3 Homework

Due: Thursday, January 31, 2013.

Instructions

- Please, review the homework grading policy outlined in the course information page.
- On the *first page* of your solution write-up you *must* make explicit which problems are to be graded for regular credit, whic problems are to be graded for extra credit, and which problems you did not attmept. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

  where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 5 of the following 7 problems
Points: 20 points per problem

1. Give the regular expressions generating the languages from Problem 1 in Homework 2, i.e.:

   (a) $\{w|w$ the length of w is at most $5\}$

   (b) $\{w|w$ is any string except aa and $aaa\}$

   (c) $\{w|w$ every odd position of w ia $a\}$

(d) $\{w|w$ contains at least two as and at most one $b\}$

(e) $\{w|w$ begins with a and ends with b, or begins with b and ends with $a\}$

2. Give the regular expressions generating the languages from Problem 2 in Homework 2, i.e.:

   (a) $\{w|w$ begins with 1, and ends with 0$\}$

   (b) $\{w|w$ contains at least three 1s $\}$

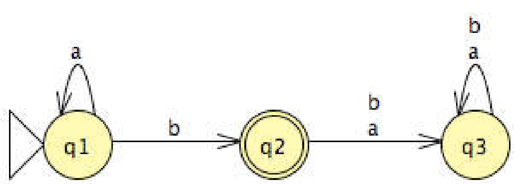
   (c) $\{w|w$ contains substring 0101$\}$ (i.e., $w = x0101y$

   (d) $\{w|w$ has length at least 3 and its third symbol is a 0$\}$

   (e) $\{w|w$ does not contain the substring 011$\}$

3. Use the procedure described in Lemma 1.60 in the text to convert the following DFA to a regular expression in two different ways:

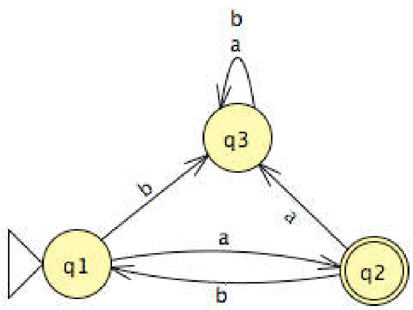
   - eliminating first state 3, then state 2, then state 1
   - eliminating first state 1, then state 2, then state 3



   Show the resulting GNFA after each step, and do not try to simplify your answer (except for eliminating all instances of $\emptyset$ in unions and all instances of ϵ in concatenations.

4. Use the procedure described in Lemma 1.60 in the text to convert the following DFA to a regular expression in two different ways:

- eliminating first state 3, then state 2, then state 1
- eliminating first state 1, then state 2, then state 3



Show the resulting GNFA after each step, and do not try to simplify your answer (except for eliminating all instances of $\emptyset$ in unions and all instances of ϵ in concatenations.

5. For any string $w = w_1w_2...w_n$, the reverse of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n...w_2w_1$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}} | w \in A\}$. Show that if A is regular, then so is $A^{\mathcal{R}}$.

6. *Problem 1.42* For languages A and B, let the *perfect shuffle* of A and B be the language $\{w | w = a_1b_1...a_kb_k$, where $a_1...a_k \in A$ and $b_1..b_k \in B$, for each $a_i, b_i \in \Sigma\}$.

   Show that the class of regular languages is closed under *perfect shuffle*.

7. Prove or disprove the following:

   (a) Every subset of a regular language is a regular language.

   (b) Every subset of a nonregular language is a nonregular language.

   (c) If A is a regular language and B is a language such that AB is regular, then B is regular.

   (d) If A is a regular language and B is a language such that $A - B$ is regular, then B is regular.

   (e) For any language A and its complement A' the language $A \cup A'$ is regular.