College of Computer & Information Science
Northeastern University
CSG714: Theory of Computation

Spring 2009
Handout 11
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Problem of the Week – 6

Minimizing NFAs

This problem is taken from Sipser's text. Given a regular language (as a DFA or a regular expression), we know that there is a polynomial-time algorithm to compute the smallest DFA, i.e., with the smallest number of states. (In class we discussed how the Myhill-Nerode Theorem gives the number of states in the smallest DFA; one can compute such a DFA in polynomial time using a dynamic programming algorithm.) The minimization problem may be much harder for NFAs, however, as we see below.

For a CNF ϕ with m variables and n clauses, give a polynomial-time algorithm to construct an NFA with $O(nm)$ states that accepts every nonsatisfying assignment, where each assignment is represented as a Boolean string of length m. Then, prove that the problem of minimizing NFAs – i.e., given an NFA, find an equivalent NFA with the smallest number of states – cannot be done in polynomial time unless P $\neq$ NP.