College of Computer & Information Science — Fall 2007
Northeastern University — Handout 3
CSG250: Wireless Networks — 24 September 2007

Sample Solution to Problem Set 1

1. (10 points) Applying low-pass and bandpass filters to a digital signal

A square periodic signal is represented as the following sum of sinusoids:

$$s(t) = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} cos((2k+1)\pi t).$$

(Note that this is just a rewriting of the formula we discussed in class.)

(a) Suppose that the signal is applied to an ideal low-pass filter with bandwidth 15 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 5 Hz; for 3 Hz. What happens as the bandwidth increases.

(b) Suppose that the signal is applied to a bandpass filter that passes the frequencies from 5 to 9 Hz. Plot the output from the filter and compare to the original signal.

For your plots, use an appropriate plotting tool. One such tool is gnuplot, available in Unix.

Answer:

(a) The non-zero frequency components of the signal $s(t)$ correspond to the frequencies 1/2 Hz, 3/2 Hz, 5/2 Hz, ... If the signal is passed through an ideal low-pass filter with bandwidth 15Hz, we obtain the output signal as:

$$\frac{2}{\pi} \sum_{k=0}^{14} \frac{(-1)^k}{2k+1} cos((2k+1)\pi t).$$

Signals can be obtained for the other cases similarly. On plotting, we obtain Figures 1, 2, and 3, respectively.

(b) See Figure 4.

2. (4 points) Bandwidth, signal element, and SNR

A digital signaling system is required to operate at 38.4 Kbps. If a signal element encodes a 8-bit word, what is the minimum required bandwidth of the channel. What signal-to-noise ratio is required to achieve the desired capacity on the bandwidth that you have computed?

Answer: We need capacity $C \geq 38400$ bps. We have $M = 2^8$. Nyquist's sampling theorem says that $C = 2B \log_2 M$. Therefore, $B \geq 38400/(16) = 2400$Hz.

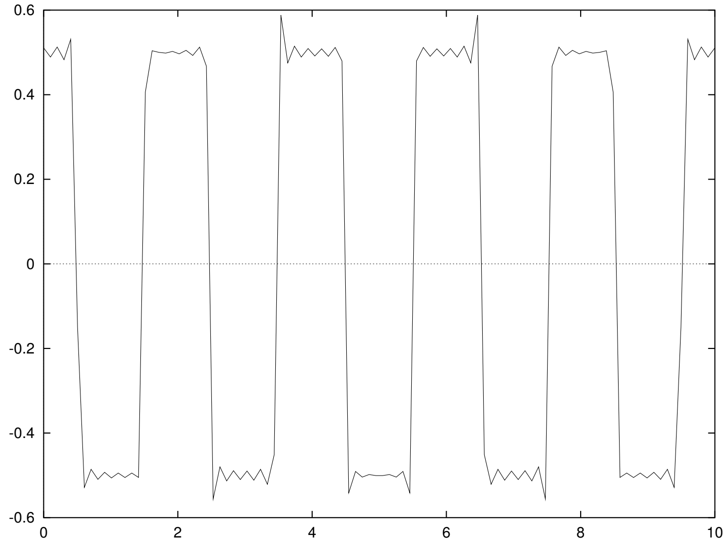

Figure 1: Passing through a low-pass filter of 15Hz bandwidth.

Using Shannon's capacity theorem, we have $C = B\log_2(1 + \text{SNR})$. Therefore $\text{SNR} \geq 2^{C/B} - 1 = 65535$. In decibels, we obtain $SNR_{dB} = 10 \cdot \log_{10}(65535) = 48.16$ dB.

3. (6 points) Effect of transmission frequency and distance on attenuation

Under the free-space path-loss model, find the transmit power required to obtain a received power of 1 dBm for a wireless system with isotropic antennas (gain = 1) and a carrier frequency $f = 5$ GHz, assuming a distance $d = 10$ m. Repeat for $d = 100$ m, keeping $f = 5$ GHz. Repeat for $f = 50$ GHz, keeping $d = 10$ m.

