Mobility Models for Ad hoc Network Simulation

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***Abstract*— In this paper, we propose a novel general technique, based on renewal theory, for analyzing mobility models in ad hoc networks. Our technique enables an accurate derivation of the steady state distribution functions for node movement parameters such as distance and speed. We first apply our technique to the random waypoint model and provide alternative proofs for previous claims about the discrepancy between the steady state average speed and the average speed associated with the simulated distribution [1]. Our main contribution is a new methodology for simulating mobility which guarantees steady state for node movement distributions from the start of the simulation. Our methodology enables the correct and efficient simulation of a desired steady state distribution, and can be implemented in a manner transparent to the user. We support our claims through both formal proofs as well as extensive simulations.**

I. Introduction

The defining characteristics of mobile ad hoc networks include the lack of a fixed communication infrastructure, limited resources for computation and energy, and the mobility of the network nodes. Consequently, ad hoc network protocols need to be decentralized, need to use limited computational and energy power, and should be robust to the movement of the nodes. While the notions of "decentralized" protocols, and energy- and computational-efficiency can be formalized and indeed has been done in previous studies (e.g., see [2], [3], [4]), the notion of robustness to mobility is still vague. One of the main hurdles in making this notion precise is the lack of analytical models for capturing mobility.

Simulations provide a valuable means to compare different protocols and study their performance in terms of efficiency and robustness. Indeed, network simulation environments such as ns-2 [5], GloMoSim [6], Qualnet [7], or Opnet [8] are the most commonly used tool to evaluate and compare the performance of mobile ad hoc networks (MANET) protocols. Since mobility models are inherently probabilistic, the assumption is that averaging over sufficiently many runs will lead to a good estimate. One of the problems with existing simulations of the underlying mobility models is that the simulation may not capture the steady state behavior since the convergence time may exceed the length of the simulation. This was noted in a recent study of the random waypoint model that established that even though the speed of a mobile station is chosen uniformly at random from a given interval, the speed of a node in steady state is not necessarily from the same distribution; indeed, it was argued that the two may significantly differ [1].

The *random waypoint* model, which is widely used in simulating protocols designed for mobile ad hoc networks [9], [10], [11], is defined as follows. Each node picks a random destination uniformly within an underlying physical space, and travels with a speed v whose value is uniformly chosen in the interval $(0, V_{max}]$. V_{max} is some parameter that can be set to reflect the degree of mobility. Upon reaching the destination, the node pauses for a time period Z, and the process repeats itself afterwards. Several variants of the random waypoint model have also been studied [12], [13], [14], [15].

Our results. In this paper, we present a new framework for analyzing random waypoint-like models and show how the framework can be used for providing accurate simulations of the steady state behavior. Our contributions are twofold:

- **Analytical:** We present the random waypoint model as a renewal process and derive an accurate characterization of the steady-state distributions of speed and residual distance, given arbitrary distributions for speed and distance of individual node movements. Our analysis provides a derivation of the steady state average speed for the random waypoint model and confirms previous claims that there may be significant discrepancies between the steady state average and the average specified by the user [1].
- **Methodological:** A useful property of our analysis is that we are able to quantitatively characterize the steady state *distributions* rather than specific parameters thereof, such as the expectation or particular moments. We exploit this generality to provide a novel methodology for simulating mobility with arbitrary steady state distributions. Importantly, our methodology guarantees steady state for node movement distributions from the *start of the simulation*, thus enabling the efficient simulation of large-scale ad hoc networks (see Theorem 3 and Corollary 3.1). Furthermore, it can be implemented in a manner transparent to the user.
  We support our claims through both formal proofs as well as extensive simulations. Finally, we illustrate the generality of our analysis and methodology by applying them to the random waypoint model with arbitrary pause time distributions.

Related work. As discussed above, simulation and experimental studies often model the motion of a node as a mobility vector, that gives the direction and speed of the node. Each node independently chooses a mobility vector that defines its motion for a period of time ("movement epoch"), after which a new random mobility vector is assigned (e.g., see [16], [17]). Recently, a mobility model that allows for varying speeds during a movement epoch has also been proposed [18]. Models for group movement, whereby a group of nodes may move in the same general direction have also been recently studied [19], [16]. Idealized models of mobility have been studied in [20] where the effect of mobility on the capacity of ad hoc networks was analyzed.

The random waypoint is the most commonly used mobility model for simulation purpose. Recently several studies of this model have revealed various flaws or unexpected properties. In [1], it has been shown that the average speed of mobile nodes decays with time. This is due to the fact that low speed nodes spend more time traveling to their destination than high speed nodes. Other mobility models such as the one proposed in [21] have been shown to suffer from the same speed decay phenomenon. In [22], it has been shown through simulation that the random waypoint model does not lead to a uniform distribution of nodes location. This result has been analytically confirmed in [23], where it was shown that the nodes distribution is higher within the center of the simulation area. It was shown that the variation in density depends on the nodes average speed and pause time, and that higher speeds lead to a more uniform distribution. It was also shown that increasing the nodes speed results in an increased network connectivity. Recently, stochastic properties of certain characteristics of the random waypoint model, including the length, duration, and the direction of a movement epoch, have been analyzed [24], [25].

Recent theoretical studies of ad hoc networks have measured mobility in terms of the changes in the underlying transmission graph. For instance, we can analyze the robustness of an ad hoc network routing protocol by considering the amount of work needed to be done when an elementary change in the transmission graph occurs; that is, when an edge is removed or added or the neighborhood of a node changes [26], [27]. Another interesting model for capturing node mobility is the recently proposed adversarial network model [28], in which an adversary may alter the underlying graph in an unpredictable manner. Arbitrary node movements can be represented by adversarial changes in topology.

