6 Homework

Due: Wednesday, October 31, 2007.

Instructions

- Please, review the homework grading policy outlined in the course information page.
- On the *first page* of your solution write-up you *must* make explicit which problems are to be graded for regular credit, whic problems are to be graded for extra credit, and which problems you did not attmept. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

  where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 4 of the following 6 problems
Points: 25 points per problem

1. Do the Problem 2.30 (a)
2. Do the Problem 2.31
3. Consider the following language:

   $\{a^i b^j c^k | i, j, k \geq 0 \text{ and } k = min(i, j)\}$

   Assume that this language is context-free, and let p be the pumping length for it. Let $s = a^p b^p c^p$.

(a) Show that $s \in L$ and that $|s| \geq p$.

(b) Show that s can be split into five pieces, $s = uvxyz$, such that conditions 2 and 3 of the Pumping Lemma for context-free languages are satisfied and $uv^ixy^iz \in L$ for all $i \geq 1$ (so s can be pumped up any number of times).

(c) Show that s can also be split into five pieces, $s = u'v'x'y'z'$, such that conditions 2 and 3 of the Pumping Lemma are satisfied and $u'x'z' \in L$ (so s can also be pumped down).

(d) Prove that, nevertheless, L violates the Pumping Lemma for context-free languages, so it is not context-free.

4. Prove that if a unary language violates the Pumping Lemma for regular languages then it cannot be context-free.

   Hint: Concatenation of strings over a unary alphabet is commutative.

   Note: Unary language has the alphabet that consists of a single symbol (not counting ϵ).

5. Give both an informal implementation-level description and a state transition diagram for a Turing Machine that decides the language $\{w | w \in \{a, b\}^*$ and w is a palindrome$\}$.

6. For the Turing Machine Example 5 of the course handout, show the sequence of the TM configurations for the following strings:

   (a) *abba*

   (b) *abab*

   (c) *baba*

   (d) *baab*