Problem of the Week – 6

Minimizing NFAs

This problem is taken from Sipser’s text. Given a regular language (as a DFA or a regular expression), we know that there is a polynomial-time algorithm to compute the smallest DFA, i.e., with the smallest number of states. (In class we discussed how the Myhill-Nerode Theorem gives the number of states in the smallest DFA; one can compute such a DFA in polynomial time using a dynamic programming algorithm.) The minimization problem may be much harder for NFAs, however, as we see below.

For a CNF $\phi$ with $m$ variables and $n$ clauses, give a polynomial-time algorithm to construct an NFA with $O(nm)$ states that accepts every nonsatisfying assignment, where each assignment is represented as a Boolean string of length $m$. Then, prove that the problem of minimizing NFAs – i.e., given an NFA, find an equivalent NFA with the smallest number of states – cannot be done in polynomial time unless $P \neq NP$. 