Outline of the paper. In Section II, we consider a slightly modified random waypoint model with zero pause time. We derive the steady state distribution functions for the nodes speed and residual travel distance. In Section III, we propose a technique to simulate nodes mobility such that, from the beginning of the simulation, the nodes' observed speed is uniformly distributed within the interval (V_{min}, V_{max}). In Section IV, we show through simulation that our technique addresses the drawbacks associated with previous simulations of the random waypoint model, and confirms our theoretical results. In Section V, we extend our results to the case where the pause time is specified by a probability distribution functions. In Section VI, we discuss the general applicability of our approach and results, and conclude in Section VII.

II. Characteristics of nodes' movement at steady state

We start with the definition of a simple mobility model.

Mobility Model S1A: Each node's movement is characterized by nonoverlapping *time periods* $X_1, X_2, \cdots$. During each period X_i, the node travels with some random speed V_i for some random distance D_i. The distribution functions (*df*s, in short) of distance D and speed V are user-specified, and denoted by $F_D(x)$ and $F_V(x)$ [1]. Within each period, D_i and V_i are selected independently. D_i's are mutually independent, so are V_i. Hence, the random variables X_i, defined as $\frac{D_i}{V_i}$, are also mutually independent. The df for time period X, denoted as $F_X(\cdot)$, is determined by F_V and F_D and will be discussed again later. Due to its frequent usage, we denote the mean of X as $\mu = E[X]$.

In model S1A, the random variable X_i's are independently and identically distributed (i.i.d.) with common distribution function $F_X(x)$. Thus, $\{X_i\}$ is a *renewal process* (see, e.g., [29], chap XI). The connection of such mobility models to renewal processes was also observed by [23]. We also note that the random waypoint model with zero pause time can be regarded as a special case of S1A if the underlying physical space is assumed to be a homogeneous metric space (for example, a two-dimensional torus, or the surface of a sphere). In this case, we can specify the speed distribution $F_V(\cdot)$ as a uniform one. Many studies on ad hoc networks adopt a random waypoint model in which the underlying space is a rectangle. Such a space is not homogeneous and hence is not covered by model S1A. We discuss this case in Section IV.

A. Speed distribution at steady state

Denote by V^t the speed of the node at time instant $t > 0$, and let $F_V^t(\cdot)$ and $f_V^t(\cdot)$ be its corresponding cumulative distribution function (cdf, in short) and probability distribution function (pdf, in short). We are interested in the analytic expressions of these functions, especially as $t \to \infty$. We have the following result.

Theorem 1: In Model S1A, V^t has the following limit probability functions (cdf and pdf):

$$\lim_{t\to\infty} F_V^t(z) = \frac{E[D]}{E[X]} \cdot \int_0^z \frac{1}{x}\,\mathrm{d}F_V(x) \qquad (1)$$

$$\lim_{t\to\infty} f_V^t(z) = \frac{E[D]}{E[X]} \cdot \frac{1}{z} f_V(z) \qquad (2)$$

where $E[D]$ and $E[X]$ are the means of distance D and renewal period X, respectively.

Proof: The proof is a standard renewal argument followed by application of the Renewal Theorem. (see, e.g., [29], p363).

[1]Unless explicitly specified, all random variables considered in this article are continuous and nonnegative.

Write $A_z(t) = \Pr\{V^t > z\}$, conditioning on the time $X_1 = x$ of the first renewal,

$$\Pr\{V^t > z | X_1 = x\} = \begin{cases} \Pr\{V_1 > z | X_1 = x\} & \text{if } t < x \\ A_z(t-x) & \text{if } 0 < x \le t \end{cases}$$

Then by the law of total probability,

$$\begin{aligned} A_z(t) &= \int_0^\infty \Pr\{V^t > z | X_1 = x\} \, \mathrm{d}F_X(x) \\ &= \Pr\{V_1 > z, X_1 > t\} \\ &\quad + \int_0^t A_z(t-x) \, \mathrm{d}F_X(x) \end{aligned} \tag{3}$$

Now,

$$\begin{aligned} &\Pr\{V_1 > z, X_1 > t\} \\ &= \int_z^\infty \Pr\{X_1 > t | V_1 = y\} \, \mathrm{d}F_V(y) \\ &= \int_z^\infty \Pr\{V_1 \times X_1 > ty | V_1 = y\} \, \mathrm{d}F_V(y) \\ &= \int_z^\infty \Pr\{D_1 > ty | V_1 = y\} \, \mathrm{d}F_V(y) \\ &= \int_z^\infty \Pr\{D_1 > ty\} \, \mathrm{d}F_V(y) \\ &= \int_z^\infty [1 - F_D(ty)] \, \mathrm{d}F_V(y) \end{aligned} \tag{4}$$

Hence we have the renewal equation

$$A_z(t) = \int_z^\infty [1 - F_D(ty)] \, \mathrm{d}F_V(y) + \int_0^t A_z(t-x) \, \mathrm{d}F_X(x)$$

Assume $\mu = E[X] < \infty$, according to renewal theorem,

$$\begin{aligned} &\lim_{t\to\infty} A_z(t) \\ &= \mu^{-1} \int_0^\infty \int_z^\infty [1 - F_D(ty)] \, \mathrm{d}F_V(y) \, \mathrm{d}t \\ &= \mu^{-1} \int_z^\infty \int_0^\infty [1 - F_D(ty)] \, \mathrm{d}t \, \mathrm{d}F_V(y) \\ &= \mu^{-1} \int_z^\infty \frac{1}{y} \int_0^\infty [1 - F_D(ty)] \, \mathrm{d}(ty) \, \mathrm{d}F_V(y) \\ &= \mu^{-1} \int_z^\infty \frac{1}{y} \times E[D] \, \mathrm{d}F_V(y) \\ &= \frac{E[D]}{E[X]} \cdot \int_z^\infty \frac{1}{y} \, \mathrm{d}F_V(y) \end{aligned}$$

Equation 2 follows directly from Equation 1 and this completes the proof. ■

For convenience, we also write $\lim_{t\to\infty} F_V^t(z)$ as $F_V^\infty(z)$ and $\lim_{t\to\infty} f_V^t(z)$ as $f_V^\infty(z)$. It is not hard to see that $f_V^\infty(z)$ is a proper probability density function, with $\int_0^\infty f_V^\infty(z) = 1$ (e.g., consider the derivation of $E[X]$ in terms of D and V).

