

## Problems of the Week – 10 and 11

### 10. Using up all Scrabble letters

You have a set of  $n$  letter tiles, each tile having exactly one letter of the English alphabet. (Each letter may appear multiple times; thus you may have 5 of As, 2 of Bs, 7 of Cs, etc.) You also have a limited vocabulary of  $m$  English words. You want to determine whether you can make a collection of English words that use up all of the letter tiles, with each tile appearing exactly once in this collection, and each word coming from your vocabulary of  $m$  words. (A word may appear multiple times in your collection.)

Show that this problem is NP-complete.

### 11. Monotone QBF

A SAT formula is set to be monotone if changing the value of any variable from 0 to 1 does not change the value of the formula from 1 to 0; that is, increasing the value of a variable can never decrease the value of the formula. Suppose you are given a formula

$$F = \exists x_1 \forall x_2 \cdots \exists x_n \phi(x_1, x_2, \dots, x_n),$$

where  $\phi$  is monotone. The Monotone QBF formula is the problem of determining whether  $F$  is true.

Do one of the following: (a) Show that Monotone QBF is PSPACE-complete; or (b) give a polynomial-time algorithm for Monotone QBF.