College of Computer & Information Science — Fall 2007
Northeastern University — Handout 4
CSG250: Wireless Networks — 10 October 2007

Sample Solutions for Problem Set 2

(Problem numbers indicated below refer to the problems in the second edition of the course text. The problem numbers in parentheses refer to the problems in the first edition of the course text.)

1. (10 points) Frequency-hopping spread spectrum

Problem 7.4 (Chapter 7, Problem 4).

(a) The period is 15.

(b) The FSK used is clearly MFSK since the same bit is modulated on a different frequency within the same channel. The given data is consistent with MFSK, with number of levels $M = 4$. In the channel given by frequencies $\{f_0, f_1, f_2, f_3\}$, the association is $00 \rightarrow f_0, 01 \rightarrow f_1, 10 \rightarrow f_2, 11 \rightarrow f_3$.

(c) $L = 2$

(d) $M = 2^L = 4$

(e) $k = 3$

(f) This is a slow FHSS since the hopping rate, 1 per 4 time units is slower than the symbol rate, which is 1 per two time units.

(g) $2^k = 8$

(h) When we consider the dehopped frequencies, we can replace f_i by f_j where $j = i \bmod 4$ (by projecting all frequencies to the first channel). So the variation of the dehopped frequency with time can be given by:

$$f_1, f_1, f_3, f_3, f_3, f_3, f_2, f_2, f_0, f_0, f_2, f_2, f_1, f_1, f_3, f_3, f_2, f_2, f_2, f_2.$$

2. (6 points) Spreading codes in CDMA

Problem 7.7 (Chapter 7, Problem 7).

Answer: C0 = 1110010; C1 = 0111001; C2 = 1011100; C3 = 0101110; C4 = 0010111; C5 = 1001011; C6 = 1100101. The output for C1 is $-5+1+1-3-1+3-3 = -7$, and the output for C2 is $5-1+1-3+1+3+3 = 9$. So the values assigned are 0 and 1, respectively.

3. (4 points) Generation of m-sequences

Problem 7.12 (Chapter 7, Problem 12).

Answer: The answers are given in Tables 1 and 2, respectively.

State	B_4	B_3	B_2	B_1	B_0	$B_0 \oplus B_3$	output
0	1	0	0	0	0	0	0
1	0	1	0	0	0	1	0
2	1	0	1	0	0	0	0
3	0	1	0	1	0	1	0
4	1	0	1	0	1	1	1
5	1	1	0	1	0	1	0
6	1	1	1	0	1	0	1
7	0	1	1	1	0	1	0
8	1	0	1	1	1	1	1
9	1	1	0	1	1	0	1
10	0	1	1	0	1	0	1
11	0	0	1	1	0	0	0
12	0	0	0	1	1	1	1
13	1	0	0	0	1	1	1
14	1	1	0	0	0	1	0
15	1	1	1	0	0	1	0
16	1	1	1	1	0	1	0
17	1	1	1	1	1	0	1
18	0	1	1	1	1	0	1
19	0	0	1	1	1	1	1
20	1	0	0	1	1	1	1
21	1	1	0	0	1	0	1
22	0	1	1	0	0	1	0
23	1	0	1	1	0	0	0
24	0	1	0	1	1	0	1
25	0	0	1	0	1	1	1
26	1	0	0	1	0	0	0
27	0	1	0	0	1	0	1
28	0	0	1	0	0	0	0
29	0	0	0	1	0	0	0
30	0	0	0	0	1	1	1
31=0	1	0	0	0	0	0	0

Table 1: Answer for 2(a)

State	B_4	B_3	B_2	B_1	B_0	$B_0 \oplus B_1 \oplus B_3 \oplus B_4$	output
0	1	0	0	0	0	1	0
1	1	1	0	0	0	0	0
2	0	1	1	0	0	1	0
3	1	0	1	1	0	0	0
4	0	1	0	1	1	1	1
5	1	0	1	0	1	0	1
6	0	1	0	1	0	0	0
7	0	0	1	0	1	1	1
8	1	0	0	1	0	0	0
9	0	1	0	0	1	0	1
10	0	0	1	0	0	0	0
11	0	0	0	1	0	1	0
12	1	0	0	0	1	0	1
13	0	1	0	0	0	1	0
14	1	0	1	0	0	1	0
15	1	1	0	1	0	1	0
16	1	1	1	0	1	1	1
17	1	1	1	1	0	1	0
18	1	1	1	1	1	0	1
19	0	1	1	1	1	1	1
20	1	0	1	1	1	1	1
21	1	1	0	1	1	0	1
22	0	1	1	0	1	0	1
23	0	0	1	1	0	1	0
24	1	0	0	1	1	1	1
25	1	1	0	0	1	1	1
26	1	1	1	0	0	0	0
27	0	1	1	1	0	0	0
28	0	0	1	1	1	0	1
29	0	0	0	1	1	0	1
30	0	0	0	0	1	1	1
31=0	1	0	0	0	0	1	0