Corollary 1.1: The mean of the speed at time ∞ is

$$E[V^\infty] = \int_0^\infty z f_V^\infty(z) dz = \frac{E[D]}{E[X]} \tag{5}$$

From the remarks following Theorem 1, as well as Corollary 1.1, one can immediately see that the distribution of speed V^t as $t \to \infty$ is solely determined by the provided speed distribution $F_V(\cdot)$, and independent of the distance or traveling time. It may help to consider the fact that, in Model S1A, for each renewal period X_i, the nodes select D_i and V_i independently according to their own dfs.

More importantly, Equation 2 tells us qualitatively that $f_V^\infty(z)$ is proportional to $f_V(z)/z$, i.e., after the simulation runs for a while, the speeds of the inspected nodes are biased toward smaller values. This phenomenon was first pointed out by [1]. Intuitively, nodes with higher speeds tend to finish the corresponding renewal period more quickly, while lower speeds tend to stay on. Hence, the speeds V^t we inspect at large time t are biased toward smaller values.

We now apply Theorem 1 to the random waypoint model to gain some insights. For illustration purpose, we use the slightly modified random waypoint used in the analysis of [1]. Let's call it **Model RW1**, where the destination is chosen uniformly within a disk of radius R_{max} centering at the picking node (implicit assumption: boundless region). The speed is uniformly chosen from $[V_{min}, V_{max}]$ with $V_{max} > V_{min} > 0$. And it was derived in [1] that $E[D] = \frac{2}{3}R_{max}, E[X] = \frac{2R_{max}}{3(V_{max}-V_{min})} \ln(\frac{V_{max}}{V_{min}})$ and $f_V(x) = \frac{1}{V_{max}-V_{min}}$.

According to Corollary 1.1,

$$E[V^\infty] = \frac{V_{max} - V_{min}}{\ln(\frac{V_{max}}{V_{min}})},$$

And it is easy to see that as $V_{min} \to 0$, $E[V^\infty] \to 0$, this is rather different from the normally expected mean speed of $\frac{V_{max}}{2}$. We mention that the above average is equal to the time average speed $\lim_{T\to\infty} \frac{1}{T} \int_0^T v^t dt$ given in [1].

Considering the uniform speed distribution in Model RW1, we have

Corollary 1.2: Under model RW1, we have:

$$f_v^\infty(z) = \frac{c}{z} \qquad (z \in [V_{min}, V_{max}]) \tag{6}$$

where the constant $c = 1/\ln \frac{V_{max}}{V_{min}}$.

Hence, as the simulation progresses, the distribution of the speed deviates far away from originally expected (uniform).

Our experimental results, described in Section IV, agree with the analytical results obtained here. In Section III, we address the problem of distribution deviation. But first, we use our approach based on renewal theory to analyze the distribution of the residual distance.

B. Residual distance and residual time at steady state

Other than the speed V^t, another characteristic of nodes' movement at a given time t is the remaining distance to travel during the period X_i that contains t. We call this quantity *residual distance* and denote it by γ^t. For example, suppose renewal period X_i starts at time t_0 and ends at time t_1, then $\gamma^{t_0} = D_i$ and $\gamma^{\frac{t_0+t_1}{2}} = \frac{D_i}{2}$. We have the following result, whose proof is similar to that of Theorem 1, and provided in the Appendix.

Theorem 2: In Model S1A, residual distance γ^t has the following limit probability functions (cdf and pdf):

$$\lim_{t\to\infty} F_\gamma^t(z) = \frac{1}{E[D]} \cdot \int_0^z [1 - F_D(x)]\,\mathrm{d}x \tag{7}$$

$$\lim_{t\to\infty} f_\gamma^t(z) = \frac{1}{E[D]} \cdot [1 - F_D(z)] \tag{8}$$

Observe that the distance limit distribution is also solely determined by the provided distance df $F_D(\cdot)$. For convenience, we also write $\lim\limits_{t\to\infty} F_\gamma^t(z)$ as $F_\gamma^\infty(z)$ and $\lim\limits_{t\to\infty} f_\gamma^t(z)$ as $f_\gamma^\infty(z)$. It can be easily verified that $f_\gamma^\infty(z)$ is a proper probability density function.

Similar to the definition of residual distance, let *residual waiting time* β^t be the remaining traveling time from instant t to the end of X_i for $t \in X_i$. It is known that β^t has the following distribution function [29], [30]. (This can also be derived directly using the technique demonstrated in the proof of Theorem 1):

$$G(x) = \lim_{t\to\infty} \Pr\{\beta^t \le x\} = \mu^{-1} \int_0^x [1 - F_X(y)]\,\mathrm{d}y \tag{9}$$

recall that $F_X(y)$ stands for the cdf of renewal period X.

III. Simulation with stable speed distribution

As observed in Section II-A, as the simulation progresses, the expected speed of the nodes decreases. This phenomenon is quantitatively characterized in Equations 1 and 2. In what follows, we address this problem and propose a solution based on the concept of *stationary renewal process* [29], [30].

If the simulation began indefinitely far in the past ($-\infty$), then at $t = 0$, the node would have V^t with a limit distribution given by Theorem 1. And to the user, it appears that the simulation has a stable speed distribution immediately from the start. This gives her the advantage of saving time originally necessary for the moving process to converge to some stable speed distribution. This is especially useful for simulating large mobile networks and thus makes the simulation process more scalable. In contrast, the experimental results in Section IV as well as those of [1] indicate that for commonly-used input parameters the convergence time of the original random waypoint model may exceed hundreds of simulation seconds, which is a significant fraction of the length of typical simulations [10].

