College of Computer & Information Science
Northeastern University
CSU690: Algorithms and Data

Spring 2005
Handout 2
10 January 2005

Problem Set 1 (due Wednesday, January 19)

1. (6 points) Twenty questions

A friend challenges you to find out a positive integer number n that she has on her mind. For any guess you make, she is willing to tell you which of the following three possibilities is true: n is less than, greater than, or equal to your guess. You have no clue as to how large n is and would like to determine n with the smallest number of guesses.

Describe a guessing procedure for determining n. What is the number of guesses your procedure makes, as a function of n? You must aim at keeping this function as small as possible.

2. (1 + 5 + 5 + 3 = 14 points) Bubblesort

Problem 2-2, page 38.

3. (1 + 3 + 6 = 10 points) Inversions and insertion sort

Parts (a) through (c), Problem 2-4, pages 39-40.

Hint for part (c): If $T(A)$ is the running time of insertion sort on input A and $I(A)$ is the number of inversions in A, then prove that $T(A)$ is $\Theta(I(A) + n)$.

4. (10 points) Ordering functions

Arrange the following functions in order from the slowest growing function to the fastest growing function. Briefly justify your answers. (*Hint:* It may help to plot the functions and obtain an estimate of their relative growth rates. It may also help to express each function as a power of 2 and then compare.)

$$n^{1/2} \quad n^2 + \lg n \quad n(\lg n)^5 \quad (\lg\lg n)^2 \quad \lg n$$

5. (3 + 3 + 4 = 10 points) Properties of asymptotic notation

Let $f(n)$, $g(n)$, and $h(n)$ be asymptotically positive and monotonically increasing functions. Prove or disprove each of the following conjectures.

(a) $f(n) + g(n) = \Theta(\min\{f(n), g(n)\})$.

(b) If $f(n) = O(h(n))$ and $g(n) = \Omega(h(n))$, then $f(n) = O(g(n))$.

(c) $f(n^2) = \Omega((f(n))^2)$.