6 Homework

Due: Thursday, February 28, 2013.

Instructions

- Please, review the homework grading policy outlined in the course information page.
- On the *first page* of your solution write-up you *must* make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

  where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 5 of the following 6 problems
Points: 20 points per problem

1. Do Problem 2.30 parts a and d

   Use the pumping lemma to show that the following languages are not context free.

   (a) (a) $\{0^n 1^n 0^n 1^n | n \geq 0\}$

   (b) (d) $\{t_1 \# t_2 \# ... \# t_k | k \geq 2$, each $t_i \in \{a, b\}^*$, and $t_i = t_j$ for some $i \neq j\}$

2. Do Problem 2.31 or the Problem 2.32

   (2.31) Let B be the language of all palindromes over $\{0, 1\}$ containing equal number of 0s and 1s. Show that B is not context-free.

   (2.32) Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* |$ in w, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s $\}$. Show that C is not context-free.

3. Do Problem 2.34:

   Consider the language $B = L(G)$, where $G = (V, \Sigma, R, R)$ is the following grammar: $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

   $S \to TT|U$

   $T \to 0T|T0|\#$

   $U \to 0U00|\#$

   The pumping lemma for context-free languages, Theorem 2.34, states the existence of a pumping length p for B.

   What is the minimum p that works in the pumping lemma for the language B. Justify your answer.

4. Do Problem 2.44

   If A and B are languages, define $A \diamond B = \{xy \in A$ and $y \in B$ and $|x| = |y|\}$. Show that if A and B are regular languages, then $A \diamond B$ is a CFL.

5. Do Problem 2.54 (skipping part b)

   2.54 Let G be the following grammar:

   $S \to T \dashv$

   $T \to TaTb|TbTa|\epsilon$

   - (a) Show that $L(G) = \{w \dashv |w$ contains equal number of a's and b's $\}$. Use proof by induction on the length of w.
   - (c)Describe the DPDA that recognizes $L(G)$.

6. Do Problem 3.2 parts (b) and (d) and Problem 3.5:

   (a) *3.2* This exercise concerns TM M_1, whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that M_1 enters when started on the indicated input string:

- (b) 1#1
- (d) 10#11

(b) *3.5* Examine the formal definition of a Turing Machine to answer the following questions, and explain your reasoning:

- (a) Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?
- (b) Can the tape alphabet Γ be the same as the input alphabet Σ?
- (c) Can a Turing machine's head *ever* be in the same location in two successive steps?
- (d) Can a Turing machine contain just a single state?