## Problem Set 6 (due Wednesday, April 13)

## 1. (15 points) Bottleneck bandwidth

One can model a communication network by a weighted directed graph $G$, in which the vertices of $G$ represent the nodes of the network, the edges of $G$ represent the communication links of the network and the weight of an edge is the bandwidth of the link. We define the bottleneck bandwidth of a path $p$ as the bandwidth of the minimum-bandwidth link in $p$.

Design and analyze an efficient algorithm to determine a path with the largest bottleneck bandwidth from a given node $s$ to a given node $t$. If no path from $s$ to $t$ exists, then your algorithm must indicate so. Briefly justify the correctness of your algorithm. You may assume that the bandwidth of every link is positive. (Hint: Modify Dijkstra's algorithm.)

## 2. (15 points) Arbitrage

Problem 24-3, page 615.

## 3. (10 points) Shortest paths on vertex-weighted graphs

Suppose you are given a connected undirected graph with weights on vertices (rather than on edges) and you are asked to compute the single-source shortest paths from a given source vertex. Here, the length of a path is defined as the sum of the weights on the vertices comprising the path. Give an algorithm for solving this problem by reducing it to the standard single-source shortest paths problem on directed graphs with weights on edges.

## 4. (10 points) Bellman-Ford algorithm

Exercise 25.1-5, page 628.

## 5. (10 points) Floyd-Warshall algorithm

Exercise 25.2-1, page 634.

