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Types for Untyped Languages:

- Ruby [Furr et al 09]
- Perl [Tang 06]
- Thorn [Wrigstad et al 09]
- ActionScript [Adobe 06]

- Typed Racket

Types for Untyped Languages:

- Reynolds 68
- Cartwright 75
- Wright & Cartwright 94
- Henglein & Rehof 95

"Some account should be taken of the premises in conditional expressions."

Reynolds, 1968

"Type testing predicates aggravate the loss of static type information."

Henglein & Rehof, 1995

Types and Predicates

```
(define-type Peano (U 'Z (List 'S Peano)))

(: convert : Peano -> Number)
(define (convert n)
  (cond [(symbol? n) 0]
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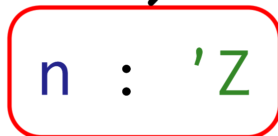

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```

```
n : (List 'S Peano)
```

```
(: combine :
   (U String Symbol) (U String Symbol) -> String)

(define (combine s s*)
  (cond [(and (string? s) (string? s*))
         (string-append s s*)]
        [(string? s)
         (string-append s (symbol->string s*))]
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         (string-append (symbol->string s) s*)]
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s : String

s* : String

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s : String

s* : Symbol

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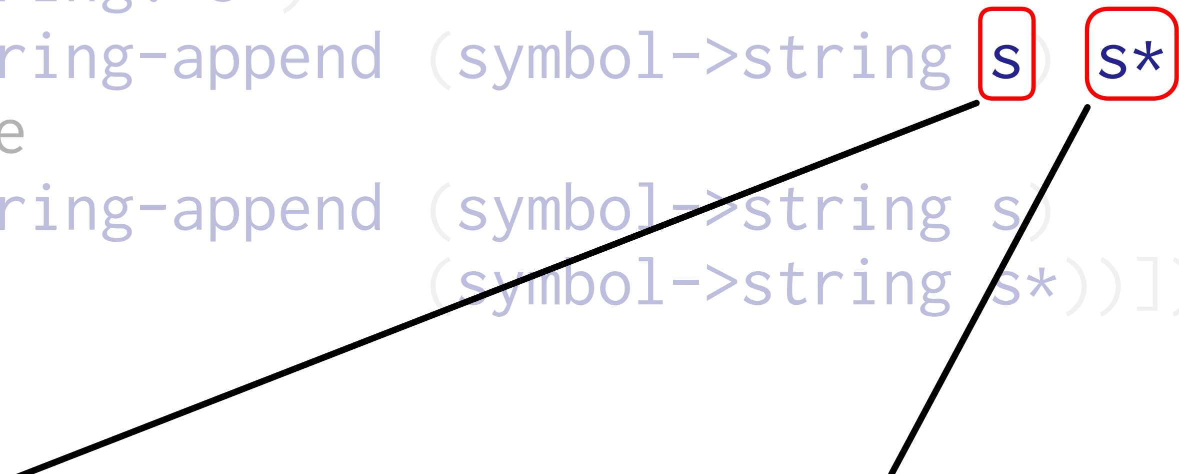

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```

s : Symbol

s* : Symbol

Types and Propositions

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```

```
⊢ (List 'S Peano) @ n
```

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```

$\vdash \overline{\texttt{Symbol @ n}}$

$\vdash$ `(List 'S Peano) @ n`

```
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(: convert : Peano -> Number)
(define (convert n)
  (cond [(symbol? n) 0]
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```

$$\frac{\vdash \texttt{Peano @ n} \qquad \vdash \overline{\texttt{Symbol}} \texttt{ @ n}}{\vdash \texttt{(List 'S Peano) @ n}}$$

```
(define-type Peano (U 'Z (List 'S Peano)))

(: convert : Peano -> Number)
(define (convert n)
  (cond [(symbol? n) 0]
        [else (add1 (convert (rest n)))]))
```

$$\frac{\vdash \texttt{(U 'Z (List 'S Peano))} \ @\ \texttt{n} \qquad \vdash \overline{\texttt{Symbol} \ @\ \texttt{n}}}{\vdash \texttt{(List 'S Peano)} \ @\ \texttt{n}}$$

