Pumping Lemma for Context-Free Languages

If L is a context-free language, then there is a number p (called a *pumping length* for L) such that any string $s \in L$ with $|s| \geq p$ can be split into $s = uvxyz$ so that the following conditions are satisfied:

1. for each $i \geq 0$, $uv^ixy^iz \in L$;
2. $|vy| > 0$; and
3. $|vxy| \leq p$.

Remarks:

- Condition 2 is equivalent to requiring that at least one of v and y be non-empty.
- If both v and y were allowed to be ε, then all the strings uv^ixy^iz would be equal to the original string s and the result would be trivial.
- Because of condition 2, p must be at least 1.
- If p is a pumping length for L, then so is any $p' > p$, since any string satisfying $|s| \geq p'$ must also satisfy $|s| \geq p$ when $p' > p$. This is why we call p *a* pumping length for L and not *the* pumping length for L.
- Using $i \geq 2$ in condition 1 is called "pumping up" the string s.
- Using $i = 0$ in condition 1 is called "pumping down" the string s.
- The Pumping Lemma may be satisfied vacuously, if there are no strings longer than a certain length (which can happen only when L is finite). In this case, any p larger than the length of the longest string in L is a pumping length for L.