The natural question now is, how can we let our simulation begin indefinitely in the past? Notice that the (movement) state of a node at time $t \to \infty$ is completely characterized by V^t, γ^t and β^t, whose distributions are already obtained by our previous discussion. For the first period X_0, we pick distance D_0 according to $F_\gamma^\infty(z)$ and speed V_0 according to $F_V^\infty(z)$, and let $X_0 = \frac{D_0}{V_0}$. As for the following periods $X_1, X_2, \cdots$, we use Model S1A ($F_V(\cdot)$ and $F_D(\cdot)$). We thus obtain a renewal moving process which *effectively* started indefinitely far in the past. In the following, we rigorously justify the preceding intuition.

Mobility Model S1B: For the first period X_0 (start from $t = 0$), the traveling speed, distance (hence traveling time) are random chosen according to $F_V^\infty(z)$ and $F_\gamma^\infty(z)$ as described above, and the remaining periods are exactly the same as in Model 1A, using distributions $F_V(\cdot)$ and $F_D(\cdot)$.

We proceed to prove that the given process in Model S1B is stationary, i.e., X_0 has the distribution function given by (9), the limit cdf of residual time β^t.

First, given the definition of $X = \frac{D}{V}$, where D and V are independent, it is not hard to see that

$$F_X(t) = \Pr\{X \le t\} = \int_0^\infty F_D(tx)\,\mathrm{d}F_V(x) \tag{10}$$

Lemma 3.1: Model S1B is stationary, that is, X_0 has the cdf given by (9).

Proof:

$$\begin{aligned}
&\Pr\{X_0 \le t\} = \Pr\{\frac{D_0}{V_0} \le t\} \\
= \;& \int_0^\infty F_\gamma^\infty(tx)\,\mathrm{d}F_v^\infty(x) \\
= \;& \int_0^\infty \int_0^{tx} \mathrm{d}F_\gamma^\infty(y)\,\mathrm{d}F_v^\infty(x) \\
= \;& \int_0^\infty \int_0^{tx} \frac{1 - F_D(y)}{E[D]}\,\mathrm{d}y \quad \frac{E[D]}{E[X]}\frac{1}{x}\,\mathrm{d}F_V(x) \\
= \;& \mu^{-1} \int_0^\infty \frac{1}{x} \int_0^{tx} [1 - F_D(y)]\,\mathrm{d}y \;\; \mathrm{d}F_V(x) \\
& \text{(change of variable: } z = y/x) \\
= \;& \mu^{-1} \int_0^\infty \int_0^t [1 - F_D(xz)]\,\mathrm{d}z \;\; \mathrm{d}F_V(x) \\
= \;& \mu^{-1} \int_0^t \int_0^\infty [1 - F_D(xz)]\,\mathrm{d}F_V(x) \;\; \mathrm{d}z
\end{aligned}$$

$$= \mu^{-1} \int_0^t \left\{ 1 - \int_0^\infty F_D(xz)\,\mathrm{d}F_V(x) \right\} \mathrm{d}z$$
$$= \mu^{-1} \int_0^t [1 - F_X(z)]\,\mathrm{d}z$$

■

We have proved that the process $\{X_i\}(i = 0, 1, \ldots)$ in Model S1B is stationary. For this process, we denote the speed at time t as V_t^S, and we have the following main result, whose proof is provided in the Appendix:

Theorem 3: At any time $t > 0$, V_t^S of Model S1B has the limit distribution of V^t in the original Model S1A.

$$\Pr\{V_t^S \le x\} = \frac{E[D]}{E[X]} \cdot \int_0^x \frac{1}{y}\,\mathrm{d}F_V(y) \tag{11}$$

Similarly for γ_t^S, at any time $t > 0$:

$$\Pr\{\gamma_t^S \le x\} = \frac{1}{E[D]} \cdot \int_0^x [1 - F_D(y)]\,\mathrm{d}y \tag{12}$$

We stress that the identities are valid for any time $t > 0$. This is quite different from Theorems 1 and 2, which are only for large time t.

Theorem 3 provides a methodology for simulating mobility with arbitrary steady state distributions for speed (or residual distance). If one wants to achieve a target (stable) distribution function of speed $\Pr\{V_t^S \le x\}$, then all she has to do is to derive an appropriate df $F_V(y)$ by solving Equation 11, and then apply Model S1B. As an illustration for this methodology, we consider the interesting case in which the desired steady state distribution for speed is uniform. We invoke Theorem 3 to obtain the following result.

Corollary 3.1: For Model S1B, if $f_V(y) = ky, y \in [V_{min}, V_{max}], k = \frac{2}{V_{max}^2 - V_{min}^2}$, the speed V_t^S is uniformly distributed within $[V_{min}, V_{max}]$ at any time $t > 0$.

Proof: By the definition of $f_V(y)$, we have $\mathrm{d}F_V(y) = kydy$. Plugging this term into Equation 11, replacing k by the value specified, and calculating the integral yields the desired claim. ■

IV. Simulation Results

In this section, we present some Monte Carlo simulation results. One may need to simulate according to some prescribed distribution function $F(x)$. One way of doing this is by the *inverse transformation method*, which is described briefly as follows. Suppose the given $F(x)$ is strictly increasing[2], and u is some uniform random variable in $(0, 1)$. Then the random variable $X = F^{-1}(u)$ has cdf $F(x)$ because:

$$\begin{aligned} \Pr\{X \le x\} &= \Pr\{F^{-1}(u) \le x\} \\ &= \Pr\{F(F^{-1}(u)) \le F(x)\} \\ &= \Pr\{u \le F(x)\} = F(x) \end{aligned}$$

[2]All dfs in this paper satisfy this property.

