Problem Set 5 (due Tuesday, April 7)

0. Problem 4 of PS 4.

1. (10 points) Probability Amplification

Problem 10.11 of Sipser.

2. (10 points) Cover time of a random walk

In class, we showed that a random walk on an undirected graph takes an expected $O(n^3)$ steps to reach all nodes of the graph. In this exercise, we prove this is asymptotically tight.

Consider a graph with $2n$ nodes, n of which form a clique and the other n form a path. A node of the clique and an endpoint of the path are connected by an edge. Show that the random walk algorithm applied to this graph takes expected $\Omega(n^3)$ time to reach all nodes.

3. (10 points) Probabilistic method for SAT

Suppose you have a SAT formula with fewer than n^k clauses, each with at least $klog(n)$ distinct variables. Show that the formula has a satisfying truth assignment. Give a polynomial-time algorithm to compute a satisfying assignment.

4. (10 points) SAT, RP, and BPP

Show that if SAT is in BPP, then SAT is in RP.

5. (20 points) BP- and R-machines

Given a string w, it is decidable to determine whether w is a valid description of a nondeterministic Turing machine (in a specified language). Similarly, given a string w, it is decidable to determine whether w is a valid description of a deterministic Turing machine.

Let us see whether similar properties hold for Turing machines that form the probabilistic complexity classes we have studied. Define a *BP-machine* to be a nondeterministic Turing machine M in which for every string w, either at least two-third of its computation paths on w accept or at least two-third of its computation paths on w reject. Define an *R-machine* to be a nondeterministic Turing machine M in which for every string w, either at least half of its computation paths on w accept or all of its computation paths on w reject.

Is the set of all BP-machines decidable? Recognizable?

Is the set of all R-machines decidable? Recognizable?