Rumors and Routes

Rajmohan Rajaraman

Northeastern University, Boston

May 2012

Outline

- Rumor spreading
  - Bounds on cover time
- Small-world networks
  - Low-degree low-diameter models
  - Decentralized search or network navigability

Rumor Spreading

- Information dissemination in a network
- Push paradigm:
  - Every node that has the rumor forwards it to a random neighbor
  - Theorem: Push completes in O(n log(n)) steps whp
- Pull paradigm:
  - Every node that does not have the rumor currently queries a random neighbor
- Push-pull paradigm

Push-Pull Rumor Spreading

- Conductance of a graph

$$\phi = \min_{S \subset V, S \neq \emptyset} \frac{e(S, V-S)}{\min(e(S), e(V-S))}$$

- Theorem [Giakkoupis 11]: The push-pull process completes in O(log(n)/Φ) steps with high probability
  - Weaker bounds given earlier by [Chierichetti-Lattanzi-Panconesi 09, 10]

Small-World Networks

- Six degrees of separation:
  - Diameter of "real" even very large networks appears to be small
  - How do we model them?
- G(n,p) graphs, with $p = \Omega(\log(n)/n)$:
  - Diameter $O(\log(n))$
  - Average degree $\Omega(\log(n))$
- Random 3-regular graphs are expanders
- Small-world graphs:
  - Model with constant degree, locality, and low diameter

Models with Low Diameter

- [Watts-Strogatz 00]
  - Consider a grid over n nodes
    - Underlying low-degree graph capturing locality
  - For each node, add a long-range edge
    - To a neighbor chosen uniformly at random
- Theorem [Flaxman-Frieze 04]: If we add a random "1-out" edge for each node in any connected graph G, we obtain an expander whp.

Decentralized Search

- Myopic shortest paths:
  - If long-range takes you closer to destination, use long-range path
  - Otherwise follow along shortest path in short-range subgraph
- Exercise: Though the Watts-Strogatz model yields low-diameter, myopic navigation takes long paths
- Alternative model [Kleinberg 00]: Suppose long range edge from u to v with probability proportional to $1/d(u,v)^{\alpha}$, where d(u,v) is short-range distance
  - Theorem: If $\alpha = 2$, then myopic navigation completes in $O(\log^2 n)$ steps

Take Away Messages

- Combination of local strategies can yield exponential improvement
- A single random edge per node can yield an expander
- Small diameter may not imply navigability
  - Appropriate long-range contact distributions lead to good navigability
- Techniques:
  - Random graph models
  - Martingales and large deviations
  - Emphasis on local algorithms for search