College of Computer & Information Science
Northeastern University
CSG714: Theory of Computation

Spring 2009
Handout 2
9 January 2009

Problem Set 1 (due Friday, January 23)

1. (10 points) Neural nets and finite state automata

This problem is taken from the text by Aho, Hopcroft, and Ullman. Figure 1 depicts a neural network. Each neuron (the triangles depicted in the figure) has excitatory (clear circles) and inhibitory (filled circles) synapses. A neuron produces a 1-output if the number of excitatory synapses with 1-inputs exceeds the number of inhibitory synapses with 1-inputs by at least the threshold of the neuron (the number inside the triangle).

Determine a finite state automaton whose behavior is equivalent to that of the given neural network. The final states of the automaton correspond to a 1-output of the network. Assume that there is sufficient time between changes in input value for signals to propagate and for the network to reach a stable configuration. Assume that the initial values of x, y, and z are all 0.

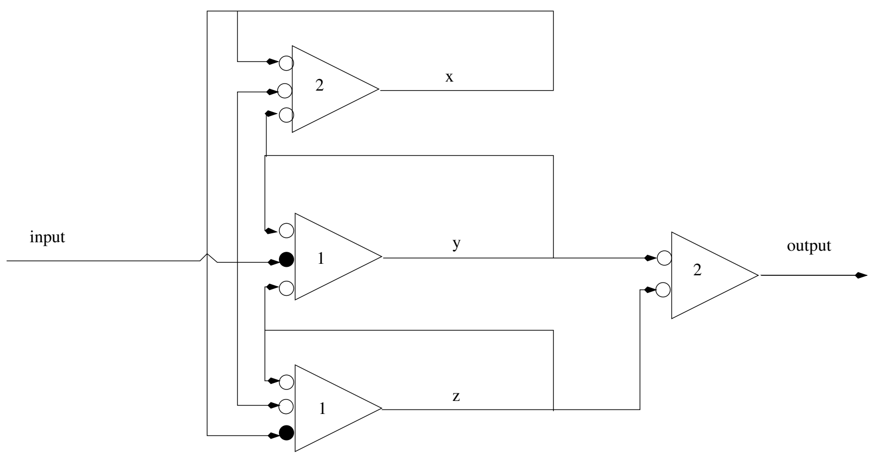

Figure 1: A neural network.

2. ($3 \times 5 = 15$ points) Number of states in FSAs

Consider the language L_k defined over the alphabet $\{0,1\}$, for a given integer $k > 0$.

$$L_k = \{w \in \{0,1\}^* : w \text{ has a 1 in the } k\text{th position from the end}\}.$$

(a) Describe an NFA for L_k, given integer $k > 0$.

(b) Describe a DFA for L_k, given integer $k > 0$. How many states does your DFA have?

(c) Show that the number of states in any DFA for L_k is exponential in k while there exists an NFA for L_k whose number of states is linear in k.

3. ($2 \times 7 = 14$ points) Regular languages

For each of the languages below, determine whether it is regular. Justify your answer.

(a) $\{uwu^R : u, w \in \{0,1\}^+\}$.

(b) $\{uu^Rw : u, w \in \{0,1\}^+\}$.

4. ($2 \times 8 = 16$ points) Context-free grammars

Design a context-free grammar for each of the following languages over the alphabet $\{a, b, c\}$.

(a) Set of all strings with twice as many b's as a's.

(b) The set $\{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

5. ($3 \times 5 = 15$ points) Context-free languages and PDAs

(a) Show that the following language is not context-free.

$$\{a^i b^j c^k : 0 \leq i \leq j \leq k\}$$

(b) Show that the following language is context-free, by constructing a (nondeterministic) PDA that accepts the language.

$$\{a^n b^n : n \geq 1\} \cup \{a^n b^{2n} : n \geq 1\}.$$

(c) Prove that there does not exist any *deterministic* PDA that accepts the language of (b).

(*Remark:* This is a challenging problem. One approach in to argue that any DPDA cannot "differentiate" between all strings of the form $a^n b^n$. That is, for two such strings, it will end up in the same configuration. Then, argue that such a machine cannot possibly accept the above language.)