Table 2: Answer for 2(b)

4. (4 points) Parity codes

Consider a simple linear block code where each codeword consists of three data bits and one parity bit.

(a) Find all codewords in this code.

Answer: 0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111.

(b) Find the minimum distance of the code.

Answer: The minimum distance of the code is 2. In every pair of codewords, the two words differ in at least two bits. And there are two codewords, e.g., 0000 and 0011, that differ in exactly two positions.

5. (4 points) Walsh codes

Demonstrate that codes in a 8×8 Walsh matrix are orthogonal to each other. What are the advantages and limitations of using Walsh codes in spread spectrum applications?

Answer: Rather than just demonstrating this for a specific Walsh matric, we prove this for a general $2^n \times 2^n$ Walsh matrix, $n \geq 1$.

The proof is by induction on n. The claim is trivially true for the base case $n = 1$. For the induction step, let us assume that the claim is true for a $2^{n-1} \times 2^{n-1}$ Walsh matrix. Consider any two distinct row vectors w_i and w_j of a $2^n \times 2^n$ Walsh matrix. We consider three cases.

The first case is when $i, j \leq 2^{n-1}$. In this case, $w_i \cdot w_j$ is the sum of two quantities, each of which is the dot-product of the ith and jth row vectors of the $2^{n-1} \times 2^{n-1}$ Walsh matrix. By the induction hypothesis, both these quantities are zero, thus establishing the induction step for this case.

The second case is when $i, j > 2^{n-1}$. In this case, $w_i \cdot w_j$ is the sum of two quantities, the first of which is the dot-product of row $i - 2^{n-1}$ and row $j - 2^{n-1}$ of the $2^{n-1} \times 2^{n-1}$ Walsh matrix, and the second is the corresponding dot-product of the complement matrix. By the induction hypothesis, both these quantities are zero, thus establishing the induction step for this case.

The final case is when $i \leq 2^{n-1}$ and $j > 2^{n-1}$. Again, the dot-product is the sum of two quantities, the first being the dot-product of row i and row $j - 2^{n-1}$ of the $2^{n-1} \times 2^{n-1}$ Walsh matrix, while the other being the dot-product of row i of the $2^{n-1} \times 2^{n-1}$ Walsh matrix and row $j - 2^{n-1}$ of the complement of the $2^{n-1} \times 2^{n-1}$ Walsh matrix. Since these quantities complement each other, we obtain a dot-product of 0, thus completing the proof.

Since Walsh codes provide perfectly orthogonal codes, they avoid interference of any kind of two users using two different codewords of a Walsh code. So they are effective in spread spectrum scenarios where the multiple users are well-synchronized. The cross-correlations between different shifts of the Walsh sequences are not zero (and could be high), so Walsh sequences do not provide good spread spectrum codes in scenarios where tight synchronization is not provided.

6. (10 points) Convolutional encoding

Consider the convolutional encoder with $n = 3$, $k = 1$, and $K = 3$, defined by $v_{n1} = u_n$, $v_{n2} = u_n \oplus u_{n-1} \oplus u_{n-2}$, and $v_{n3} = u_n \oplus u_{n-2}$.

(a) Draw a shift-register diagram for the encoder.

(a) Draw a trellis diagram for the encoder.

Answer: See Figure 1.

7. (4 points) Block error correction codes

Problem 8.11 (Chapter 8, Problem 11).

Answer: Each received data is mapped to the closest codeword, in terms of its Hamming distance (that is, the number of bits in which the received data and the codeword differ).