Answer: In free space, we have the path-loss model as:

$$\frac{P_t}{P_r} = \frac{(4\pi f d)^2}{(G_r G_t c^2)}.$$

Expressing the same in decibels and noting that the gains are 1, we obtain:

$$10\log P_t = 10\log[(4\pi f d)2/c^2] + 10\log P_r$$

For $d = 10$m, $f = 5$ GHz:

$$\begin{aligned} 10\log P_t &= 10\log[(4\pi \cdot 5 \cdot 10^9 \cdot 10)^2/c^2] + 1 \text{ dBm} \\ &= 66.42 + 1 \text{ dBm} \\ &= 67.42 \text{ dBm} \end{aligned}$$

For $d = 100$m $f = 5$ GHz:

$$\begin{aligned} 10\log P_t &= 10 log[(4\pi \cdot 5 \cdot 10^9 \cdot 100)^2/c^2] + 1 \text{ dBm} \\ &= 86.42 + 1 \text{ dBm} \\ &= 87.42 \text{ dBm.} \end{aligned}$$

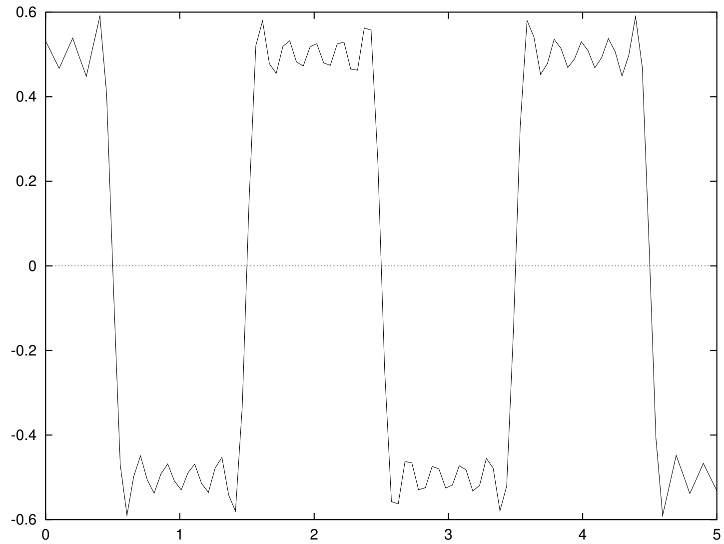

Figure 2: Passing through a low-pass filter of 5Hz bandwidth.

For $d = 10$ m $f = 50$ GHz:

$$\begin{aligned} 10 \log P_t &= 10 \log[(4\pi \cdot 50 \cdot 10^9 \cdot 10)^2 / c^2] + 1 \text{ dBm} \\ &= 86.42 + 1 \text{ dBm} \\ &= 87.42 \text{ dBm} \end{aligned}$$

4. (5 points) Received power using half-wave dipoles

Problem 5.9.

Answer: The relationship between transmitted power P_t and received power P_r is:

$$\frac{P_t}{P_r} = \frac{16\pi^2 d^2}{G_r G_t \lambda^2},$$

where d is the distance and G_r and G_t are the gains of the receiving and transmitting antennas, respectively. The two gains are given to be 3 dB, which is a factor of $10^{3/10} = 2$. The wavelength λ is given to be $3 \times 10^8 / 10^8 = 3$m. The distance is 10^4m and $P_t = 1$W. Substituting these numbers we get:

$$P_r = \frac{2 \cdot 2 \cdot 3^2 \cdot 1}{16\pi^2 \cdot 10^8} = 2.28 \times 10^{-9} W.$$

5. (7 points) Modulation schemes and E_b/N_0

Problem 6.2.

Answer: Let T_s be the signal element period, T_b be the bit period, and A be the amplitude of the signal.

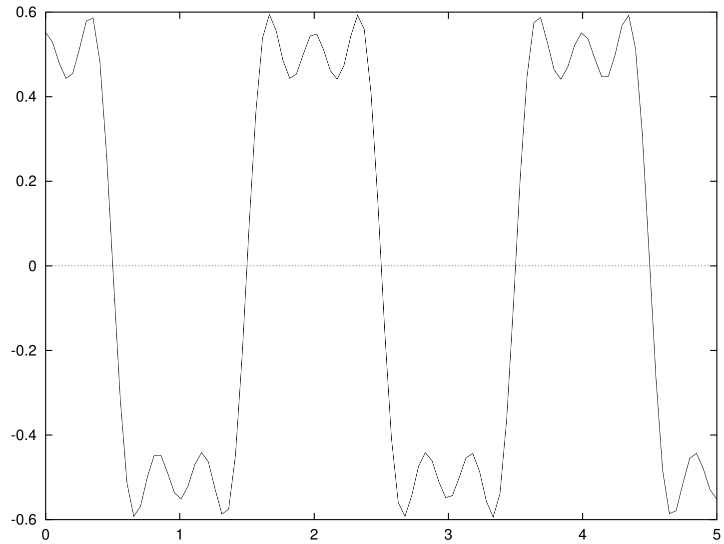

Figure 3: Passing through a low-pass filter of 3Hz bandwidth.