A. Random Waypoint with Specified Distance Distribution Function

For the sake of testing the theoretical results developed in the previous sections, Monte Carlo simulations were performed under two scenarios, where $R_{max} = 1000\ m$ and $V_{min} = 0, V_{max} = 20\ m/s$:

- Original: (cf. Model S1A) During each renewal period, the node N picks distance D according to $F_D(r) = \frac{r^2}{R_{max}^2}$. (This corresponds to the model used in [1], where destination is uniformly picked from a disk centered at N with radius R_{max}). The speed distribution is uniform within $[V_{min}, V_{max}]$
- Modified: (cf. Model S1B) For the first period X_0, pick distance D_0 according to $F_d^0(r) = \frac{3}{2R_{max}}(r - \frac{r^3}{3R_{max}^2})$. Pick V_0 uniformly from $[V_{min}, V_{max}]$. For the following periods $X_i, i > 0$, pick D according to $F_D(r) = \frac{r^2}{R_{max}^2}$ and speed according to $F_V(x) = \frac{x^2 - V_{min}^2}{V_{max}^2 - V_{min}^2}$.

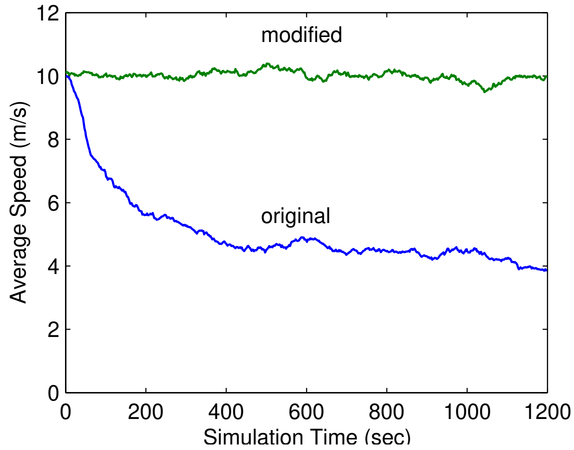

Fig. 1. Average speed of simplified random waypoint. (1000 nodes)

Figure 1 gives the average speed decay over time. It is clear that the average speed in Model S1B is stable right from the start of simulation, while that in Model S1A, which is close to the original random waypoint, keeps decreasing over time. In particular, within 200 seconds, the average speed in Model S1A drops about 50% from the expected $10\ m/s$.

In fact, the evolvement of the speed V^t distribution behaves as predicted. Figure 2 shows the V^t pdf (from histogram plot) of Model S1A at the start of the simulation ($t = 1$) as well as at $t = 850$. We find that at $t = 850$ seconds the pdf of the speed V^t already converges reasonably close to the limit distribution predicted by Corollary 1.2.

On the other hand, V^t's pdf in Model S1B is stable and uniform all along within a simulation of length 1200 seconds. For illustration, Figure 3 shows the plots at two time instants 0 and 850. And this agrees with what Corollary 3.1 predicted.

B. Random Waypoint Model with uniform and stable speeds

In this section, we discuss the simulation of the random waypoint model to obtain a stable speed distribution within

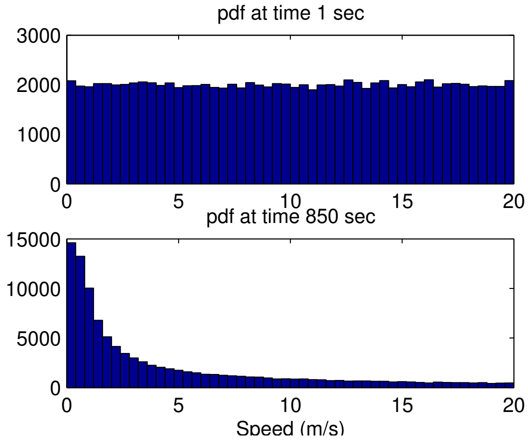

Fig. 2. Speed density histogram (pdf) (original, $100,000$ sample points)

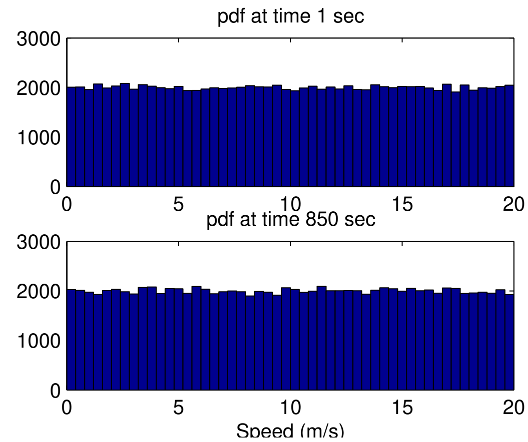

Fig. 3. Speed density histogram (pdf) (modified, $100,000$ sample points)

the entire simulation window. We consider the model when the nodes move within a rectangle area of $n \times n$ (typical values of n are 1000 m or 1500 m, say). Strictly speaking, the distance D_i in each period X_i is not independent. For example, for nodes at the corner of the rectangle, the next distance is biased toward larger average values than nodes at the center of the region, because the destination is selected uniformly within the entire region. Hence, the random waypoint model over a non-homogeneous space (such as a rectangle) is not a renewal process as discussed and analyzed previously.

The formal analysis of the random waypoint within a bounded rectangle is trickier due to the so-called "edge-effects". Fortunately, the dependence among distance D_i is *weak*, and it is expected that the process can still be *reasonably approximated* by the renewal process we developed. To confirm this, we appeal to simulation with the following setup (rectangle: $1500\ m \times 1500\ m$ and $V_{min} = 0, V_{max} = 20\ m/s$):

- Original: This is the unmodified random waypoint. Pick destination uniformly within the simulation region. Pick speed uniformly from (V_{min}, V_{max}).
- Modified: Pick destination uniformly within the simulation region. Pick first period' speed uniformly from (V_{min}, V_{max}), pick the following periods' speeds according to $F_V(x) = \frac{x^2 - V_{min}^2}{V_{max}^2 - V_{min}^2}$.

