Sample solution to Spring 2005 Midterm

Part I ($5 \times 3 = 15$ points)

1. In Pulse Code Modulation (PCM) and Delta Modulation (DM), what are the drawbacks of using linear encoding (equally-spaced quantization levels or equal delta steps)? How can these be addressed?

Answer: The problem with equally-spaced quanitzation levels in PCM is that the mean absolute error in the samples is the same, regardless of the signal level. Consequently, lower amplitude values are affected more adversely than higher amplitude values. One way to address is to use a greater number of quantization steps for signals of low amplitude and a smaller number for signals of large amplitude. Another technique is that of companding.

The equal delta steps in DM result in slope overload and quantizing noise. These can be addressed by having a continuously variable delta modulation, like in Bluetooth.

2. Walsh codes have excellent correlation properties. Yet, they are not used as spreading codes for upstream communication (mobile to base station). Provide a plausible justification for this choice.

Answer: Walsh codes have poor autocorrelation properties, and poor cross correlation properties if the codes are not accurately synchronized. Since perfect synchronization cannot be guaranteed among the mobile stations, using Walsh codes for upstream communication may make it difficult for the base station to demultiplex the incoming data.

3. In the Distributed Coordination Function (DCF) protocol of IEEE 802.11, why does a node wait only SIFS time units (after the last data packet reception) before sending an ACK, while waiting DIFS time units before attempting a data transmission. How critical is this in DCF with an RTS/CTS mechanism? In the Point Coordination Function (PCF)?

Answer: Sending an ACK after SIFS prevents a collision with a potential data packet transmission, since other nodes will wait for DIFS time units before attempting a transmission. This is critical when RTS/CTS is not used since ACK collisions require the original data packet to be retransmitted as well.

When an RTS/CTS mechanism is used, this is less critical since nodes are made aware of the imminent communication and can wait for the entire data-ACK exchange as well as any inter-frame spacing. If all nodes are not in the transmission range of every node, however, then this rule is useful when a node can sense an ongoing transmission but not receive it accurately.

PCF also has a contention phase in which the same problem as above could occur if spacing between a data packet and an ACK nears DIFS. Sending an ACK after SIFS also prevents the point coordinator from taking over the channel before the ACK for a preceding data transmission has been sent (and received).

4. True or False: Interleaving increases the number of bit errors that can be detected or corrected. Justify your answer and provide the motivation for using interleaving in wireless communication.

Answer: False. Interleaving does not increment the number of bit errors that can be detected or corrected because it does not add any redundancy to the original data. Interleaving is simply a mapping from a space to itself. Nevertheless, interleaving helps tremendously in addressing errors of a specific kind: burst errors. It helps spread a burst error among multiple each fragments, each fragment consisting of small number of errors that may be corrected (or detected) using other error correction and detection techniques.

5. Consider the handoff procedure in cellular systems that is based on relative signal strength with threshold; that is, a mobile switches from one cell to another if (1) the signal at the current BS is sufficiently weak (less than a predefined threshold) and (2) the other signal is stronger than the two. Discuss the drawbacks of this scheme, when the threshold is too low or too high.

Answer: If the threshold is too high, then the first condition is always true, and the mobile may repeatedly switch between two base stations (the ping-pong effect). If the threshold is too low, then the mobile may continue to be connected to a base station even when link quality is poor, even when a better alternative is available.

Part II

6. (4 + 3 = 7 points) Antennas and propagation

A microwave transmitter with an output of 0.5 W at 2 GHz is used in a transmission system, where both the transmitting and receiving antennas are parabolas, each 1 m in diameter. Suppose the two antennas are directionally aligned and are 10 kms apart.

(a) What is the effective radiated power of the transmitted signal, in W and dB?

Answer: The gain of a parabolic antenna is given by $0.56 \cdot 4\pi\pi r^2/\lambda^2$, where r is the radius of the antenna and λ is the wavelength. We have $r = 0.5$m and $\lambda = 3 \times 10^8/(2 \times 10^9) = 0.15$m. Substituting the numbers, we obtain the gain of each antenna to be 245.39.

Effective radiated power of the transmitted signal is $0.5 \cdot 245 = 122.5$W $= 20.88$ dB.

