7 Homework

Due: Monday, November 19, 2007.

Instructions

• Please, review the homework grading policy outlined in the course information page.

• On the first page of your solution write-up you must make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ...
|---------|---|---|---|---|---|---|---|---|---|---
| Credit  | RC| RC| RC| EC| RC| EC| NA| NA| EC|...

where “RC” denotes “regular credit”, “EC” denotes “extra credit”, and “NA” denotes “not attempted”. Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.

• You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 5 of the following 7 problems

Points: 20 points per problem

1. • Prove that the collection of decidable languages is closed under concatenation and star.

• Prove that the collection of Turing-recognizable languages is closed under concatenation and star.

For this problem, give only informal high-level description of any required Turing Machines. Hint: You may find it helpful to use non-deterministic and/or multi-tape Turing Machines.
2. Given an arbitrary Turing machine (or Turing machine variant) $M$, let $M'$ be the same machine but with the accept and reject states swapped. Is it possible that there exists strings accepted by:

i both $M$ and $M'$; or

ii neither $M$ nor $M'$;

when

a $M$ is a (deterministic) decider?

b $M$ is a (deterministic) recognizer?

c $M$ is a (nondeterministic) decider?

d $M$ is a (nondeterministic) recognizer?

Note that 8 answers are required. Justify all answers.

3. • Do the Problem 4.2  
   • Do the Problem 4.12

4. Do the Problem 4.3

5. Do the Problem 4.4

6. Prove that $\text{ONE}_{\text{DFA}} = \{ < D > | D \text{ is a DFA and } |L(D)| = 1 \}$ is decidable.

7. Do the Problem 4.26