College of Computer & Information Science | Spring 2009
Northeastern University | Handout 10
CSG714: Theory of Computation | 10 February 2009

Problem Set 3 (due Friday, February 20)

1. ($5 \times 4 = 20$ points) Complexity measures

What is a reasonable complexity measure? In class, we have discussed time and space as two reasonable complexity measures, when using Turing machines as our computational models. In a 1967 JACM paper, Manuel Blum considered this question and defined a *complexity measure* Φ to be a function that maps Turing machine-input pairs to nonnegative integers and is decidable. Formally, Φ is a complexity measure if $\Phi(M, x)$ is defined whenever x is a valid input to M and it is decidable given M, x, and k, whether $\Phi(M, x) = k$. By this definition, which of these are complexity measures?

- Time, the number of steps during a computation.
- Space, the number of cells used on the worktape of a 2-tape Turing machine (assuming that the first input tape is read-only).
- Reversals, the number of times during a computation that the head must change direction of motion.
- Waste, the number of times during a computation that the head writes the same symbol as just read by the head.
- Work, the number of times during a computation that the head writes a different symbol than the one just read by the head.

2. (10 points) Crossing sequences and regular languages

Show the following claim, left unproved in class. Let L be a language and M be a deterministic single-tape Turing Machine that decides L. For a given string x, let $c_i(x)$ denote the sequence of states that the M is in as it crosses cell i to $i+1$, or vice versa, during its computation on x. And let $c(x)$ denote the length of the longest crossing sequence among $c_i(x)$, $1 \le i \le |x|$.

Show that if there exists an integer k, such that $c(x) \le k$ for all x, then L is regular.

(*Hint:* Think NFAs.)

3. (10 points) SAT formula with no negations

Suppose you have a SAT formula in conjunctive normal form, in which very literal is a nonnegated variable (e.g., $((x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_4) \wedge (x_2 \vee x_3 \vee x_5)))$. It is easy to see that ϕ is satisfiable. Given such a formula, a natural goal is to select the mimimum number of variables that need to be set true to satisfy the formula. Show that the following language is NP-complete.

$$\{\langle \phi, k \rangle : \phi \text{ has no negated literals and is satisfied by setting at most } k \text{ variables true}\}$$

4. (10 points) A flexible variant of Post Correspondence Problem

Consider the following variant of the Post Correspondence Problem. We are given two sets of binary strings $\{t_1, \ldots, t_m\}$ and $\{b_1, \ldots, b_n\}$. The goal is to determine whether there exists a string w such that (a) w can be written as $t_{i_1} t_{i_2} \cdots t_{i_k}$ for some $i_1, \ldots i_k \in \{1, \ldots, m\}$ and (b) w can be written as $b_{j_1} b_{j_2} \cdots b_{j_\ell}$ for some $j_1, \ldots, j_\ell \in \{1, \ldots, n\}$.

Show that the above problem is in NP. Why does the same argument not work for the Post Correspondence Problem?

5. (10 points) Minesweeper

Problem 7.30 in Sipser's text.

6. (10 points) Difference hierarchy

Problem 7.45 in Sipser's text.