Sample Solution to Quiz 1

1. Recall that a directed graph is *strongly connected* if there is a path from every node to every other node in the graph. Also, the *diameter* of a strongly connected graph is the maximum distance between any two nodes in the graph. Recall that the *distance* from one node \( u \) to another node \( v \) is the number of hops in the shortest path from \( u \) to \( v \).

(a) Draw a strongly connected directed graph with 6 nodes that has the smallest possible diameter.

**Answer:**

![Figure 1: A strongly connected directed graph of diameter 1.](image)

(b) Draw a strongly connected directed graph with 6 nodes that has the largest possible diameter. For extra credit, argue why your graph has the largest possible diameter among all strongly connected graphs of 6 nodes.

**Answer:**

![Figure 2: A strongly connected directed graph of diameter 5.](image)

The maximum distance between any two nodes in the above graph is 5 (e.g., from node 1 to 2). Therefore, the diameter is 5. We claim that this is the maximum possible.
The shortest path between any two nodes in a strongly connected graph of \( n \) nodes has no cycles (we can remove them to obtain a shorter path, otherwise). Since each hop on a path with no cycles leads to a new node, it follows that the shortest path between any two nodes has at most \( n - 1 \) hops. Therefore, the distance between any two nodes in a strongly connected directed graph of \( n \) nodes is at most \( n - 1 \). We thus obtain that the diameter of a strongly connected directed graph of \( n \) nodes is at most \( n - 1 \). In particular, the diameter of a strongly connected directed graph of 6 nodes is at most 5, which is what we set out to prove.

2. Give brief definitions of the terms web crawler and PageRank that appear in the article on Google that was assigned as reading last week.

**Answer:** The first two definitions are from the *American National Standard for Telecommunications: Glossary of Telecommunication Terms*, available at

http://www.atis.org/tg2k/t1g2k.html

*Web crawler:* “A robot that searches the world wide web for new and updated web pages. Found pages are categorized by subject and placed in a database. Typically, an associated search engine will access that database.”

*Robot:* “A relatively small and focused computer application that (a) runs continuously, in the background (i.e., simultaneously), as other programs are being run, and (b) responds automatically to a user’s activity.”

*PageRank:* A rank assigned to a page that indicates the importance of the page and is used by search engines to determine the order in which pages relevant to a particular query are listed. The PageRank of a page depends on the links into the page and the PageRanks of the pages that link to the page.