Problem Set 1 (due Friday, January 23)

1. (10 points) Neural nets and finite state automata

This problem is taken from the text by Aho, Hopcroft, and Ullman. Figure 1 depicts a neural network. Each neuron (the triangles depicted in the figure) has excitatory (clear circles) and inhibitory (filled circles) synapses. A neuron produces a 1-output if the number of excitatory synapses with 1-inputs exceeds the number of inhibitory synapses with 1-inputs by at least the threshold of the neuron (the number inside the triangle).

Determine a finite state automaton whose behavior is equivalent to that of the given neural network. The final states of the automaton correspond to a 1-output of the network. Assume that there is sufficient time between changes in input value for signals to propagate and for the network to reach a stable configuration. Assume that the initial values of x, y, and z are all 0.

![Figure 1: A neural network.](image)

2. (3 × 5 = 15 points) Number of states in FSAs

Consider the language $L_k$ defined over the alphabet $\{0, 1\}$, for a given integer $k > 0$.

$$L_k = \{w \in \{0, 1\}^* : w \text{ has a } 1 \text{ in the } k\text{th position from the end}\}.$$

(a) Describe an NFA for $L_k$, given integer $k > 0$.

(b) Describe a DFA for $L_k$, given integer $k > 0$. How many states does your DFA have?

(c) Show that the number of states in any DFA for $L_k$ is exponential in $k$ while there exists an NFA for $L_k$ whose number of states is linear in $k$. 
3. (2 × 7 = 14 points) Regular languages

For each of the languages below, determine whether it is regular. Justify your answer.

(a) \( \{u w u^R : u, w \in \{0, 1\}^+ \} \).

(b) \( \{u u^R w : u, w \in \{0, 1\}^+ \} \).

4. (2 × 8 = 16 points) Context-free grammars

Design a context-free grammar for each of the following languages over the alphabet \( \{a, b, c\} \).

(a) Set of all strings with twice as many b’s as a’s.

(b) The set \( \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k \} \).

5. (3 × 5 = 15 points) Context-free languages and PDAs

(a) Show that the following language is not context-free.

\[ \{a^i b^j c^k : 0 \leq i \leq j \leq k \} \]

(b) Show that the following language is context-free, by constructing a (nondeterministic) PDA that accepts the language.

\[ \{a^n b^n : n \geq 1 \} \cup \{a^n b^{2n} : n \geq 1 \} \]

(c) Prove that there does not exist any deterministic PDA that accepts the language of (b).

(Remark: This is a challenging problem. One approach in to argue that any DPDA cannot “differentiate” between all strings of the form \( a^n b^n \). That is, for two such strings, it will end up in the same configuration. Then, argue that such a machine cannot possibly accept the above language.)