Suppose the minimum distance of a given code is at least $2t+1$. Let D be the original data that is transmitted, and let R be the received data; so D is a codeword. If R has at most t bit errors, then it differs from D in at most t bits. If the distance between any two codewords is at least $2t+1$, then we argue that D is the unique codeword closest to R. Suppose there are two codewords, say X and Y that differ from R in at most t bits. Then, X and Y differ in at most $t+t=2t$ bit positions, a contradiction. Thus, R is decoded as D, showing that the code can correct any errors of up to and including t bit errors.

8. (8 points) Interleaving

Problem 8.19 (Chapter 8, Problem 19).

(a) A burst of m contiguous channel bit errors results is spread across the m rows, so each row gets only one error.

(b) Any burst of bm errors results is averaged out across the m rows. So each row has either $\lfloor b \rfloor$ or $\lceil b \rceil$ errors. Consequently, when we view the bits in row-major order (as in the output), the separation between the last bit of the burst in a row from the first bit in the following row is at least $n - \lceil b \rceil$.

(c) Single bit errors spaced m bits apart all reside on a single row, thus yielding a single burst of length n in the output.

(d) The last bit of the first column gets filled at time $(n-1)m+1$. The last bit of the first row is received at time $(m-1)n+1$. Hence, the end-to-end delay is $2nm - n - m + 2$.

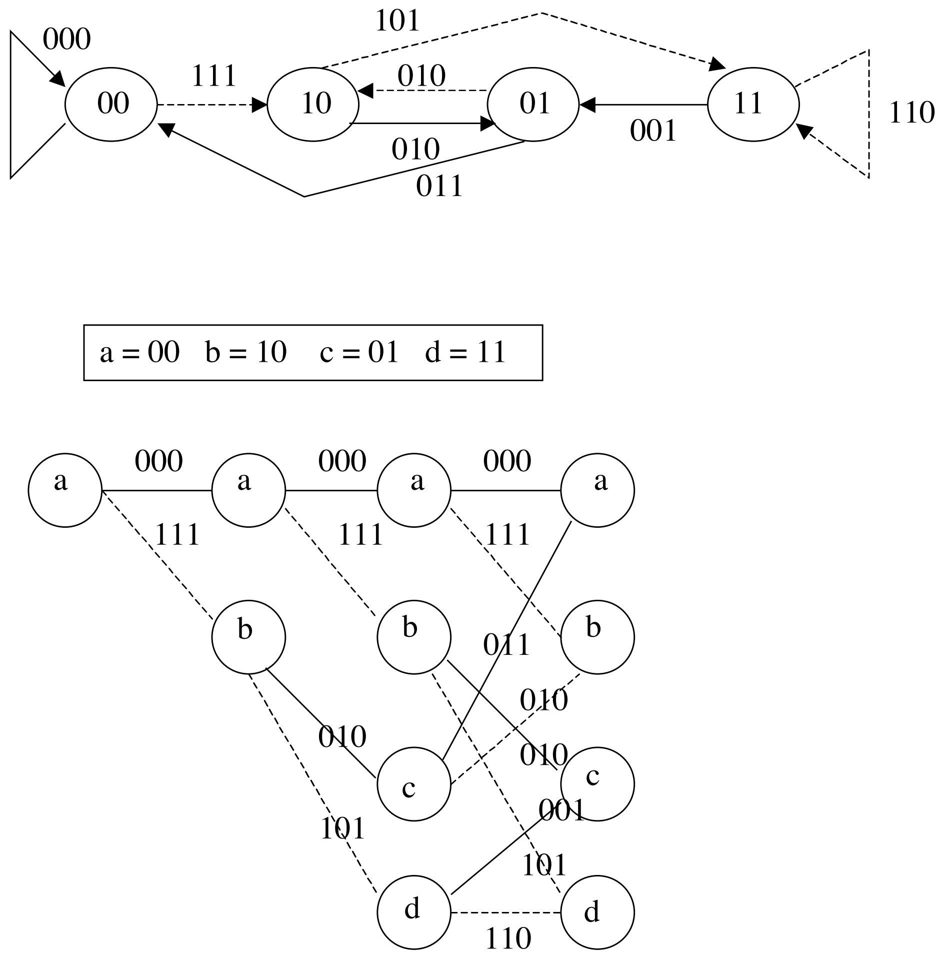

Figure 1: Problem 6