Problem Set 5 (due Monday, March 28)

1. **Weighted activity selection**

Consider a weighted version of the activity selection problem, in which each activity has a *weight*, in addition to the start and finish times. (For example, the weight may signify the importance of the activity.) The goal is to select a maximum-weight set of mutually compatible activities, where the weight of a set of activities is the sum of the weights of the activities in the set.

(a) (5 points) Consider a greedy algorithm that repeatedly performs the following step until no more activities can be selected: select an activity that has the maximum ratio of weight over length among all activities that do not overlap with the activities selected thus far.

Give a counterexample to show that the above greedy algorithm will not yield an optimal solution for the weighed activity selection problem.

(b) (15 points) Use dynamic programming to solve the weighted activity selection problem. Analyze the running time of your algorithm.

2. (15 points) **Edit distance**

Part (a) of Problem 15–3, pages 364–366.

3. (10 points) **Depth-first search**

Exercise 22.3-2, page 547.

4. (10 points) **Connected components**

Exercise 22.3-11, page 549.

5. (10 points) **Topological sort**

Exercise 22.4-5, page 552.

6. (10 points) **Minimum spanning tree**

Let $G$ be an undirected weighted graph. Show the weight of the maximum-weight edge of any MST of $G$ is minimum among all spanning trees of $G$.

(*Hint:* Consider two trees $T_1$ and $T_2$, where $T_1$ is an MST. If the maximum-weight edge of $T_1$ has higher weight than the maximum-weight edge of $T_2$, then argue that you can replace this edge of $T_1$ with another edge of $T_2$, thus yielding a contradiction.)