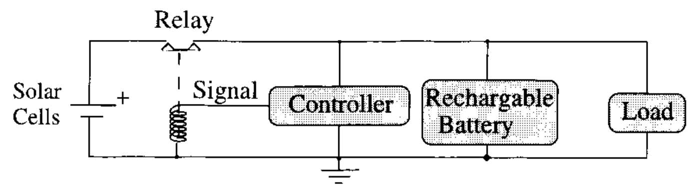

Figure 1.4: The hybrid circuit example.

Let us consider a particular example from [Iwasaki *et al.*, 1995] (we borrow the example, the qualitative model, the intended environment and a tentative solution). Figure 1.4 shows a simple circuit in which electric power is provided to a *load* either by a *solar cells* array or by a *re-chargeable battery*.
A part of the continuous behavior of this system is described as follows:

C0: "If the sun is shining and the relay is closed then the solar array acts as a constant current source and the battery accumulates charge."

The discrete events are specified as follows:

D1: "If the relay is closed, when the signal from the controller goes high, then the relay opens."

D2: "If the relay is open, when the signal from the controller goes low, then the relay closes."

D3: "If the signal is low, when the controller detects that the charge level in the battery has reached the threshold q_2, then the controller turns on the signal to the relay."

Now, let us consider a particular predicted qualitative behavior. A qualitative behaviour is described as a sequence of states that hold alternatively at an instant and throughout a period. The transition from one state to another is produced either by a continuous or by a discrete change. Quoting Iwasaki *et al* [Iwasaki *et al.*, 1995]: "...we would like to model discrete events as being instantaneous". Problems arise when sequences of them occur. For instance "the signal goes high and *immediately after* the relay closes". The following predicted behavior and the explanation about why sequences of discrete events are problematic are borrowed from [Iwasaki *et al.*, 1995]. Given the initial state of our example where the *signal* is *low*, the *relay* is *closed* and the sun is shining, the intended environment would be as follows:

s_1	(t_1, t_2)	$Q_{BA} < q_2$	$signal = low$	$relay = closed$
s_2	t_2	$Q_{BA} = q_2$	$signal = low$	$relay = closed$
$s_{2.1}$	$t_{2.1}$	$Q_{BA} =?$	$signal = high$	$relay = closed$
$s_{2.2}$	$t_{2.2}$	$Q_{BA} =?$	$signal = high$	$relay = open$
s_3	$(t_{2.2}, \text{-})$	$Q_{BA} < q_2$	$signal = high$	$relay = open$

The state $s_{2.1}$ is produced by the signal going high, and $s_{2.2}$ by the relay closing.

It is not clear how to model neither the times of s_2, $s_{2.1}$ and $s_{2.2}$ nor the time spans and the discrete events between them. If we assume that discrete events take no time, we encounter the following logical problem: "The antecedent for rules specifying discrete events often includes the negation of the consequence; this leads to a contradiction when events are treated as implications." An alternative is assuming that discrete changes take a very little period. It is problematic too since the value of every continuous variable that changes concurrently becomes unknown after a sequence of actions. In the example, the charge of the battery would keep continuously increasing for a short period. After a number of discrete events these small variations accumulate and complicate the computation of parameter values.

Several solutions have been proposed to solve this quandary. They are based on complicating the model of time by either introducing *mythical* time ([Nishida and Doshita, 1987], *direct method*), extending the real numbers with infinitessimals ([Nishida and Doshita, 1987], *approximation method*)[Alur *et al.*, 1993], or using non-standard analysis [Iwasaki *et al.*, 1995]. Next we show that none of these is necessary. We use our theory of instants/periods and continuous fluents/discrete fluents/events as follows:

- Discrete events are modelled as instantaneous events.
- Continuous/discrete quantities are modelled as continuous/discrete fluents.

Since HOLDS_{on} is defined as holding at the inner points only, the value of a fluent that changes because of an instantaneous event is not defined at the time that the event's time unless there is some specific knowledge about it. The sequence of states representing the intended environment becomes simpler:

s_1	(t_1, t_2)	$Q_{BA} < q_2$	*signal* = *low*	*relay* = *closed*
s_2	t_2	$Q_{BA} = q_2$	*signal* =?	*relay* =?
s_3	$(t_2, \text{-})$	$Q_{BA} < q_2$	*signal* = *high*	*relay* = *open*

Indeed, this solution is much simpler than the previously proposed techniques. The formalization of the environment is as follows:

$\widetilde{\text{HOLDS}}_{on}(Q_{BA} < q_2, p_1)$	
$\text{HOLDS}^{\neg}_{on}(\textit{signal} = \textit{low}, p_2)$	
$\text{HOLDS}^{\neg}_{on}(\textit{relay} = \textit{closed}, p_3)$	
$\widetilde{\text{HOLDS}}_{at}(Q_{BA} = q_2, \texttt{end}(p_1))$	
$\text{OCCURS}_{at}(\textit{turn\_on}(\textit{signal}), \texttt{end}(p_1))$	$\texttt{end}(p_1) = \texttt{end}(p_2)$
$\text{HOLDS}^{\neg}_{on}(\textit{signal} = \textit{high}, p_4)$	$\texttt{Meets}(p_2, p_4)$
$\text{OCCURS}_{at}(\textit{open}(\textit{relay})), \texttt{end}(p_3))$	$\texttt{end}(p_2) = \texttt{end}(p_3)$
$\text{HOLDS}^{\neg}_{on}(\textit{relay} = \textit{open}, p_5)$	$\texttt{Meets}(p_3, p_5)$

1.12 Concluding Remarks

A theory of time and temporal incidence is the foundation for a proper temporal representation, independently of both the temporal qualification method and the underlying representation language. In this chapter we identified the problematic issues that need to be addressed, namely the expression of *instantaneous events* and *fluents*, the *dividing instant problem* (DIP) and the formalization of the properties *for non-instantaneous holding of fluents*.

In this chapter we have surveyed the most relevant theories of time in Artificial Intelligence and we have discussed the pros and cons of each of them. Also, we presented a brief overview of temporal incidence theories and proposes a theory of *temporal incidence* called $\mathcal{CD}$ defined upon a theory of instants and periods (such as $\mathcal{IP}$) whose key insight is the distinction between continuous and discrete fluents.