(b) What is the available signal power out of the receiving antenna?

Answer:

$$P_r = \frac{245.39^2 \cdot 0.15^2 \cdot 0.5}{16\pi^2 \cdot 10^2 \cdot 10^6} = 4.2 \times 10^{-8} W$$

7. (2 + 2 + 3 = 7 points) CDMA codes

Consider a CDMA system in which users A and B have codes (1 1 1 -1 -1 1 -1) and (1 -1 -1 -1 1 1 -1), respectively.

(a) Show the output of the transmission from A at the receiver if A transmits a data bit 1 and if B does not transmit.

Answer: Let A's code be c_A and TB's be c_B. The output of A's transmission is $c_A \cdot c_A$, which is 7, thus indicating a 1.

(b) Show the output of the transmission from A at the receiver if A transmits a data bit 1 and if B transmits a data bit 1, assuming that the received power from A is same as that from B.

Answer: The output is $(c_A + c_B) \cdot c_A = 7 + 1 = 8$, indicating a 1.

(c) Show the output of the transmission from A at the receiver if A transmits a data bit 1 and if B transmits a data bit 1, assuming that the received power from B is twice as that from A. This can be represented by showing the received signal component from A as consisting of elements of magnitude 1 $(+1, -1)$ and the received signal component from B as consisting of elements of magnitude 2 $(+2, -2)$.

Answer: The output is $(c_A + 2c_B) \cdot c_A = 7 + 2 = 9$, again suggesting a 1.

8. (5 points) CRC

Prove that the CRC coding technique can detect any burst errors for which the length of the burst is less than or equal to the length of the frame check sequence (number of redundant bits), as long as the most significant and least significant bits of the CRC code are both 1.

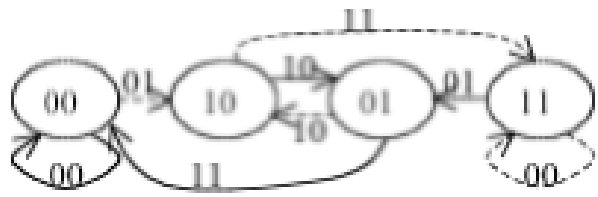
Answer: We are given a degree-k data polynomial $D(X)$ and a degree $n-k$ CRC polynomial $P(X)$. Let the encoded data be represented by the polynomial $Z(X)$. Thus, $Z(X)$ is a multiple of $P(X)$. Suppose, we encounter an error given by $E(X)$, which is represented by $X^i + X^{i+1} + \ldots + X^{i+\ell}$, where ℓ is at most $n-k-1$. Thus, we have

$$E(X) = X^i(1 + X + X^2 + \ldots + X^{\ell}).$$

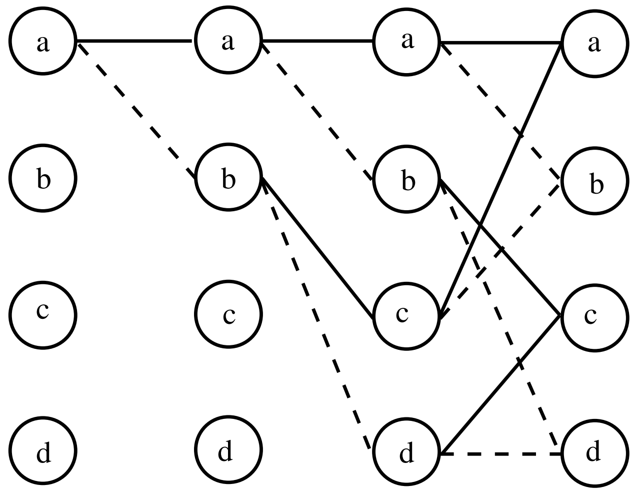
An $n-k$ degree polynomial $P(K)$ with the highest and lowest coefficient being 1 cannot divide $E(X)$ since it is not possible for the highest order and lowest order terms of any multiple of $P(X)$ to be X^i and $X^{i+\ell}$.

9. (6 + 2 = 8 points) Convolutional encoding

Draw a trellis diagram for the encoder whose state transition diagram is given below. (A regular arrow denotes an input bit of 0 and a dotted arrow denotes an input bit of 1.)



