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CS U390, Spring 2007 (Instructor: Clinger)
Homework 1
Assigned:  Thursday, 18 January 2007 (corrected 20 January)
Due:       Wednesday, 24 January 2007
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Good students should get at least 50 of the 100 possible points.

1. [5 pts] Write down the formal (5-tuple) description of the first DFA pictured in exercise 1.1 on page 83 of the textbook.

2. [4 pts] Draw the state transition diagram for the DFA whose formal description is

   $$(\{1,2,3,4\}, \{a,b,c\}, \delta, 1, \{1,4\})$$

   where δ is the function listed within the following table:

	a	b	c
1	2	1	1
2	2	3	1
3	2	3	4
4	2	1	4

3. [8 pts] For each of the following languages draw the state transition diagram for a DFA with alphabet $\{0,1\}$ that recognizes the language.

   (a) $\{1, 10, 11, 100\}$

   (b) $\{w \mid w$ contains at least two 1s$\}$

   (c) $\{w \mid w$ contains at most three 0s$\}$

   (d) $\{w \mid w$ starts with 1 and ends with 1$\}$

   (e) $\{w \mid w$ contains an even number of 0s and an even number of 1s$\}$

   (f) $\{w \mid w$ is a binary numeral that is divisible by 3$\}$

   (g) $\{w \mid w$ is a binary numeral that is divisible by 7$\}$

   (h) $\{w \mid w$ there exist strings x and y such that $w = x1101y\}$

4. [10 pts] Prove that the regular languages are closed under intersection.

5. [8 pts] For each of the following languages, write down a regular expression whose value is the language.

   (a) $\{10, 11, 101, 111\}$

   (b) $\{w \mid w$ is a binary numeral that contains at least three 0s$\}$

   (c) $\{w \mid w$ is a binary numeral divisible by eight$\}$

   (d) $\{w \mid w$ is a binary numeral that starts with 10 and ends with 01$\}$

   (e) $\{w \mid w$ is a binary numeral that contains an odd number of 0s and an odd number of 1s$\}$

   (f) $\{w \mid w$ is a binary numeral that is divisible by 3$\}$

(g) $\{w \mid w$ is a binary numeral that is divisible by $5\}$

(h) $\{w \mid w$ is a binary numeral and there exist strings x and y such that $w = x001y\}$

6. [5 pts] Do exercise 1.16(a) in the textbook.

7. [5 pts] Do exercise 1.16(b) in the textbook.

8. [5 pts] Do exercise 1.22(a) in the textbook.

9. [5 pts] Do exercise 1.22(b) in the textbook.

10. [5 pts] Prove the following theorem. If B is a language over an alphabet Σ, and $B = B^*$, then $BBBB \subseteq B$.

11. [10 pts] Let $L = \{w \mid w$ is a binary numeral with at least as many 0s as 1s$\}$. Is L a regular language? If L is regular, prove it is regular. If L is not regular, prove it is not regular.

12. [10 pts] Do problem 1.31 in the textbook.

13. [10 pts] Let $L = \{w \mid x$ is a binary numeral, y is the reverse of x, and $w = xy\}$. Is L a regular language? If L is regular, prove it is regular. If L is not regular, prove it is not regular.

14. [10 pts] Do problem 1.37 in the textbook.