3 Homework

Due: Thursday, January 31, 2013.

Instructions

• Please, review the homework grading policy outlined in the course information page.

• On the first page of your solution write-up you must make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ...
|---------|---|---|---|---|---|---|---|---|---|---
| Credit  | RC | RC | EC | RC | EC | NA | NA | EC | NA | ...

where “RC” denotes “regular credit”, “EC” denotes “extra credit”, and “NA” denotes “not attempted”. Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.

• You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 5 of the following 7 problems
Points: 20 points per problem

1. Give the regular expressions generating the languages from Problem 1 in Homework 2, i.e.:

   (a) \{w | w the length of w is at most 5\}
   (b) \{w | w is any string except aa and aaa\}
   (c) \{w | w every odd position of w ia a\}
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(d) \{w | \text{w contains at least two a's and at most one b}\}

(e) \{w | \text{w begins with a and ends with b, or begins with b and ends with \text{a}}\}

2. Give the regular expressions generating the languages from Problem 2 in Homework 2, i.e.:

(a) \{w | \text{w begins with 1, and ends with 0}\}

(b) \{w | \text{w contains at least three 1's}\}

(c) \{w | \text{w contains substring 0101}\} \text{ (i.e., } w = x0101y\}

(d) \{w | \text{w has length at least 3 and its third symbol is a 0}\}

(e) \{w | \text{w does not contain the substring 011}\}

3. Use the procedure described in Lemma 1.60 in the text to convert the following DFA to a regular expression in two different ways:

- eliminating first state 3, then state 2, then state 1
- eliminating first state 1, then state 2, then state 3

Show the resulting GNFA after each step, and do not try to simplify your answer (except for eliminating all instances of \(\emptyset\) in unions and all instances of \(\epsilon\) in concatenations).

4. Use the procedure described in Lemma 1.60 in the text to convert the following DFA to a regular expression in two different ways:
• eliminating first state 3, then state 2, then state 1
• eliminating first state 1, then state 2, then state 3

Show the resulting GNFA after each step, and do not try to simplify your answer (except for eliminating all instances of $\emptyset$ in unions and all instances of $\epsilon$ in concatenations.

5. For any string $w = w_1w_2...w_n$, the reverse of $w$, written $w^R$, is the string $w$ in reverse order, $w_n...w_1$. For any language $A$, let $A^R = \{w^R | w \in A\}$. Show that if $A$ is regular, then so is $A^R$.

6. Problem 1.42 For languages $A$ and $B$, let the perfect shuffle of $A$ and $B$ be the language $\{w | w = a_1b_1...a_kb_k, \text{ where } a_1...a_k \in A \text{ and } b_1..b_k \in B, \text{ for each } a_i, b_i \in \Sigma\}$. Show that the class of regular languages is closed under perfect shuffle.

7. Prove or disprove the following:

(a) Every subset of a regular language is a regular language.
(b) Every subset of a nonregular language is a nonregular language.
(c) If $A$ is a regular language and $B$ is a language such that $AB$ is regular, then $B$ is regular.
(d) If $A$ is a regular language and $B$ is a language such that $A - B$ is regular, then $B$ is regular.
(e) For any language $A$ and its complement $A'$ the language $A \cup A'$ is regular.