Formal Definition of a Deterministic Finite Automaton (DFA)

A *deterministic finite automaton* M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set (whose elements are called *states*);
- Σ is a finite set (called the *input alphabet*);
- $\delta : Q \times \Sigma \longrightarrow Q$ (called the *transition function*);
- $q_0 \in Q$ (called the *start state*); and
- $F \subseteq Q$ (whose elements are called *accept states*).

Observations:

- Q must contain at least one state: the start state.
- There is always exactly one start state.
- F could be the empty set Φ.

Computation Performed by a DFA

Let $w = a_1 a_2 \ldots a_n$ be a string, with $a_i \in \Sigma$ for $i = 1, 2, \ldots, n$.
Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA.
Then M *accepts* w if there exists a sequence of states $(r_0, r_1, r_2, \ldots, r_n) \in Q^{n+1}$ such that

1. $r_0 = q_0$;
2. $r_i = \delta(r_{i-1}, a_i)$ for $i = 1, 2, \ldots, n$; and
3. $r_n \in F$.

The *language recognized by* M is $L(M) = \{w \mid M \text{ accepts } w\}$.

Formal Definition of a Nondeterministic Finite Automaton (NFA)

Recall that for any set S, the power set 2^S is the set of all subsets of S.

Also, given a finite set Σ (representing an alphabet), let $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$.

A *nondeterministic finite automaton* N is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set (whose elements are called *states*);
- Σ is a finite set (called the *input alphabet*);
- $\delta : Q \times \Sigma_\varepsilon \longrightarrow 2^Q$ (called the *transition function*);
- $q_0 \in Q$ (called the *start state*); and
- $F \subseteq Q$ (whose elements are called *accept states*).

Computation Performed by an NFA

Let $w = a_1 a_2 \ldots a_n$ be a string, with $a_i \in \Sigma$ for $i = 1, 2, \ldots, n$.
Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA.
Then N *accepts* w if there exists a sequence of states $(r_0, r_1, r_2, \ldots, r_n) \in Q^{n+1}$ such that

1. $r_0 = q_0$;
2. $r_i \in \delta(r_{i-1}, a_i)$ for $i = 1, 2, \ldots, n$; and
3. $r_n \in F$.

The *language recognized by* N is $L(N) = \{w \mid N \text{ accepts } w\}$.