10 Homework

Due: Thursday, April 11, 2013.

Problems

Required: 4 of the following 5 problems
Points: 25 points per problem

1. a Show that $P$ is closed under complement and concatenation.
   
b Let $A$ be a decidable language and let $D$ be a polytime decider for it. Consider the following algorithm for deciding whether a given non-empty string $s$ of length $n$ belongs to $A^*$: For every possible way of splitting $s$ into non-empty substrings $s = s_1s_2...s_k$, run $D$ on each substring $s_i$ in that split and accept iff all substrings are accepted by $D$ for some split. Derive an exact expression for how many possible such splits there are as a function of $n = |s|$. Use this to conclude that this algorithm does not run in polynomial time, even though $D$ does.

   c What does the result of part b imply about the closure of $P$ under the star operation?

2. Do the Problem 7.10
   Show that $ALL_{DFA}$ is in $P$.

3. Do the Problem 7.13
   Let $MODEXP = \{ <a, b, c, p> | a, b, c, p \text{ are binary integers such that } a^b \equiv c \pmod{p} \}$.
   Show that $MODEXP \in P$.
   Note that the most obvious algorithm does not run in polynomial time. Hint: Try it first where $b$ is a power of 2.
4. Do the Problem 7.24
   Let $CNF_k = \{<\phi> | \phi \text{ is a satisfiable cnf-formula where each variable appears in at most } k \text{ places}\}$.
   
   (a) Show that $CNF_2 \in P$.

   (b) Show that $CNF_3$ is NP-complete.

5. Do the Problem 7.27
   A cut in undirected graph is a separation of the vertices $V$ into two disjoint subsets $S$ and $T$. The size of the cut is the number of edges that have one endpoint in $S$ and the other in $T$. Let
   $MAXCUT = \{<G,k> | G \text{ has a cut of size } k \text{ or more}\}$.
   Show that $MAXCUT$ is NP-Complete.