Problem Set 5 (due Friday, April 4)

1. (10 + 10 = 20 points) Use of the padding technique in separation results

Problems 9.13 and 9.14 in Sipser’s text.

2. (10 points) Alternating machines

Assume $S(n) \geq \log(n)$ and is space-constructible. Prove that

$$\bigcup_{k>0} (\Sigma_k \text{SPACE}(S(n)) \cup \Pi_k \text{SPACE}(S(n))) = \text{NSPACE}(S(n)).$$

That is, prove that any language decidable by alternating Turing machines with a fixed number of alternations in space $S(n)$ is also decidable by a nondeterministic Turing machine in space $S(n)$.

3. (10 points) The class PolyLog

Define the class PolyLog as $\bigcup_{k \geq 0} \text{SPACE}(\log^k n)$.

(a) Prove that PolyLog does not have any language that is complete with respect to logspace reductions. That is, show that there does not exist any language $A \in \text{PolyLog}$ such that every language $B$ in PolyLog reduces to $A$ in logarithmic space.

(b) Using (a), argue that PolyLog $\neq P$.

4. (10 points) Classes NP, BPP, and RP

Problem 10.19 in Sipser’s text: Show that if $\text{NP} \subseteq \text{BPP}$, then $\text{NP} = \text{RP}$.

5. (10 points) Amplification lemma for IP

Fix any real $\varepsilon > 0$. Show that if a language $A$ is in IP then there exists a prover $P$ and a polynomial-time verifier $V$ such that:

- $w \in A$ implies that $\Pr[V \leftrightarrow P \text{ accepts } w] \geq 1 - \varepsilon$.
- for any prover $\tilde{P}$, $w \notin A$ implies that $\Pr[V \leftrightarrow \tilde{P} \text{ accepts } w] \leq \varepsilon$.

How small can $\varepsilon$ be made, as a function of $|w|$?