4 Homework

Due: Monday, February 11, 2013.

Instructions

- Please, review the homework grading policy outlined in the course information page.
- On the *first page* of your solution write-up you *must* make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

  where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: Five of the following 6 problems
Points: 20 points per problem

1. (Problem 1.55, but for the different languages)

   The pumping lemma says that every regular language has a pumping length p, such thath every string in the language can be pumped if it has length p or more. If p is a pumping length for teh language A, so is any length $p' \geq p$. The **minimum pumping length** for A is the smallest p that is the pumping length for A. For example, of $A = 01^*$, the minimum pumping length is 2. The reason is that the string $s = 0$

is in A and has length 1 yet s cannot be pumped; but any string in A of length 2 or more contains 1 and hence can be pumped by dividing it so that $x = 0, y = 1$ and z is the rest.

For each of the following languages, give the minimum pumping length and justify your answer.

(a) ab^*a
(b) $aab \cup a^*b^*$
(c) $(abab)^*$
(d) ϵ
(e) $ababa$
(f) a^*bbba^*

2. (a) (Problem 1.47)
Let $\Sigma = \{1, \#\}$ *and let*
$Y = \{w | w = x_1\#x_2\#...\#x_k$ *for* $k \geq 0$, *each* $x_i \in 1^*$, *and* $x_i \neq x_j$ *for* $i \neq j\}$.
Prove that Y *is not regular.*

(b) (Problem 1.48)
Let $\Sigma = \{0, 1\}$ *and let*
$D = \{w | w =$ *contains an equal number of ocurrences of substrings* 01 *and* 10$\}$.
Thus $101 \in D$ *because* 101 *contains a single* 01 *and a single* 10, *but* $1010 \notin D$ *because 1010 contains two* 10*s and one* 01. *Show that* D *is a regular language.*

3. (Problem 1.62)
Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let D_k be the language of all strings that have at least one a among the last k symbols. Thus $D_k = \Sigma^* a(\Sigma \cup \epsilon)^{k-1}$. Describe a DFA with at most $k + 1$ states that recognizes D_k in terms of both a state diagram and a formal description.

4. (Problem 2.1 for the given strings)
The CFG G_4 is given as follows:

$$
\begin{aligned}
E &\rightarrow E + T | T \\
T &\rightarrow T \times F | F \\
F &\rightarrow (E) | a
\end{aligned}
$$

Give parse trees and derivations for each string:

- $a + a$
- $(a \times a)$
- $a + a \times a$
- $(a + a) \times a$

5. Give the context-free grammars that generate the following languages:

   - 2.4 (e) $\{w | w = w^R$, that is, w is a palindrome $\}$
   - 2.6 (b) The complement of the language $\{a^n b^n | n \geq 0\}$
   - 2.6 (d)
     $\{x_1\#x_2\#...\#x_k | k \geq 1, x)i \in \{a, b\}^*$, and for some i, j
     we have $x_i = x_j^R\}$

6. Do the following:

   (a) (Problem 2.9)
   Give the CFG that generates the language
   $A = \{a^i b^j c^k | i = j$ or $j = k$ where $i, j, k \geq 0\}$

   Is your grammar ambiguous? Why, or why not?

(b) (Problem 2.13)

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

$S \rightarrow TT|U$

$T \rightarrow 0T|T0|\#$

$U \rightarrow 0U00|\#$

(a) Describe $L(G)$ in English.

(b) Prove that $L(G)$ is not regular.