## Sample Solution to Problem Set 3

(The problem numbers from the text are the same in both the first and second editions.)

## 1. (6 points) The DCF mode of IEEE 802.11

An ad Hoc network running 1 Mbps IEEE 802.11 has 4 nodes: N1, N2, N3, N4. Assume that at the beginning the channel is idle (no transmission), and that at time $10 \mu s$, N2 has a packet to be sent to N4. At instant $20 \mu s$, both N1 and N3 have a packet to be sent to N4. Assume that the random number generator (for backoff) will give the following values for $\mathrm{N} 1: 2,5, \ldots$; for $\mathrm{N} 2: 3$, $3, \ldots$; and for N3: $1,4, \ldots$ (These are in slots.)

Show the execution of the DCF mode of IEEE 802.11 for the above transmissions. Assume that SIFS is $10 \mu \mathrm{~s}$, DIFS $50 \mu \mathrm{~s}$, and slot time is $20 \mu \mathrm{~s}$. Assume that we don't use RTS/CTS or fragmentation, and that all data packets have the same length of 125 bytes and that the Ack packet has length 25 bytes. Furthermore the channel bit error rate is assumed to be 0 , and 802.11 provides the maximum possible throughput when there are no collisions.

Answer: We have the following sequence of events, labeled by the time (in $\mu \mathrm{s}$ ) of their occurrence. We assume below that if a node does not hear any transmission for DIFS period, immediately after its packet arrives, it does not use the backoff counter. If you assumed otherwise, you will get a different answer, which is also fine.

- 10: A packet arrives at N2. It senses the channel free and waits for DIFS time units.
- 20: A packet arrives at each of N1 and N3. They both sense the channel free and wait for DIFS time units.
- 60: N2 transmits its packet, noting that the channel is free.
- 60: N1 and N3 select a backoff time of $2 \cdot 20=40$ and $1 \cdot 20=20$ time units, respectively. (The factor of 20 corresponds to the slot time.) Their backoff times will not start until (at least) DIFS time units after the transmission of N2 ends.
- 1060: N2 completes its transmission. N4 waits for SIFS time units to send its ACK.
- 1070: N4 sends its ACK. The backoff counters of N1 and N3 are still frozen.
- 1270: N2 receives the ACK.
- 1320: DIFS units have passed since last transmission. So the backoff counters of N1 and N3 resume (begin).
- 1340: N3's backoff counter times out. So N3 transmits its packet. N1 freezes its backoff counter.
- 2340: N3's transmission completes. N4 waits for SIFS before sending ACK.
- 2350: N4 sends ACK.
- 2550: ACK transmission complete.
- 2600: N1 resumes its backoff counter.
- 2620: N2's backoff counter times out and it transmits its packet.
- 3620: N2's packet received at N4. It waits for SIFS before sending ACK.
- 3630: ACK sent by N4.
- 3830: ACK received by N2.


## 2. (8 points) Mobile IP

Chapter 12, Problem 2.
Answer: We list the key parts of the IP headers.
(a) IP datagram arriving at R1 from Internet, using IP-in-IP encapsulation, the IP header is as follows: Destination address in the outer header is R1 and the destination address in the inner header is the home address of A . The source address in the inner header is the IP address of the correspondent node while that in the outer header is the IP address of the home agent.
Furthermore, both the inner and outer headers contain all IP header fields.
For the MAC frame leaving R1 into LAN X, the MAC header contains the destination address as the MAC address of A on LAN X. The IP header is the same as the inner IP header of the above frame.

This is followed by the transport layer header and data.
(b) When minimal encapsulation is used, the common fields in the outer and inner headers are combined. In particular, there are no flags in the inner header and no fragment offset either.

The destination address in the outer header is R 1 and the destination address in the inner header is the home address of A . The source address in the inner header is the IP address of the correspondent node while that in the outer header is the IP address of the home agent.
For the MAC frame, the header info is the same as above.
This is followed by transport layer header and data.
(c) For the packet arriving into R1, the answer is the same as (a) except that the destination address in the outer header is A's IP address in LAN X. For the MAC frame in LAN X, the MAC destination address is the MAC address of A in LAN X. Following any LLC information, the IP packet contains both the outer and inner IP headers since R1 does not strip out the outer header (it is not a foreign agent for A ).
(d) For the packet arriving into R1, the answer is the same as (b) except that the destination address in the outer header is A's IP address in LAN X. For the MAC frame in LAN X, the MAC destination address is the MAC address of A in LAN X. Following any LLC information, the IP packet contains both the outer and inner IP headers as the packet arriving to router R1 since R1 does not strip out the outer header (it is not a foreign agent for $A$ ).

## 3. (8 points) Organization of cells in a cellular networks

Problem 10.7.

## Answer:

- The total number of available channels is $K=33000 / 50=660$. For a frequency reuse factor $N$, each cell can use $K / N$ channels.
For $N=4$, we get 165 channels per cell. For $N=7$, we get 94 channels per cell. For $N=12$, we get 55 channels per cell.
- 32 MHz is available for voice channels for a total of 640 channels. For $N=4$, we can have 160 voice channels and one control channel per cell. For $N=7$, we can have 4 cells with 91 voice channels and 3 cells with 92 voice channels, and one control channel per cell. For $N=12$, we can have 8 cells with 53 voice channels and 4 cells with 54 voice channels, and one control channel per cell.


## 4. (8 points) Performance analysis of cellular networks

Problem 10.8.

