

CS U390, Spring 2007 (Instructor: Clinger)

Homework 0

Assigned: Thursday, 11 January 2007

Due: Wednesday, 17 January 2007

1. [5 pts] For each of the following set operations, specify the result by listing its elements inside curly braces.

(a)  $\{1, 2\} \cup \{2, 3, 4\} =$

(b)  $\{1, 2\} \cap \{2, 3, 4\} =$

(c)  $\{1, 2\} - \{2, 3, 4\} =$

(d)  $\{2, 3, 4\} - \{1, 2\} =$

(e)  $\{1, 2\} \times \{2, 3, 4\} =$

2. [6 pts] Write out each of the following power sets by listing their elements inside curly braces.

(a)  $\mathcal{P}(\{1, 2, 3\}) =$

(b)  $\mathcal{P}(\{1\}) =$

(c)  $\mathcal{P}(\emptyset) =$

3. [5 pts] If  $S$  is any set, then we use the notation  $|S|$  to indicate the number of elements in  $S$ . Suppose  $A$ ,  $B$ , and  $C$  are sets with  $|A| = 4$ ,  $|B| = 7$ , and  $|C| = 11$ . Compute the number of elements in each of the following sets.

(a)  $|A \times A| =$

(b)  $|B \times C| =$

(c)  $|\mathcal{P}(A)| =$

(d)  $|\mathcal{P}(B)| =$

(e)  $|\mathcal{P}(A \times B)| =$

4. [10 pts] For any  $n \in \mathcal{N}$ , we say that  $n$  is odd if and only if there exists  $m \in \mathcal{N}$  such that  $n = 2m + 1$ . From this definition, give a rigorous proof that the product of two odd numbers is odd.

5. [10 pts] Prove that there is no largest natural number. (In other words, prove that there is no  $l \in \mathcal{N}$  such that  $l \geq n$  for all  $n \in \mathcal{N}$ . Hint: for all  $n \in \mathcal{N}$ ,  $n + 1 > n$ .)