1 Homework

Due: Thursday, January 17, 2013.

Instructions

- This assignment will not be accepted late. Also, the general grading scheme shown on the course web page does not apply: all problems will be graded completely.
- Hand in a paper copy of your solution before the end of the class on Thursday. Write legibly. If I cannot read something, I cannot grade it.
- You must also write down on the top of your submission with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

1. (10 pts total)

   (a) (5 pts) For each of these, give the resulting set by listing out all its elements:
   - $\{a, c, e\} \cap \{b, c, d, e\}$
   - $\{a, c, e\} \cup \{b, c, d, e\}$
   - $\{a, c, e\} - \{a, d, e\}$
   - $\{b, c\} \times \{a, b, e\}$

   (b) (5 pts) Given any set S, the power of S, written as $\mathcal{P}(S)$ or 2^S, is the set of all subsets of S. Write out $2^{\{a,b,c\}}$.

2. (10 pts total)

   (a) (5 pts) How many elements did you find in $2^{\{a,b,c\}}$? How many elements are there in $2^{\{a,b\}}$? (It is not necessary to list all of them.) In general, if S is a finite set containing n elements (which we write as $|S| = n$), make a reasonable conjecture based on these examples for a formula for $|2^S|$ in terms of n.

   (b) (5 pts) Give a rigorous proof that your formula is correct for any $n \geq 0$.

A brief explanation of the description of infinite sets:

There are essentially two forms of notation we use to describe infinite sets in this class:

- using ellipses (i.e., ...); or
- using set-builder notation.

Here are two examples, described using ellipses:

- $\mathcal{N}$ = the set of all natural numbers = $\{0, 1, 2, 3, ...\}$; and
- $\mathcal{Z}$ = the set of all integers = $\{... - 3, -2, -1, 0, 1, 2, 3, ...\}$.

Here is another example, which we define using both methods: The set of all natural numbers that are perfect squares is

$\{0, 1, 4, 9, 16, 25, ...\} = \{n | n = m^2 \text{ for some } m \in \mathcal{N}\} = \{n^2 | n \in \mathcal{N}\}$

Note that whenever a set is infinite, only the set-builder notation gives a mathematically rigorous specifications of that set. If a set is infinite (or even finite but has more elements than we want to list out), the use of ellipsis is simply a convenience designed to help our intuitive understanding, but is not as mathematically precise as set-builder notation.

3. (15 pts total)

   Define the set $\mathcal{N}^{2mod3}$ of all natural numbers that can be written as $3n + 2$ for some natural number n. These are the numbers that produce a remainder 2 when divided by 3.

   A set S is said to be *closed* under an operation if the result of applying the operation to one or more elements of that set is always in the set. (How many elements the operation is applied to depends on how many operands that operation takes.)

   - (5 pts) Is $\mathcal{N}$ closed under addition? Is it closed under subtraction? Explain briefly (no rigorous proof is required).
   - Prove or disprove (rigorously):
     - (5 pts) $\mathcal{N}^{2mod3}$ is closed under addition.
     - (5 pts) $\mathcal{N}^{2mod3}$ is closed under multiplication.

4. (10 pts total)

   Define the set $\mathcal{N}^{even}$ of all even natural numbers.

   Define the set $\mathcal{N}^{0mod3}$ of all natural numbers that can be written as $3n$ for some natural number n.

   Define the set $\mathcal{N}^{0mod6}$ of all natural numbers that can be written as $6n$ for some natural number n.

   - (5 pts) Give a rigorous proof that the set $\mathcal{N}^{even} - \mathcal{N}^{0mod3}$ is not empty.
   - (5 pts) Give a rigorous proof that the set $\mathcal{N}^{even} \cap \mathcal{N}^{0mod3} = \mathcal{N}^{0mod6}$

5. (15 pts total)

   A Farside Country has several bus companies that provide transportation throughout the country. The minister of transportation proudly claims that at least one of the bus companies provides wifi service in every express bus that carries at least 45 passengers.

   (a) (5 pts) An unhappy citizen of Farside Country claims that this is not true because he knows that the FlyingCarpet Company runs the express bus between the cities Bango and Mingo with seating for 50 passengers and does not provide wifi service on that bus.

   Does this logic refute the claim? Explain clearly why or why not.

   (b) (5 pts) Another unhappy citizen asserts that this claim is not true because she knows that every company uses at least one bus that is never used on an express route.

   Does this logic refute the claim? Explain clearly why or why not.

   (c) (5 pts) If neither of these assertions refutes the minister's claim, explain exactly what needs to be done to prove that the claim is false.

6. (15 pts total)

   Suppose in Java you have the following interface and class definitions: (assume the standard full constructors for classes)

```
interface Comparable{
  int compareTo(Comparable that); }
```

```
class Pet implements Comparable{
  String name;
  Person owner;
  ...
}

class Person{ ... }

interface IShape{ }

class Circle implements IShape{ ... }

class Rectangle implements IShape{ ... }

interface ILoS extends Comparable{ }

class MtLoS implements ILoS{ ... }

class ConsLoS implements ILoS{
  IShape first;
  ILoS rest;
  ...
}
```

- (5 pts) Each class and data definitions defines a class of data objects. Write down a lit of all subset relationships that are known from these definitions. For example, $Circle \subset IShape$.
- (5 pts) Suppose the programmer defines the following variables and follows with the shown assignments. For each assignment indicate whether it is a valid statement. If not, explain why the compiler will reject it.

```
// Definitions:
IShape c;
IShape s;
Circle cir;
Owner bob;
Pet poodle;
Comparable cmp1;
Comparable cmp2;
ILoS lst1;
ILoS lst2;
ILoS lst3;
ILoS lst4;
```

```
MtLoS mt;

// Assignments:
c = new Circle(...);
c = new Rectangle(...);
s = new Circle(...);
s = new Rectangle(...);
cir = new Circle(...);
cir = new rectangle(...);
bob = new Owner(...);
poodle = new Pet("Pooch",bob);
cmp1 = poodle;
cmp1 = bob;
cmp1 = new Circle(...);
cmp2 = mt;
ls1 = new MtLos();
ls2 = new ConsLoS(cmp1, ls1);
ls3 = new ConsLoS(cir, ls1);
ls4 = new ConsLoS(s, ls1);
mt = ls3;
```

- (5 pts) If we eliminate all invalid assignments, show for each variable in a valid assignment the class that represents its current runtime type. (When the object represented by its variable invokes a method, the search for the method begins in this class.)

7. (10 pts total)

   (a) (5 pts)
   Give the state transition diagram for the FA whose formal description is $(\{1,2,3,4\}, \{a,b\}, \delta, 1, \{1,4\})$, where δ is given by the following table:

	a	b
1	1	2
2	3	4
3	2	1
4	2	4

(b) (5 pts)
Give a formal description of the FA1 shown in Figure 1:

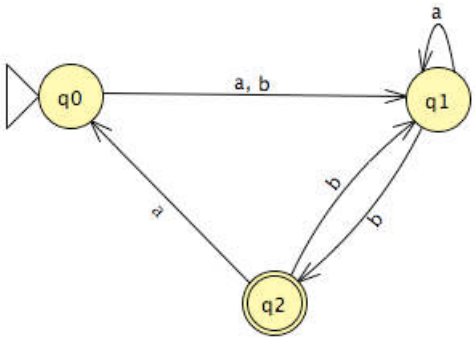

Figure 1: FA1