Figure 4 gives the average speed decay. The modified random waypoint has a very stable average speed, while the average speed in the unmodified model decreases steadily.

Figures 5, 6 show the speed density function at two time instants ($t = 1$ sec and $t = 850$ sec). It can be seen that the V^t in the original random waypoint again converges to some pdf like c/v, while that of the modified model is close to some uniform distribution with only slight bias toward smaller value.

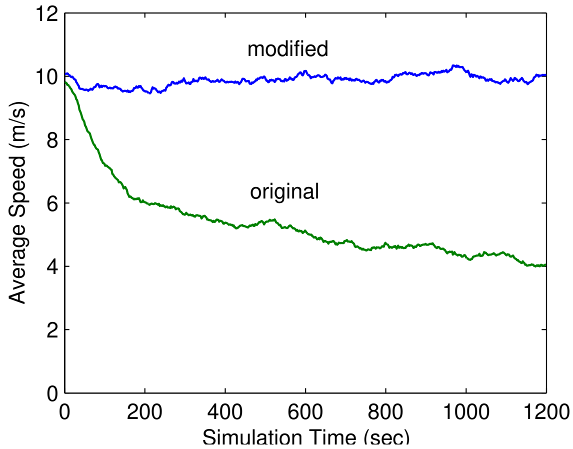

Fig. 4. Average speed of random waypoint. (1000 nodes)

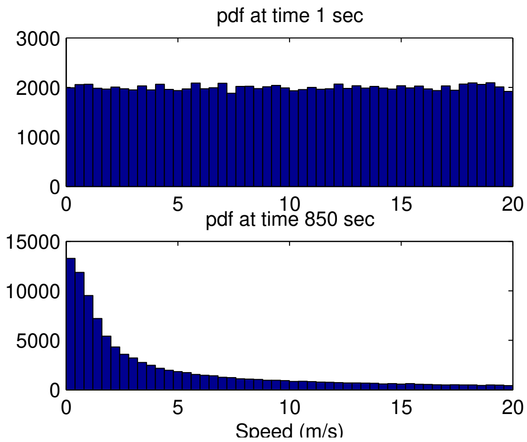

Fig. 5. Speed density histogram (pdf) (original, $100,000$ sample points)

V. Mobility Models With Nonzero Pause Time

In the previous sections, we analyze the random waypoint model with zero pause time. In this section, we extend our analysis and methodology to the random waypoint model with nonzero pause time.

Extended Mobility Model E2A: This is an extended model of S1A, where each node pauses for a random Z unit time after each move. The mobile intervals are determined in exactly the same way as in model S1A. The pause time Z is independent of D, V (hence X) with distribution function $F_z(\cdot)$.

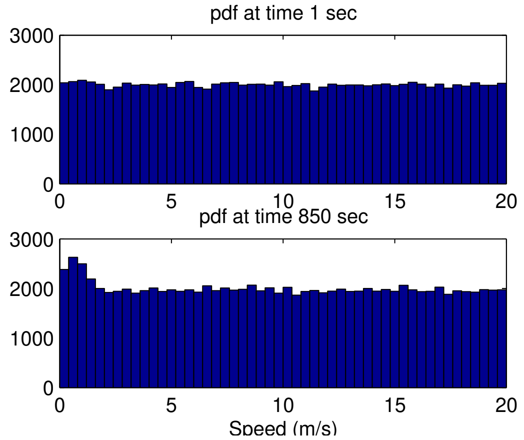

Fig. 6. Speed density histogram (pdf) (modified, $100,000$ sample points)

If we see each combined period of X_i and Z_i as a new variable Y_i, it is evident that $\{Y_i\}$ is also a renewal process. Hence the results we developed previously carry over with only slight modification.

Let $\mu_1 = E[X]$, $\mu_2 = E[Z]$, and $\mu = E[Y] = \mu_1 + \mu_2$. Then, $\lim_{t\to\infty} \Pr\{t \text{ falls within some } X_i\} = \frac{\mu_1}{\mu}$ (see, e.g., [30]), which we denote by α. The quantities speed V^t, residual distance γ^t and residual time β^t are defined as in Model S1A, relative to process $\{Y_i\}$.

Parallel to Theorems 1, 2, we have

Theorem 4: The process $\{Y_i\}$ associated with Model E2A has the following limit distributions:

$$\begin{aligned} & \lim_{t\to\infty} \Pr\{V^t \le z\} \\ = \ & \alpha \cdot \frac{E[D]}{E[X]} \cdot \int_0^z \frac{1}{x} \, \mathrm{d}F_V(x) + (1-\alpha) \quad (13) \\ & \lim_{t\to\infty} \Pr\{\gamma^t \le z\} \\ = \ & \alpha \cdot \frac{1}{E[D]} \cdot \int_0^z [1 - F_D(x)] \, \mathrm{d}x + (1-\alpha) \quad (14) \\ & \lim_{t\to\infty} \Pr\{\beta^t \le z\} \\ = \ & \mu^{-1} \int_0^z [1 - F_Y(x)] dx \quad (15) \end{aligned}$$

where $F_Y(x)$ is the cdf of $Y (= X + Z)$, which has the following identity in terms of $F_X(\cdot)$ and $F_Z(\cdot)$:

$$F_Y(t) = \Pr\{X + Z \le t\} = F_X \star F_Z(t) \quad (16)$$

Here, $\star$ is the convolution between dfs F_X and F_Z: $F_X \star F_Z(t) = \int_0^t F_X(t-x) dF_Z(x)$.

The proof of the above theorem is technically similar to those of Theorems 1 and 2 (a standard renewal argument to build a renewal equation, followed by application of Renewal Theorem) and hence omitted. Here, we provide an intuitive argument to interpret the results. Let's consider Equation 13. The first term of the right hand side is the probability of t falling within some mobile interval X_i times the quantity given by Equation 1, and the second term is the probability of t falling within some pause interval Z_i when the speed is 0. A moment of reflection will tell that the sum is exactly the probability $\Pr\{V^t \le z\}$. The interpretation for the other two equations is similar.

