

Problem of the Week – 2

Languages about integers

Consider the collection of languages given by sets of nonnegative integers. That is, consider the collection $\{L : L \subseteq Z\}$, where Z is the set of nonnegative integers. Thus, for instance, the set of all primes, the set of all even numbers, the set of all sums of squares, etc., are in this collection.

- (a) Show that the above collection of languages can be viewed as a collection of languages over a single-element alphabet.

(Remark: This is an easy observation.)

- (b) Prove that any context-free language over a single-element alphabet is also a regular language.

(Remark: Note that together with part (a), this implies that any set of integers recognized by a push-down automaton can also be recognized by a finite state automaton.)