

Multiparty Generation of an RSA Modulus

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<http://ia.cr/2020/370>

In this work

*Multiparty RSA modulus sampling
with malicious security
against a dishonest majority*

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- *First with cubic communication
complexity in modulus length*

Why RSA Moduli?

Useful For

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- **VDFs and Timelock Puzzles**
[Rivest-Shamir-Wagner-96], [Boneh-Bonneau-Bünz-Fisch-18],
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- **Efficient zk-SNARKs**
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Local Biprime Sampling

1. Choose random $p \leftarrow \mathbb{Z}_{2^k}$
2. If p is not prime, repeat step 1
3. Sample q in the same way as p
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Miller-Rabin Test

$$a^{p-1} \pmod p$$

MPC-unfriendly as p is secret

History of the Problem

1997

- 
- [Boneh-Franklin-97/01]
 - [Frankel-MacKenzie-Yung-98]
 - [Poupard-Stern-98]
 - [Gilboa-99]
 - [Malkin-Wu-Boneh-99]
 - [Algesheimer-Camenish-Shoup-02]
 - [Damgård-Mikkelsen-10]
 - [Gavin-12]
 - [Hazay-Mikkelsen-Rabin-Toft-12]
 - [Frederiksen-Lindell-Osheter-Pinkas-18]
 - [Hazay-Mikkelsen-Rabin-Toft-Nicolosi-19]
 - [[This Work](#)]
 - [Chen-Hazay-Ishai-Kashnikov-Micciancio-Riviere-shelat-Muthu-Wang-20]

2020

Recent Progress

	[HMRT12] [HMRTN19]	[FLOP18]
Security	Malicious	Malicious
Parties	n	2
Corruptions	$n - 1$	1
Uses HE	Yes	No
Assumptions	DCR, DDH	OT
ZK over Crypto	Yes	Almost No
Extra Leakage	No	Yes
Covert abort(DoS)	No	Yes

Open Question

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O(s) to fix generically



Our Contribution

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Comm. Cost	?	$\tilde{O}(sk^4)$	$\tilde{O}(nsk^3)$

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[BF97]: Test biprimality of N instead of primality of p, q individually

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Shared by
most works

Protocol Overview

We leverage the **Chinese Remainder Theorem** for efficiency

- Sieving speed of OT-based trial division but without the leakage
- OT-based secure multiplication over small fields: cheaper than single equivalent large field mult
- Component-wise consistency check plugs covert DoS attack

See the longer talk for...

- More details on challenges and techniques to surmount them
- Component-wise cost analysis
- Jack's pandemic hair

Thanks!

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