We now propose Model E2B, with the motivation that the distribution of speed would be stable immediately from the start of the simulation using the model.

Extended Mobility Model E2B: The first period Y_0 is determined as follows. With probability α, determine a period X_0 as in model S1B, which is followed by some random pause time Z_0 with df $F_z(\cdot)$. With probability $(1-\alpha)$, Y_0 consists solely of the residual time of β_z^t with distribution $\mu_2^{-1} \int_0^t [1 - F_z(x)] \, dx$. The remaining $Y_i (i > 0)$ is determined with X_i the same as in Model S1B and Z_i randomly selected according to distribution $F_z(\cdot)$.

To analyze the properties of Model E2B, we prove first that Model E2B is stationary.

Lemma 5.1: The renewal process associated with $\{Y_0, Y_1, \cdots\}$ is stationary, i.e., Y_0 has the distribution function of residual time β_Y^t given by (15).

Proof:

$$\begin{aligned} & \Pr\{Y_0 \le t\} \\ = \ & \alpha \cdot \Pr\{X_0 + Z \le t\} + (1-\alpha) \cdot \Pr\{\beta_Z^\infty \le t\} \\ = \ & \alpha \cdot G \star F_Z(t) + (1-\alpha) \cdot \mu_2^{-1} \int_0^t [1 - F_Z(x)] dx \\ & (G \text{ as in (9), the cdf for } X_0) \\ = \ & \alpha \cdot \mu_1^{-1} \int_0^t F_Z(x)[1 - F_X(t-x)] dx \\ & \quad + (1-\alpha) \cdot \mu_2^{-1} \int_0^t [1 - F_Z(x)] dx \\ = \ & \mu^{-1} \int_0^t [1 - F_Z(x) F_X(t-x)] dx \end{aligned}$$

The first two equalities follow from the definition of Model E2B, the third equality follows from easy manipulation of $\star$ and the fourth from the fact that $\mu^{-1} = \alpha \cdot \mu_1^{-1} = (1-\alpha) \cdot \mu_2^{-1}$.

Comparing to Equation 15, we only need to show that

$$\int_0^t F_X \star F_Z(x) dx = \int_0^t F_X(x) F_Z(t-x) dx$$

Let the left term be $L(t)$ and the right one be $R(t)$. We have $L(0) = R(0) = 0$. It is obvious that $L'(t) = F_X \star F_Z(t)$. Straightforward computation gives

$$\begin{aligned} R'(t) &= \lim_{\Delta\to 0} \frac{R(t+\Delta) - R(t)}{\Delta} \\ &= \int_0^t F_X(x) f_Z(t-x) dx = L'(t) \end{aligned}$$

This completes the proof. ■

Given that the renewal process associated with Model E2B is a stationary one, we are in a position to state the following result, whose proof is identical to that of Theorem 3, hence omitted:

Theorem 5: For Model E2B, the speed V^t at time t and the residual distance γ^t have the following identities ($\forall t > 0$):

$$\Pr\{V^t \leq z\} = \alpha \cdot \frac{E[D]}{E[X]} \cdot \int_0^z \frac{1}{x} \, \mathrm{d}F_V(x) + (1-\alpha) \quad (17)$$

$$\Pr\{\gamma^t \leq z\} = \alpha \cdot \frac{1}{E[D]} \cdot \int_0^z [1 - F_D(x)] \, \mathrm{d}x + (1-\alpha) \quad (18)$$

VI. Discussion

In this paper, renewal theory proved to be a powerful tool to analyze and solve the problem of speed decay in the random waypoint model. It allowed us to derive the steady state distribution functions for the speed and residual distance. We remark that our analytical approach applies to a larger class of models than the random waypoint-like models that we have focused on in this paper. In fact, any mobility model that is reasonably captured by a renewal process will be amenable to our analysis. For example, a mobility model where after each pause period the node chooses a random speed value, speed direction and travel time (instead of a random destination as in the random waypoint model) can be easily shown to have a uniformly distributed speed.

Another interesting result of our approach is that we are now able to generate mobility patterns for which we control the steady state distribution. We can choose the target speed steady state distribution, then using Theorem 3, we can derive the probability distribution functions needed for models *S1B* and *E2B*. It is reasonable to believe that different mobility patterns, e.g., speed distribution, will have impacts on the performance of routing protocols. Armed with our results, one is now able to study these kinds of effects, which may not be so easy previously, and hopefully, get a better understanding of the protocols.

More generally, mobility impacts the whole network performance in various ways. In [20], it has been shown that mobility can significantly increase capacity. In [23], it has been shown that mobility has an impact on network connectivity. For future work we plan to investigate the use of the renewal argument in analyzing other multihop ad hoc networks characteristics such as their connectivity.

VII. Conclusion

In this paper, we have analyzed the steady state distribution functions of the random waypoint model. In addition to confirming the drawbacks of previous simulations of the random waypoint model, our analysis technique allowed us to derive a theoretically sound solution to the speed decay problem. Our solution has several advantages. First, the simulation reaches the steady state from the start and therefore the mobile nodes speed is from the beginning as expected by the user which is not the case of previous approaches. Second, our technique is transparent to the user. Finally, our approach provides a general framework for analyzing other mobility models. In addition to the theoretical proofs, we have also simulated our techniques and verified our claims.

