2 Homework

Due: Wednesday, September 26, 2007.

Instructions

- Please, review the homework grading policy outlined in the course information page.
- On the *first page* of your solution write-up you *must* make explicit which problems are to be graded for regular credit, whic problems are to be graded for extra credit, and which problems you did not attmept. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

  where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 5 of the following 7 problems
Points: 20 points per problem

1. Give the state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is $\{a, b\}$.
   (a) The language $\{w|w \text{ contains } baba\}$ with five states.
   (b) The language $\{w|w \text{ contains } ab \text{ or } ba \text{ (or both)}\}$ with five states.
   (c) The language $\{w|w = z_1z_2...z_k$ for $k \in \mathcal{N}$, where $z_i = ab$ or $z_i = bab$ for all $k\}$ with three states.

(d) The language $\{w|w$ contains 0 or more a-s, or contains one or more b-s and ends with $a\}$ with four states.

(e) The language $\{a\}$ with 2 states.

(f) The language $\{\epsilon\}$ with one state.

(g) The language $\{a^*\}$ with one state.

2. For the NFA N shown in figure 1

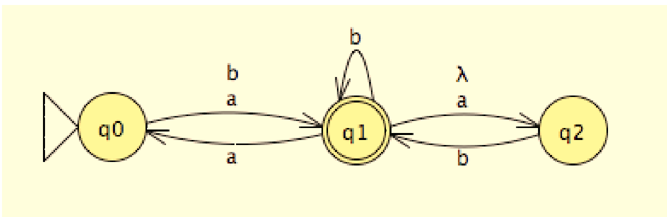

Figure 1: NFA N

(a) Give the formal description of N as a 5-tuple (according to Definition 1.37 on p. 53).

Note: The λ in the diagram represents ϵ. The JFLAP software I use to draw the diagrams uses this convention for the empty string label.

(b) For each of the following strings determine whether it is accepted by the NFA N. If it is, give the sequence of states leading to the accept state. For example, for the string ab the sequence leading to accept state may be

$q0 : a \longrightarrow q1 : b \longrightarrow q1$

If the string is not accepted, show the path through the NFA, by listing the sets of states reached after each letter is read:

$q0 : x_1 \longrightarrow \{q_{k1}, q_{k2}, ...\} : x_2 \longrightarrow ...$

i. aa

ii. $abaab$

iii. $baaba$

iv. aaa

v. $aaaa$

3. (a) Do Exercise 1.14.
   (b) Do Exercise 1.15.

4. Give the regular expressions generating the languages from Problem 2 in Homework 1, i.e.:
   (a) $\{w|w$ begins with a, contains b, and ends with $c\}$
   (b) $\{w|w$ contains at least two a-s$\}$
   (c) $\{w|w$ contains $abab\}$
   (d) $\{w|w$ has length at least 3, contains $c\}$
   (e) $\{w|w$ begins with a and ends with b, or begins with b and ends with $a\}$

5. Give the regular expressions generating the languages from Problem 3 in Homework 1, i.e.:
   (a) $\{w|w$ begins with a, and has even length or begins with b, and has odd length, or begins with $c\}$
   (b) $\{w|w$ does not contain $b\}$
   (c) $\{w|w$ does not contain a substring $cba\}$
   (d) $\{w|w$ has length less than $5\}$
   (e) $\{w|w$ every odd position is $b\}$

6. Do Problem 1.31.

7. (a) Do Problem 1.42.
   (b) Look at the definition of the *perfect shuffle* of two languages given in Problem 1.41. Use the result of Problem 1.42 to prove that the perfect shuffle of *any* two languages (whether regular or not) is regular.

   Clarification: In the definition of these "shuffle" languages, the value of k can be any integer ≥ 0.