```
(: combine :
   (U String Symbol) (U String Symbol) -> String)

(define (combine s s*)
  (cond [(and (string? s) (string? s*))
         (string-append s s*)]
        [(string? s)
         (string-append s (symbol->string s*))]
        [(string? s*)
         (string-append (symbol->string s) s*)]
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         (string-append (symbol->string s)
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```

⊢ String @ s   ⊢ String @ s*

```
(: combine :
   (U String Symbol) (U String Symbol) -> String)

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        [(string? s)
         (string-append s (symbol->string s*))]
        [(string? s*)
         (string-append (symbol->string s) s*)]
        [else
         (string-append (symbol->string s)
                        (symbol->string s*))]))
```

$$\frac{\vdash \text{String @ s} \land \text{String @ s*}}{\vdash \text{String @ s} \qquad \vdash \text{String @ s*}}$$

```
(: combine :
   (U String Symbol) (U String Symbol) -> String)

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  (cond [(and (string? s) (string? s*))
         (string-append s s*)]
        [(string? s)
         (string-append s (symbol->string s*))]
        [(string? s*)
         (string-append (symbol->string s) s*)]
        [else
         (string-append (symbol->string s)
                        (symbol->string s*))]))
```

⊢ Symbol @ s*

```
(: combine :
   (U String Symbol) (U String Symbol) -> String)

(define (combine s s*)
  (cond [(and (string? s) (string? s*))
         (string-append s s*)]
        [(string? s)
         (string-append s (symbol->string s*))]
        [(string? s*)
         (string-append (symbol->string s) s*)]
        [else
         (string-append (symbol->string s)
                        (symbol->string s*))]))
```

$$\frac{\vdash \texttt{String} \; @ \; \texttt{s} \qquad \vdash \texttt{String} \; @ \; \texttt{s} \supset \overline{\texttt{String} \; @ \; \texttt{s*}}}{\vdash \texttt{Symbol} \; @ \; \texttt{s*}}$$

```
(string?      s )
```

```
(string?      s )

String @ s
```

```
(string?      s )
```

$\texttt{String @ s} \mid \overline{\texttt{String @ s}}$

(x:Any -> Bool : String@x | $\overline{\texttt{String@x}}$)

s

(string? s)

String @ s | $\overline{\texttt{String @ s}}$

Latent Propositions

```
(x:Any -> Bool : String@x | ‾String@x‾)
```

Objects

s

```
(string? s )
```

$$\texttt{String @ s} \mid \overline{\texttt{String @ s}}$$

Propositions

Latent Propositions

(x:Any -> Bool : String@x | $\overline{\text{String@x}}$)

Objects

car(s)

(string? (car s))

String @ car(s) | $\overline{\text{String @ car(s)}}$

Propositions

```
(λ ([s : (Pair Any Any)])
  (string? (car s)))
```

$\texttt{String @ car(s)} \mid \overline{\texttt{String @ car(s)}}$

```
(λ ([s : (Pair Any Any)])
  (string? (car s)))
```

(s:(Pair Any Any) -> Bool :
 String @ car(s) | $\overline{\texttt{String @ car(s)}}$)

Propositional Logic

Judgments

$$\Gamma \vdash e : T \; ; \; \varphi_1 \mid \varphi_2 \; ; \; o$$

Judgments

$$\Gamma \vdash e : T \;;\; \varphi_1 \mid \varphi_2 \;;\; o$$

```
e ::= n | c | (λ x : T . e) | (e e) | (if e e e)
```

Judgments

$$\Gamma \vdash e : T \; ; \; \varphi_1 \mid \varphi_2 \; ; \; o$$

```
T ::= Number | (U T ...) | #t | #f | (x:T -> T : φ|φ)
```

Judgments

$$\Gamma \vdash e : T \; ; \; \varphi_1 \mid \varphi_2 \; ; \; o$$

$$\varphi \; ::= \; T@\pi(x) \mid \overline{T@\pi(x)} \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \supset \varphi_2$$

Judgments

$$\Gamma \vdash e : T \; ; \; \varphi_1 \mid \varphi_2 \; ; \; o$$

Γ ::= $x{:}T$ . . .

