College of Computer & Information Science — Spring 2008
Northeastern University — Handout 4
CSG714: Theory of Computation — 25 January 2008

Problem Set 2 (due Friday, February 8)

1. ($3 \times 7 = 21$ points) Turing-recognizability and -decidability

Each of the following three parts gives a definition, description, or some properties of a language. In each case, tell whether the language is:

A Turing-decidable

B Turing-recognizable but not Turing-decidable

C not Turing-recognizable

Justify your answers with proofs, constructions, algorithms, or examples as needed.

(a) $\{\langle M \rangle : L(M)$ is infinite$\}$.

(b) $\{\langle M \rangle : L(M)$ contains at least two strings$\}$.

(c) $\{\langle G_1, G_2 \rangle : G_1$ and G_2 are context free grammars that generate the same language$\}$.

2. (9 points) INF_CFG is decidable

Given a context-free grammar G, show that it is decidable whether the language generated by G is infinite. (*Hint:* Use the pumping lemma for context-free languages.)

3. ($2 \times 8 = 16$ points) [Sipser 5.17, 5.18] PCP over unary and binary alphabets

We have seen that Post Correspondence Problem over a finite alphabet is undecidable.

(a) Show that if the alphabet is unary, then PCP is decidable.

(b) Show that if the alphabet is binary, then PCP is undecidable.

4. (12 points) [Sipser 5.34] Reduction via computation history

Consider the problem of determining whether a PDA accepts some string of the form $\{ww | w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable.

5. (12 points) Conditions of Rice's Theorem

Show that both conditions of Rice's Theorem are necessary. That is, if we drop any one of the conditions then show that there is a counterexample to the claim of Rice's Theorem.