name——————————————————

Quiz 3 Thursday March 2, 2006 Open book. One page of notes. Five problems. Each is worth 10 points. Write on this paper.

1. Huffman trees.

   (a) Draw a Huffman tree for the following frequency table. (You need not use a priority queue to construct it—so there will be several correct answers.)

   | s | t | u | v | w | x |
   |---|---|---|---|---|---|
   | 1 | 1 | 2 | 2 | 3 | 8 |

   (b) Give the cost $B(T)$ for this tree. Show your calculation so we can give you partial credit if the number you write down is wrong.

   $$B(T) = \sum_{c \in C} f(c) d_T(c)$$

   where $f(c)$ is the frequency of $c$ and $d_T(c)$ is the codeword length for $c$.

   (c) Now, assuming the same frequencies and three-bit codes (as in Figure 16.4(a), page 387), tell what the cost of fixed length encoding is.

2. Here is a problem similar to the weighted activity scheduling. You are to find the longest increasing subsequence of a sequence of distinct positive numbers. For example, in the sequence

   ```
   20    40    30    35
   ```

   the longest increasing subsequence is

   ```
   20    30    35
   ```

   (a) Find the largest increasing subsequence in

   ```
   25    40    45    30    50    80    60    65
   ```

   (b) For a dynamic programming solution, what arrays or other data structures would you maintain?

   (c) Write pseudocode to solve this problem and print out (in reverse order) the solution of the problem (so your printed out solution should be a decreasing sequence!).

3. Assembly line problem. Given the following information, show the arrays $f_1[j], f_2[j], l_1[j]$ and $l_2[j]$ and draw the assembly line with the optimal schedule in thick arrows as on page 326.

```
enter line 1: cost 6        leave line 1: cost 5
enter line 2: cost 3        leave line 2: cost 3

station number i:      1       2       3      4
S1,i cost:             5       7       2      6
S2,i cost:             9       4       6      3

transitions:    S1,1 to S2,2 : 2     S1,2 to S2,3 : 3    S1,3 to S2,4 : 1

                S2,1 to S1,2 : 2     S2,2 to S1,3 : 4    S2,3 to S1,4 : 3
```

4. Given the following adjacency lists, draw this directed graph and show the $d$ and $\pi$ values for each vertex. Then draw the "breadth first tree" which is the tree of all vertices reachable from the source and the edges $(\pi(v), v)$. Use the vertex numbered 6 as the source vertex.

```
1--> 2 --> 3  (there is an edge from 1 to 2 and from 1 to 3)
2--> 4 --> 5
3--> 6
4--> 1 --> 3
5--> 3
6--> 4 (vertex 6 is the source)
```

5. It is claimed that the following problem has a greedy solution: Given a set $\{x_1, x_2, ..., x_n\}$ of distinct points on the real line with $x_i \leq x_{i+1}$, find the smallest set of unit-length closed intervals that contain all of the given points. Here is a proposal to start the solution with a greedy choice: pick a unit interval $I_1 = [s_1, f_1]$ where $s_1 = x_1$. Do the first two parts of showing this is a greedy solution to the problem:

   (a) Identify what subproblem is left after $I_1$ is chosen.

   (b) Given an arbitrary unknown optimal solution $O$, show that a substitution can be made in it so that $I_1$ substitutes for one of the intervals in $O$ and the new set of intervals obtained this way is also an optimal solution.

   (You are *not* asked in this problem to show that the combination of the greedy choice and an optimal solution to the subproblem yields a solution to the whole problem.)