(a) We have $A = 0.005$V, $T_s = T_b = 10^{-5}$s. We obtain the power P as follows.

$$P = \frac{1}{T_s}\int_0^{T_s} s^2(t) = \frac{A^2}{2}$$

$$E_b = P \times T_b = P \times T_s = \frac{A^2}{2}T_s; N_0 = 2.5 \times 10^{-8} \times T_s$$

$$\frac{E_b}{N_0} = \frac{(A^2/2) \times T_s}{2.5 \times 10^{-8} \times T_s} = 500.$$

In dB, we obtain 27dB.

(b) For this, all the values are identical except that $T_b = T_s/2$. So we obtain $E_b/N_0 = 250$. In dB, this is 24 dB.

6. (5 points) Telephone channel bandwidth

Problem 6.6.

Answer: The required bandwidth for 2400 bps QPSK is $2R/2 = 2400$ Hz. The required bandwidth for 4800 bps MPSK with 8-level multisignaling is $2R/3 = 3200$Hz. Thus, bandwidth of the telephone channel is adequate for the former but not for the latter.

7. (5 points) Sampling

Problem 6.10.

Answer: The bandwidth of the signal is 2700 Hz. And the sampling rate is 7000 samples per second.

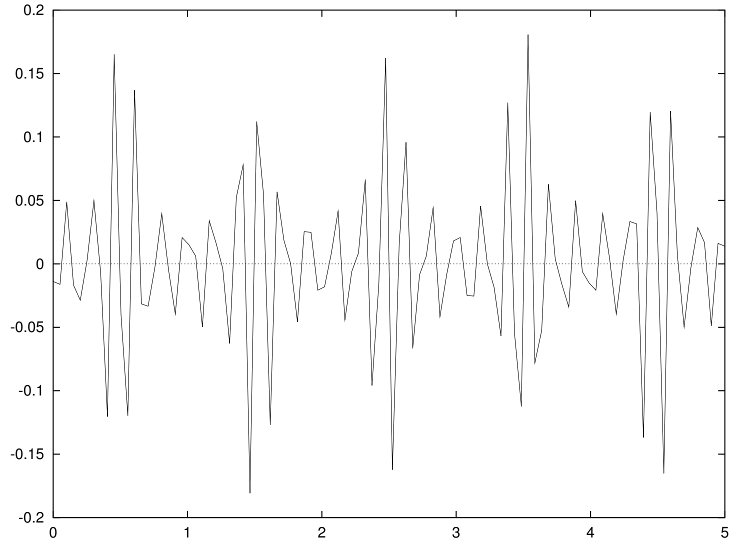

Figure 4: Passing through a 5-9Hz bandpass filter.

(a) SNR (in dB) and the number n of bits in the number of quantizing levels are related as

$$\text{SNR} = 6.02n + 1.76\text{dB}.$$

So we obtain $n \geq 4.69$. Thus, the number of quantizing levels needed is 32.

(b) With 7000 samples per second and 5 bits per sample, we obtain a data rate of 35000 bps.

8. (8 points) Delta modulation

Problem 6.13.

Answer: See Figure 5.

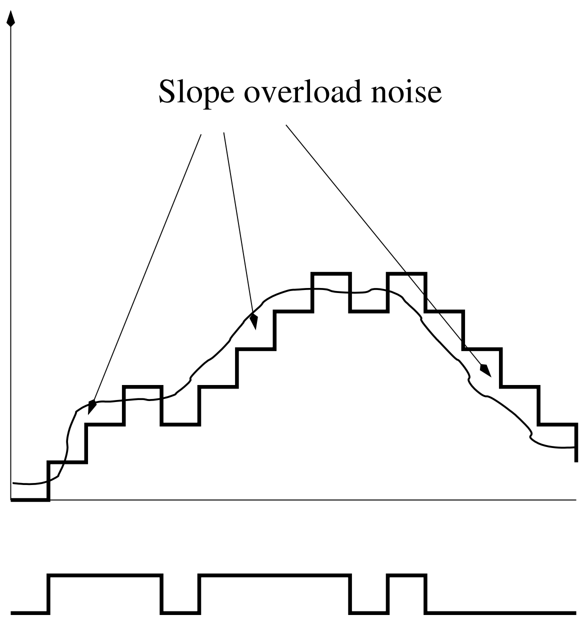

Figure 5: Delta modulation