4 Homework

Due: Monday, February 11, 2013.

Instructions

- Please, review the homework grading policy outlined in the course information page.

- On the first page of your solution write-up you must make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

<table>
<thead>
<tr>
<th>Problem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit</td>
<td>RC</td>
<td>RC</td>
<td>RC</td>
<td>EC</td>
<td>RC</td>
<td>EC</td>
<td>NA</td>
<td>NA</td>
<td>EC</td>
<td>...</td>
</tr>
</tbody>
</table>

where “RC” denotes “regular credit”, “EC” denotes “extra credit”, and “NA” denotes “not attempted”. Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.

- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: Five of the following 6 problems

Points: 20 points per problem

1. (Problem 1.55, but for the different languages)

The pumping lemma says that every regular language has a pumping length $p$, such that every string in the language can be pumped if it has length $p$ or more. If $p$ is a pumping length for the language $A$, so is any length $p' \geq p$. The minimum pumping length for $A$ is the smallest $p$ that is the pumping length for $A$. For example, of $A = 01^*$, the minimum pumping length is 2. The reason is that the string $s = 0$
is in $A$ and has length 1 yet $s$ cannot be pumped; but any string in $A$

of length 2 or more contains 1 and hence can be pumped by dividing

it so that $x = 0, y = 1$ and $z$ is the rest.

For each of the following languages, give the minimum pumping

length and justify your answer.

(a) $ab^*a$
(b) $aab \cup a^*b^*$
(c) $(abab)^*$
(d) $\epsilon$
(e) $ababa$
(f) $a^*bbba^*$

2. (a) (Problem 1.47)
Let $\Sigma = \{1, \#\}$ and let
$Y = \{w | w = x_1\#x_2\#\ldots\#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$
Prove that $Y$ is not regular.

(b) (Problem 1.48)
Let $\Sigma = \{0, 1\}$ and let
$D = \{w | w \text{ contains an equal number of occurrences of substrings } 01 \text{ and } 10\}.$

Thus $101 \in D$ because $101$ contains a single $01$ and a single $10$, but
$1010 \notin D$ because $1010$ contains two $10$s and one $01$. Show that $D$ is
a regular language.

3. (Problem 1.62)
Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let $D_k$ be the language of all strings
that have at least one $a$ among the last $k$ symbols. Thus $D_k = \Sigma^*a(\Sigma \cup
\epsilon)^{k-1}$. Describe a DFA with at most $k + 1$ states that recognizes $D_k$ in
terms of both a state diagram and a formal description.

4. (Problem 2.1 for the given strings)
The CFG $G_4$ is given as follows:

$$
E \rightarrow E + T | T \\
T \rightarrow T \times F | F \\
F \rightarrow (E) | a
$$

Give parse trees and derivations for each string:
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5. Give the context-free grammars that generate the following languages:

- 2.4 (e) \( \{ w | w = w^R, \text{ that is, } w \text{ is a palindrome} \} \)
- 2.6 (b) The complement of the language \( \{a^n b^n | n \geq 0 \} \)
- 2.6 (d) \( \{x_1 \# x_2 \# \ldots \# x_k | k \geq 1, x \in \{a, b\}^*, \text{ and for some } i, j \text{ we have } x_i = x_j^R \} \)

6. Do the following:

(a) (Problem 2.9)
Give the CFG that generates the language
\( A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \geq 0 \} \)
Is your grammar ambiguous? Why, or why not?
(b) (Problem 2.13)
Let \( G = (V, \Sigma, R, S) \) be the following grammar. \( V = \{S, T, U\}; \Sigma = \{0, \#\}; \) and \( R \) is the set of rules:

\[
\begin{align*}
S & \rightarrow TT|U \\
T & \rightarrow 0T|0|T0|\# \\
U & \rightarrow 0U00|\#
\end{align*}
\]

(a) Describe \( L(G) \) in English.
(b) Prove that \( L(G) \) is not regular.