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APPENDIX

A. PROOF OF THEOREM 2

Write $B_z(t) = \Pr\{\gamma^t > z\}$, conditioning on the time $X_1 = x$ of the first renewal,

$$\begin{aligned} & \Pr\{\gamma^t > z | X_1 = x\} \\ = & \begin{cases} \Pr\{D_1 - V_1 \times t > z | X_1 = x\} & \text{if } t < x \\ B_z(t-x) & \text{if } 0 < x \leq t \end{cases} \end{aligned}$$

Then by the law of total probability,

$$\begin{aligned} B_z(t) &= \int_0^\infty \Pr\{\gamma^t > z | X_1 = x\} \,\mathrm{d}F_X(x) \\ &= \Pr\{D_1 - V_1 t > z, X_1 > t\} \\ &\quad + \int_0^t B_z(t-x) \,\mathrm{d}F_X(x) \end{aligned} \tag{19}$$

For $z \geq 0$,

$$\begin{aligned} & \Pr\{D_1 - V_1 t > z, X_1 > t\} \\ = & \Pr\{D_1 - V_1 t > z\} \\ = & \int_0^\infty \Pr\{D_1 - V_1 t > z | V_1 = x\} \,\mathrm{d}F_V(x) \\ = & \int_0^\infty \Pr\{D_1 > z + tx\} \,\mathrm{d}F_V(x) \\ = & \int_0^\infty [1 - F_D(z + tx)]\} \,\mathrm{d}F_V(x) \end{aligned}$$

Hence we have the renewal equation,

$$B_z(t) = \int_0^\infty [1 - F_D(z+tx)]\} \,\mathrm{d}F_V(x) + \int_0^t B_z(t-x) \,\mathrm{d}F_X(x)$$

Invoking renewal theorem again,

$$\begin{aligned} & \lim_{t\to\infty} B_z(t) \\ = & \mu^{-1} \int_0^\infty \int_0^\infty [1 - F_D(z + tx)]\} \,\mathrm{d}F_V(x) \,\mathrm{d}t \\ = & \mu^{-1} \int_0^\infty \int_0^\infty [1 - F_D(z + tx)]\} \,\mathrm{d}t \,\mathrm{d}F_V(x) \\ & \text{(change of variable: } y = z + tx) \\ = & \mu^{-1} \int_0^\infty \frac{1}{x} \int_z^\infty [1 - F_D(y)]\} \,\mathrm{d}y \,\mathrm{d}F_V(x) \\ = & \mu^{-1} \int_0^\infty \frac{1}{x} \,\mathrm{d}F_V(x) \times \int_z^\infty [1 - F_D(y)]\} \,\mathrm{d}y \\ = & \frac{1}{E[D]} \cdot \int_z^\infty [1 - F_D(y)]\} \,\mathrm{d}y \\ = & 1 - \frac{1}{E[D]} \cdot \int_0^z [1 - F_D(y)]\} \,\mathrm{d}y \end{aligned}$$

B. PROOF OF THEOREM 3

The following proof follows the lines of arguments given in [30], pp 199-200.

Write $A_z^S(t) = \Pr\{V_t^S > z\}$ and $A_z(t) = \Pr\{V^t > z\}$. The standard renewal argument gives us

$$A_z^S(t) = \Pr\{V_0 > z, X_0 > t\} + \int_0^t A_z(t-x) \,\mathrm{d}G(x)$$

or writing in the following form:

$$A_z^S(t) = \Pr\{V_0 > z, X_0 > t\} + G \star A_z(t)$$

Where $\star$ is the convolution of G and $A_z(t)$. $\star$ is commutative and associative.

Previously, (3) gives

$$A_z(t) = \Pr\{V > z, X > t\} + F_X \star A_z(t)$$

which has the unique solution in the form of

$$A_z(t) = a_z(t) + M \star a_z(t)$$

where $a_z(t) = \Pr\{V > z, X > t\}$. $M(t)$ is the renewal function associated with the ordinary renewal process

$$M = \sum_{n=1}^\infty F^{n\star}$$

with $F^{1\star} = F_X(x)$ and $F^{(i+1)\star} = F^{i\star} \star F_X(x)$.

If we write $a_z^S(t) = \Pr\{V_0 > z, X_0 > t\}$, we get

$$\begin{aligned} A_z^S(t) &= a_z^S(t) + G \star [a_z(t) + M \star a_z(t)] \\ &= a_z^S(t) + (G + G \star M) \star a_z(t) \\ &= a_z^S(t) + M_S \star a_z(t) \\ &= a_z^S(t) + \int_0^t a_z(t-y)\, \mathrm{d}M_S(y) \end{aligned} \tag{20}$$

M_S is the renewal function for the delayed (in our case, stationary) renewal process, and $M_S(t) = G(t) + G \star M(t)$. Thanks to the property of being stationary renewal process (Lemma 3.1), $M_S(t) = t/\mu$ (see, e.g., [30] p199).

$$a_z^S(t) = \int_z^\infty [1 - F_\gamma^\infty(ty)]\, \mathrm{d}F_v^\infty(y)$$

$$\begin{aligned} &= \{1 - F_v^\infty(z)\} - \int_z^\infty F_\gamma^\infty(ty)\, \mathrm{d}F_v^\infty(y) \\ &= \{1 - F_v^\infty(z)\} \\ &\quad -\mu^{-1} \int_z^\infty \int_0^t [1 - F_D(xy)]\, \mathrm{d}y \ \ \mathrm{d}F_V(x) \end{aligned} \tag{21}$$

And,

$$\begin{aligned} &\int_0^t a_z(t-y)\, \mathrm{d}M_S(y) \\ &= \mu^{-1} \int_0^t a_z(t-y)\, \mathrm{d}y \\ &= \mu^{-1} \int_0^t a_z(y)\, \mathrm{d}y \\ &= \mu^{-1} \int_0^t \int_z^\infty [1 - F_D(yx)]\, \mathrm{d}F_V(x)\, \mathrm{d}y \\ &= \mu^{-1} \int_z^\infty \int_0^t [1 - F_D(yx)]\, \mathrm{d}y\, \mathrm{d}F_V(x) \end{aligned} \tag{22}$$

Substituting (21) and (22) into (20), we get the result to be shown.

The proof for γ_t^S is almost identical, hence omitted.