Answer: Below a, b, c, and d represent 00, 10, 01, and 11, respectively.



What is the encoded sequence corresponding to the information sequence 1101011, where the leftmost bit is the first bit presented at the encoder?

Answer: Assuming that the encoding starts with the state 00, we obtain 01110110101011, by tracing through the finite-state machine. Actually, the bits are presented in the state transition diagram in a flipped manner (the earlier bit in the stream appearing to the right of the later bit). So the correct answer should be 10111001010111. For grading purposes, either answer (or a different assumption of the start state) would fetch full points.

10. (4 + 4 = 8 points) MAC contention protocols

Suppose two nodes A and B are using an 802.11-like protocol and each has a large set of packets waiting in their queues to be transmitted to access point AP. For simplicity, assume that time is divided into slots and that A, B, and AP are synchronized on slot boundaries. No other nodes are transmitting in the network. The protocol details are as follows.

- Each packet takes 1 slot, an ACK takes 1 slot, and each inter-frame spacing (IFS) is 1 slot.
- A node waits for IFS time units after the completion of a packet transmission before attempting a transmission (packet or ACK) or resuming a backoff counter.
- The backoff procedure sets the backoff value to a random integer chosen uniformly from $[0, 2^{c+1} - 1]$, where c is the number of collisions incurred by the node on the current packet. Thus, the node selects a backoff value from $[0, 1]$ for the very first attempt, a backoff value from $[0, 3]$ for the second attempt, and so on.

(a) Assuming that the sequence of packets arrive at the two nodes at time 0 and the channel is free then, describe a possible sequence of transmission attempts in which B transmits 3 packet successfully, while A has no successful packet transmissions during this period.

Answer: In the following, we assume that ACK does not incur any collisions.

At time 0, both nodes set their backoff values chosen from $[0, 1]$. Suppose A selects 1 and B selects 0. Each waits until time 1.

At time 1, B transmits. At time 3 an ACK is sent. At time 4, B selects a backoff value of 0 for its second packet. A's backoff counter is frozen during times 1-5.

At time 5, B transmits. At time 7, an ACK is sent. At time 8, B selects a backoff value of 0 again. A's backoff counter is frozen during times 5-9.

At time 9 B transmits again. At time 11, an ACK is sent. At time 12, B selects a backoff value of 0 again. A's backoff counter is frozen during times 9-12.

(b) Compute the probability that B completes 2 packet transmissions successfully, the first with no collision and the second with at most one collision, before A has any successful packet transmission. Assume that ACK does not incur any collisions.

Answer: In the following, we assume that ACK does not incur any collisions. (The calculation is similar if ACKs are allowed to collide.) If each of B's packets has no collision, this implies that B selects backoff value 0 for both packets, and A selects 1 for its packet. The probability of this event is $1/8$. If the first packet of B has no collision and the second has one, then (i) B should select backoff value 0 for its first packet, (ii) A should select 1, (iii) then B should select 1 for its second packet, and (iv) following that, B should select a backoff value in $[0, 3]$ earlier than that selected by A in $[0, 3]$. The probability of each of the first three events is $1/2$ independently. The probability of the last event is $6/16$. So the probability of the second case is $(1/4) \cdot (1/2) \cdot (6/16) = 3/64$. So total probability is $11/64$.

(c) **4 bonus points:** Assume that B has an infinite number of packets to send. Argue that there is a non-zero probability that A will *never* be able to transmit its first packet.

Answer: The informal intuition is that with some probability A will incur several collisions and its backoff value will become large. Once this happens, the probability that A will transmit ahead of B keeps decreasing exponentially. As a result, the probability that all of these unfortunate events happen ad infinitum is finite. A formal argument is as follows.

Consider the scenario where A's packet has just suffered its cth collision and B's has suffered its first. A will select a backoff value in the range $[0, 2^{c+1} - 1]$, while B will select from the range $[0, 3]$. With probability $1 - 10/2^{c+3}$, B will select a backoff value smaller than A. If this happens, then the scenario will repeat with c incremented by 1. The product of $(1 - 10/2^{c+3})$, over all $c \geq 1$ is non-zero, thus yielding a non-zero probability that A will never be able to transmit a packet.