Due at the beginning of class on Thursday, 9/26.

When solving a problem you can use all the results seen in class. You should not use other sources. You are allowed to discuss problems with other students. However, each student must write up his or her own solutions and must not read or copy the solutions of others. You should try working on each problem on your own before discussing with others. If you work with others on a problem, you must note with whom you discussed the problem at the beginning of your solution write-up.

Problem 1 (NFAs with a Single Accept State) 5 pt

Show that for any regular language $L$ there exists an NFA $N = (Q, \Sigma, \delta, q_0, F)$ that recognizes $L$ and has a single accepting state $|F| = 1$.

Problem 2 (Convert NFA to DFA) 10 pts

Use the procedure described in class to convert each of the following two NFAs into an equivalent DFA. You can delete any “unreachable” states. You should label the states of the DFA with the corresponding subsets of the states of the NFA, like we did in class.

Problem 3 (Regular Expressions) 15 pt

For each of the languages given by the regular expressions below, give an example of 2 different string that are in the language, and 2 different string that are not in the language (4 strings total).
Then draw an NFA that accepts this language. (Hint: it will be helpful to rely on the procedures we gave in class for constructing NFAs for the union, concatenation and $*$ operation, but you can also simplify your final picture if you’d like.)

1. 0(10)*1
2. 0* ∪ 1*
3. 0*1*
4. ($\varepsilon$ ∪ 0)1
5. 00*11*

**Problem 4 (More Regular Expressions) 10 pt**

1. Construct regular expressions for the following languages over the alphabet $\Sigma = \{0, 1\}$.
   
   (a) $L = \{w : w$ contains the substring 010$\}$.
   
   (b) $L = \{w : w$ contains an odd number of 0s$\}$.
   
   (c) $L = \{w : w$ every odd position of $w$ contains the symbol 1$\}$.
   
   (d) $L = \{w : w$ the length of $w$ is at most $4$$\}$.

2. Suppose $R_1, R_2$ and $R_3$ are regular expressions whose languages are of finite sizes $k_1, k_2$ and $k_3$ respectively. What is the size of the language denoted by the expression $(R_1 ∪ R_2)R_3$?

**Problem 5 (Prefixes of a Regular Language) 10 pts**

Fix some alphabet $\Sigma$. For a language $L$ over $\Sigma$, define the language

$$Prefix(L) = \{w \mid \exists w' \in \Sigma^* \text{ such that } ww' \in L\}.$$ 

In other words, $Prefix(L)$ consists of all possible prefixes of the strings in $L$. Show that, if $L$ is regular, then $Prefix(L)$ is regular.

Small Hint (may ruin some of the fun - read at own risk): Given a DFA $M$ for the language $L$, construct a DFA $M'$ for $Prefix(L)$ that has the same number of states as $M$.

**Problem 6 (Suffixes of a Regular Language) Extra Credit**

Show that if $L$ is regular, then $Suffix(L) = \{w \mid \exists w' \in \Sigma^* \text{ such that } w'w \in L\}$ is also regular.