10 Homework

Due: Thursday, April 11, 2013.

Problems

Required: 4 of the following 5 problems
Points: 25 points per problem

1. a Show that **P** is closed under complement and concatenation.

   b Let A be a decidable language and let D be a polytime decider for it. Consider the following algorithm for deciding whether a given non-empty string s of length n belongs to A^*: For every possible way of splitting s into non-empty substrings $s = s_1 s_2 ... s_k$, run D on each substring s_i in that split and *accept* iff all substrings are accepted by D for some split. Derive an exact expression for how many possible such splits there are as a function of $n = |s|$. Use this to conclude that this algorithm does not run in polynomial time, even though D does.

   c What does the result of part b imply about the closure of **P** under the star operation?

2. Do the Problem 7.10

   Show that ALL_{DFA} is in P.

3. Do the Problem 7.13

   Let $MODEXP = \{< a, b, c, p > | a, b, c, \text{ and } p \text{ are binary integers such that } a^b \equiv c(mod p)\}$.

   Show that $MODEXP \in P$.

   Note that the most obvious algorithm does not run in polynomial time. Hint: Try it first where b is a power of 2.

4. Do the Problem 7.24

   Let $CNF_k = \{< \phi > | \phi$ is a satisfiable cnf-formula where each variable appears in at most k places$\}$.

   (a) Show that $CNF_2 \in P$.

   (b) Show that CNF_3 is NP-complete.

5. Do the Problem 7.27

   A cut in undirected graph is a separation of the vertices V into two disjoint subsets S and T. The size of the cut is the number of edges that have one endpoint in S and the other in T. Let

   $MAXCUT = \{< G, k > | G$ has a cut of size k or more$\}$.

   Show that $MAXCUT$ is NP-Complete.