College of Computer & Information Science — Spring 2008
Northeastern University — Handout 17
CSG714: Theory of Computation — 25 March 2008

Problem Set 5 (due Friday, April 4)

1. (10 + 10 = 20 points) Use of the padding technique in separation results

Problems 9.13 and 9.14 in Sipser's text.

2. (10 points) Alternating machines

Assume $S(n) \geq log(n)$ and is space-constructible. Prove that

$$\bigcup_{k>0} (\Sigma_k \text{SPACE}(S(n)) \cup \Pi_k \text{SPACE}(S(n))) = \text{NSPACE}(S(n)).$$

That is, prove that any language decidable by alternating Turing machines with a fixed number of alternations in space $S(n)$ is also decidable by a nondeterministic Turing machine in space $S(n)$.

3. (10 points) The class PolyLog

Define the class PolyLog as $\bigcup_{k \geq 0} \text{SPACE}(\log^k n)$.

- **(a)** Prove that PolyLog does not have any language that is complete with respect to logspace reductions. That is, show that there does not exist any language $A \in$ PolyLog such that every language B in PolyLog reduces to A in logarithmic space.
- **(b)** Using (a), argue that PolyLog $\neq$ P.

4. (10 points) Classes NP, BPP, and RP

Problem 10.19 in Sipser's text: Show that if NP $\subseteq BPP$, then NP $=$ RP.

5. (10 points) Amplification lemma for IP

Fix any real $\varepsilon > 0$. Show that if a language A is in IP then there exists a prover P and a polynomial-time verifier V such that:

- $w \in A$ implies that $\Pr[V \leftrightarrow P \text{ accepts } w] \geq 1 - \varepsilon$.
- for any prover $\widetilde{P}$, $w \notin A$ implies that $\Pr[V \leftrightarrow \widetilde{P} \text{ accepts } w] \leq \varepsilon$.

How small can ε be made, as a function of $|w|$?