2 Homework

Due: Thursday, January 24, 2013, 6:00 pm in class.

Instructions

- Please, review the homework grading policy outlined in the course information page.
- On the *first page* of your solution write-up you *must* make explicit which problems are to be graded for regular credit, whic problems are to be graded for extra credit, and which problems you did not attmept. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

  where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
- You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 4 of the following 5 problems
Points: 25 points per problem

1. Give the state diagram of DFAs recognizing the following languages. In all parts the alphabet is either $\{a, b\}$ or $\{a, b, c\}$ when the letter c is mentioned in the problem statement.

   (a) $\{w | w$ the length of w is at most 5$\}$

   (b) $\{w | w$ is any string except aa and $aaa\}$

(c) $\{w|w$ every odd position of w ia $a\}$

(d) $\{w|w$ contains at least two as and at most one $b\}$

(e) $\{w|w$ contains an even number of as and at most one $b\}$

2. Give the state diagram of DFAs recognizing the following languages. In all parts the alphabet is either $\{0,1\}$.

   (a) $\{w|w$ begins with 1, and ends with $0\}$

   (b) $\{w|w$ contains at least three 1s $\}$

   (c) $\{w|w$ contains substring $0101\}$ (i.e., $w = x0101y$ for some x and y.

   (d) $\{w|w$ has length at least 3 and its third symbol is a $0\}$

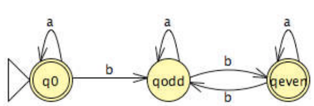
   (e) $\{w|w$ does not contain the substring $011\}$

3. Use the construction in the proof of Theorem 1.45 in Sipser ("The class of regular languages is closed under the union operation.") to give the state diagram of NFAs recognizing the union of the languages described in

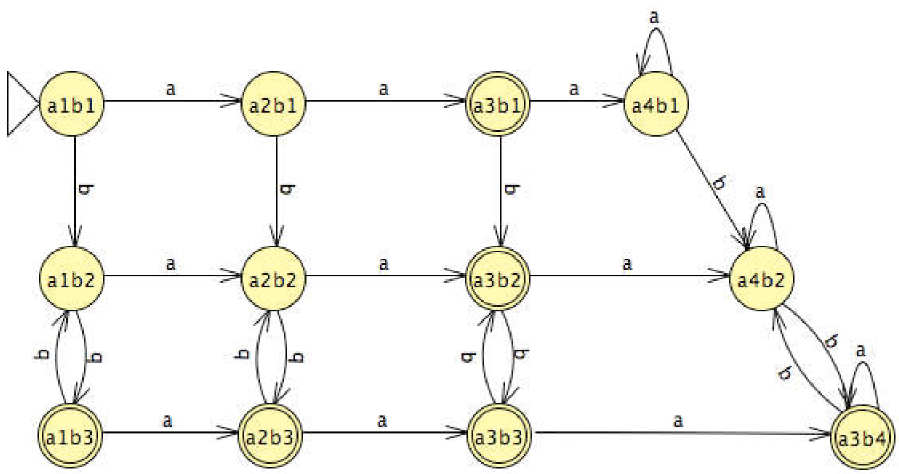
   (a) Exercises 2a and 2b

   (b) Exercises 2c and 2e

   (c) Produce the 5-tuple definition of the following DFA:



(d) Produce the 5-tuple definition of the following DFA:



4. Read the informal definition of a finite state transducer (FST) given in Exercise 1.24 (p. 87). Construct a state transition diagram for an FST whose input and output alphabets are both $\{0, 1\}$ and which works as follows: The input string represents the binary number n in reverse (i.e. with the least significant bit first). So, the string 0010 represents the decimal number 4, the string 010100 represents the decimal number 10. The output string is the input multiplied by two (but without the leading digit). So, the input 0010 would produce a result 0001, but the input 001 would produce 000 losing the leading 1. Of course, the output string has the same length as the input string.

   In addition to giving a diagram for your FST, briefly describe what each state represents.

5. Let $B_n = \{a^k | k \text{ is a multiple of } n\}$. Show that for each $n \geq 1$, the language B_n is regular.