Exam 3 — CSU 390 Theory of Computation — Fall 2007

Instructions

- The exam is open book: You may use your notes, homeworks, any handouts and solutions provided to you in this class, the text (Sipser), and any other paper-based references. You may not use any electronic devices such as laptops, cell phones, PDAs, etc.
- Please, write your answers in the blue books provided. **Show all your work** and indicate your final answers clearly.

Problem 1 [20 points]

Recall that the reverse L^R of a language L is defined by
$L^R = \{w | w^R \in L\}$,
where w^R means the reverse of the string w.

1. Is the class of decidable languages closed under the reverse operation?
2. Is the class of Turing-recognizable languages closed under the reverse operation?

Give a proof of your answers. *Hint:* The same approach can be used to provide an answer to both questions. If it involves constructing a Turing machine, give only an informal implementation-level description.

Problem 2 [20 points]

Given a string $w = a_1 a_2 ... a_n$, where each $a_i \in \{0, 1\}$, define $ECHO(w) = a_1 a_1 a_2 a_2 ... a_n a_n$. Then, for any language $L \subseteq \{0, 1\}^*$, define $ECHO(L) = \{ECHO(w) | w \in L\}$. For example, if
$L = \{\epsilon, 0, 0110, 01010\}$, then
$ECHO(L) = \{\epsilon, 00, 00111100, 0011001100\}$.

Prove that each of the following language classes is closed under application of the $ECHO$ operation to its members:

1. The class **P**.
2. The class **NP**.

For the remaining problems you will need to create appropriate mapping reductions. In all cases, the reduction is simple and straightforward. No complicated constructions are required for any of these.

Problem 3 [40 points - (20, 20)]

Prove that the following languages are undecidable:

1. $SUB_{TM} = \{< M_1, M_2 > | M_1$ and M_2 are Turing machines and $L(M_1) \subseteq L(M_2)\}$.

2. $FINITE_{TM} = \{< M > | M$ is a TM and $|L(M)|$ is finite $\}$.

   In other words, (the encoding of) a Turing machine M is in $FINITE_{TM}$ if the number of strings accepted by M is finite.

   Hint: Reduce from A_{TM} or from $\overline{A_{TM}}$, whichever you are able to do. (One may not be possible.)

Problem 4 [20 points]

A tautology is a Boolean formula that always evaluates to 1 (i.e. $TRUE$) no matter what values are assigned to its variables. For example, the formula

$\phi = (x_1 \vee \overline{x_1}) \wedge (x_2 \vee \overline{x_2})$

is a tautology.

Define

$TAUT = \{< \phi > | \phi$ is a tautology $\}$.

At least one of $TAUT$ or its set complement $\overline{TAUT}$ is NP-complete. Give an NP-completeness proof for one of them.
Hint: Reduce from SAT.