Judgments

$$\Gamma \vdash e : T \; ; \; \varphi_1 \mid \varphi_2 \; ; \; o$$

Γ ::= ~~x : T~~ T@π(x) ...

Judgments

$$\Gamma \vdash e : T \; ; \; \varphi_1 \mid \varphi_2 \; ; \; o$$

$$\Gamma ::= \sout{x : T} \;\; \sout{T@\pi(x)} \;\; \varphi \;\; \ldots$$

Judgments

$$\Gamma \vdash \varphi$$

Judgments

$$\Gamma \vdash \varphi$$

$$\overline{\texttt{Number @ } x} \lor \overline{\texttt{String @ } y}\ ,\ \texttt{Number @ } x \vdash \overline{\texttt{String @ } y}$$

Typing

```
(if e1 e2 e3)
```

Typing

$$\frac{\Gamma, \varphi_{+} \vdash e_2 : T \; ; \; \varphi_{1+} | \varphi_{1-} \; ; \; o \qquad \Gamma, \varphi_{-} \vdash e_3 : T \; ; \; \varphi_{2+} | \varphi_{2-} \; ; \; o \qquad \Gamma \vdash e_1 : T' \; ; \; \varphi_{+} | \varphi_{-} \; ; \; o'}{\texttt{(if } e_1 \; e_2 \; e_3\texttt{)}}$$

Typing

$$\dfrac{\Gamma, \varphi_{+} \vdash e_2 : T \;;\; \varphi_{1+} | \varphi_{1-} \;;\; o \qquad \Gamma, \varphi_{-} \vdash e_3 : T \;;\; \varphi_{2+} | \varphi_{2-} \;;\; o \qquad \Gamma \vdash e_1 : T' \;;\; \varphi_{+} | \varphi_{-} \;;\; o'}{\Gamma \vdash (\texttt{if}\ e_1\ e_2\ e_3) : T \;;\; \varphi_{1+} \vee \varphi_{2+} \mid \varphi_{1-} \vee \varphi_{2-} \;;\; o}$$

Typing

$$\Gamma \vdash x : T$$

Typing

$$\frac{\Gamma \vdash T_x}{\Gamma \vdash x : T}$$

Evaluation

Scaling	Adapting
Multiple Arguments	Mutable Structures
Multiple Values	Mutable Variables
User-defined Datatypes	
Local Binding	

Local Binding

$$\frac{\Gamma \vdash e_0 : S \;;\; \varphi*_+ \mid \varphi*_- \;;\; o \qquad \Gamma,\ S@x,\ \overline{\#f\ @\ x} \supset \varphi*_+ \vdash e_1 : T \;;\; \varphi_+ \mid \varphi_- \;;\; o*}{\Gamma \vdash (\texttt{let}\ [x\ e_0]\ e_1) : T[o/x] \;;\; \varphi_+ \mid \varphi_-\ [o/x] \;;\; o*[o/x]}$$

Empirical Evaluation

Estimated usage of two idioms in Racket code base (600k lines)

- Local binding with `or`: $\sim$470 uses
- Predicates with Selectors: $\sim$440 uses

Prior Work

None of the Examples

- Shivers 91, Henglein & Rehof 95, Crary et al 98, ...

Just `convert`

- Aiken et al 94, Wright 94, Flanagan 97, Komondoor et al 2005

Everything but abstraction

- Bierman et al 2010

Conclusions

Propositions can *relate* types and terms

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Propositions can *relate* types and terms

Existing programs are a source of type system ideas

Code and Documentation

http://www.racket-lang.org

samth@ccs.neu.edu

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