Written Homework 5
Due November 25, 2003 at the start of class

Reading: Rosen 3.3 Mathematical Induction
Use mathematical induction to prove the statements in the problems below. Follow the steps used in the proofs in section 3.3 of Rosen.

a) DEFINE P(n):
Let P(n) be the proposition ...

b) BASIS STEP:
P(1) is true since ...
(You may have to start with P(3) or some other starting case.)

b) INDUCTIVE STEP:
Assume that P(k) is true for positive integer k and use that to prove that P(k+1) is true.

WRITE OR TYPE YOUR PROOFS NEATLY.

1. If \( n \) is a positive integer, then
\[
1^3 + 2^3 + \cdots + n^3 = \left( \frac{n(n+1)}{2} \right)^2.
\]

2. For every natural number \( n \), \( n \mid (n^3 - n) \).

3. If \( n \) is an integer and \( n \geq 5 \), then \( 2^n \geq n^2 \).

4. If \( n \) is an integer \( n \geq 1 \), then \( \sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n} \).

5. a) Find a formula for
\[
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)}
\]
by examining the values of this expression for small values of \( n \).

b) Use mathematical induction to prove your result.