Problem of the Week – 4

Turing machines that do not attempt to move head left on the leftmost cell

Consider the problem of determining whether a given Turing machine \( M \) on a given input \( w \) ever attempts to move its head left when the head is at the leftmost cell.

(a) Show that the above problem is undecidable.

(b) In the proof of the undecidability of the Post Correspondence Problem, we claimed that we can assume that the given Turing Machine never attempts to move its head left when the head is at the leftmost cell. In particular, we claimed that we can always alter the Turing machine, if necessary. Reconcile this claim with part (a); that is, argue why we can make such an assumption and why does the assumption not contradict part (a).