## Answer:

(a) The number of 30 KHz channels is $12500 / 30=416$. Of these, 21 are control channels. So the total number of voice channels is 395 . The reuse factor being 7 , the number of voice channels per cell is $395 / 7=56$.
(b) Using interpolation, we obtain $(56-40) /(70-40)=(A-31) /(59.13-31)$. So we get $A$ as 46 Erlangs/cell.
(c) Number of calls per hour per cell $=46 \cdot 3600 / 100=1656$. The number of cells per $\mathrm{km}^{2}$ is $1 / 8$. So number of calls per hour per $\mathrm{km}^{2}$ is $1656 / 8=207$.
(d) Number of users per hour per cell equals $1656 / 1.2=1380$. Taken this amount per channel, we get $1380 / 56=24.6$.
(e) The total number of cells is $4000 / 8=500$. So the total traffic carried is $46 \cdot 500=23000$ Erlangs. The total bandwidth is 12.5 MHz and the total coverage area ia $4000 \mathrm{~km}^{2}$. So we get $\eta$ equal to 0.46 Erlangs $/ \mathrm{MHz} / \mathrm{km}^{2}$.

## 5. (20 points) Access points and handoffs

Consider a network consisting of two access points: AP1 and AP2, located at positions (-100, 0 ) and $(100,0)$ on the plane, respectively, where the distance unit is meter. A mobile unit is communicating through this network, which offers a data rate of 1 Mbps , using BPSK modulation and operating at a frequency spectrum centered around 2.4 GHz . Each access point transmits at the level of 100 mW and the noise density in the area is $10^{-16} \mathrm{~W} / \mathrm{Hz}$. Assume that all packets have the same size: 100 bytes.
(a) Draw a two-dimensional graph that (approximately) indicates the region in which the frame error rate of communication with access point is at least $10 \%$.

Answer: Consider a point $(x, y)$. Assuming the free-space loss model, we obtain that the received power at the mobile station for a transmission from AP2 equals

$$
\frac{100 \cdot 3^{2} \cdot 10^{1} 6}{\left.16 \pi^{2} \cdot 2.4^{2} \cdot 10^{18} \cdot\left((x+100)^{2}+y^{2}\right)\right)}=\frac{0.01}{(x+100)^{2}+y^{2}} \mathrm{~mW} .
$$

Similarly, the received power for a transmission from AP2 equals

$$
\frac{0.01}{(x-100)^{2}+y^{2}} m W \text {. }
$$

We thus obtain the $E_{b} / N_{0}$ values as

$$
\frac{10^{-5}}{10^{-16} \cdot 10^{6} \cdot\left((x+100)^{2}+y^{2}\right)}=\frac{10^{5}}{(x+100)^{2}+y^{2}}
$$

and

$$
\frac{10^{5}}{(x-100)^{2}+y^{2}},
$$

respectively.
Plugging this value into the BPSK BER formula, we obtain the bit-error rate for a transmission from AP1 and AP2, as received at $(x, y)$. We can compute the frame-error rate as $1-(1-B E R)^{800}$.
The frame-error rate of $10 \%$ is obtained at point $(117,0)$ from AP1 i.e., at a distance of 117 m from the access point.

The solution is depicted in Figure 1.
(b) Suppose the mobile decides to switch from one access point to another whenever the signal from the newer access point is at least $25 \%$ stronger than from the first one. Draw a twodimensional graph that indicates the points at which a mobile will switch from AP1 to AP2.

Answer: We can compare the $E_{b} / N_{0}$ values to determine the points where the signal strength of AP2 is better than that of AP1 by at least $20 \%$. We obtain the region to be defined by:

$$
\frac{10^{5}}{(x-100)^{2}+y^{2}} \geq 1.25 \frac{10^{5}}{(x+100)^{2}+y^{2}},
$$

which can rewritten as

$$
x^{2}+y^{2}-1800 x+10000 \leq 0,
$$

and as

$$
(x-900)^{2}+y^{2}-800000 \leq 0 .
$$

The solution is depicted in Figure 2.
(c) Repeat part (b) for the case when the mobile decides to switch whenever the signal from the newer access point is at least $25 \%$ stronger than from the first one and the frame error rate for communication with the new access point is at most $10 \%$. An approximate curve will suffice.

Answer: All we need is the intersection of the two regions above. Thus, the mobile will switch from AP1 to AP2 when it enters the inside of both the circles. (I would like to add that from a practical standpoint, this is a very strong condition for handoff. More likely, handoff will be done when the newer access has a stronger signal and the frame error rate from the previous access point exceeds a certain limit.)
The solution is depicted in Figure 3.

You may adopt a free-space loss model for idealized isotropic antenna (ignoring gains) and may use the following formula for the Bit-Error-Rate (BER) of BPSK modulation:

$$
B E R=\int_{\sqrt{2 E_{b} / N_{0}}}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-\frac{u^{2}}{2}} d u .
$$

(To calculate the above numerically, you can use the complement of the error function used in statistical analysis - e.g., the ERFC function in Excel.)


Figure 1: Frame error rate at most $10 \%$ for transmission from AP2 in the region inside the circle.


Figure 2: Signal strength from AP2 is stronger than that from AP1 by at least $25 \%$ in the region inside the circle (and including the circle boundary).


Figure 3: Assuming the mobile is connected to AP1, it will switch to AP2 when it enters the intersection of both the circles.

