CS5800 Pre-requisites Optional Self-Assess Exam

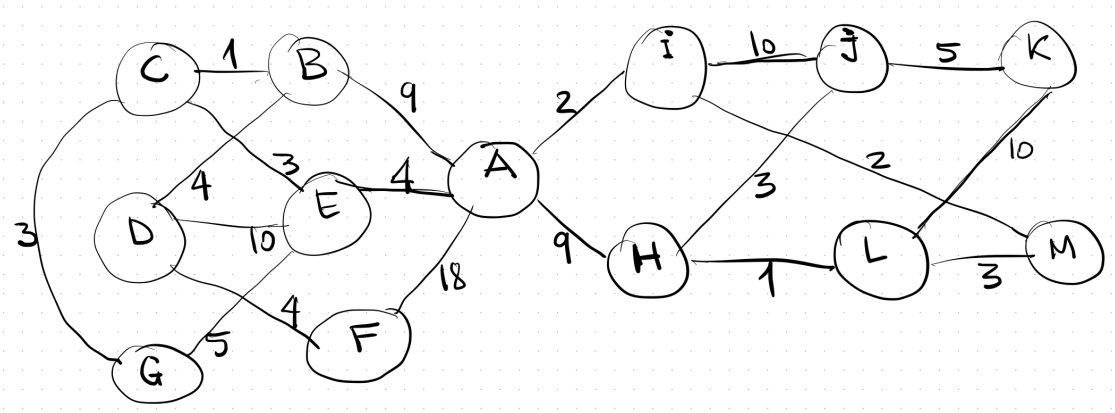
Instructions.

- This is an optional exam to test you readiness for 5800, especially on rigorous reasoning. It is based on the UG-level Algorithms prerequisites, in particular the CS1800 discrete-math course. It is 240 (or more) minutes test; there are 6 questions questions totaling 300 points.
- It is very important to justify your answers, because (1) partial credit will be given for correct arguments even if the final answer is incorrect, and (2) correct final answers with no arguments are worth very little.

Question	Your Score	Max
1 Graph Quiz		50
2 Merge-5-sort Recurrence		50
3 Set Counting		50
4 ★ Rearrange ABRACADABRA		50

Name: _____

Problem 1 Graph Quiz



Wherever possible below, select the node which is earlier in the alphabet first (e.g. prefer visiting node A first over node B, when the search allows you to visit either). For all points except last one (Dijkstra) ignore the edge weights.

i Show the adj-List representation of the graph.

ii Starting at A, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

iii How many edges have to be deleted to obtain a tree?

iv Starting at A, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

v Construct an max-edge Eulerian Tour: you have to stop in the same vertex you started, can pass through each edge at most once, want to pass through as many edges as possible. Make an informal argument for max-edges: that no Eulerian Tour can have more edges than yours.

vi What is the Shortest Path (by sum of weights) from D to L ?

Problem 2 Merge-5-Sort

A) Consider the following variant of the popular sorting Algorithm, now called Merge-5-Sort:

- split array in 5 parts of length $n/5$ each
- sort recursively each part using Merge-5-Sort
- MERGE the 5 sorted chunks into the output

If $T(n)$ is the runtime for Merge-5-Sort of array size n, write the recurrence following the logic above. Assume the MERGE procedure in last step can be implemented in linear time just as it is on regular MergeSort.

B) Prove by induction that $T(n) \leq Cn\log(n)$, for large enough n and some constant $C > 0$

C) An Alien God allows your code to call his magic "fussion-qbit-routine" and sort the middle $n/5$ part of any array instantly (in constant time); so now you only have to recursively call the procedure on the first two and the last two $n/5$ parts. If R(n) is the run time for array size n, what is the recurrence for R(n)?

D) Prove by induction that $R(n) \leq Dn$, for large enough n and some constant $D > 0$

Problem 3 Set Counting

There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

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Problem 4 Rearrange ABRACADABRA

A) How many different orderings are there of the letters in the word:

ABRACADABRA

Please assume that each instance of a character is indistinguishable from all others above (e.g. with this assumption there are 3 other orderings of 'EGG': 'EGG', 'GEG', and 'GGE' as we do not distinguish between copies of 'g').

Please leave any counting operations (factorial, combination, permutation etc) in your response (e.g. write 3! instead of 6 and 2! instead of 2).

(B) How many orderings, with restriction that the 5 "A" cannot be chunk "AAAAA" anywhere in the sequence?

C)★ How many orderings, with restriction that any 2 "A" cannot be adjacent to each other?