# Sample solution to Midterm 

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\text { Part I }(4 \times 4=16 \text { points })
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1. Explain how the RTS/CTS mechanism alleviates the hidden terminal problem? Why is it important to include the length of the data packet in both the RTS and CTS packets?

Answer: The hidden terminal problem occurs when a node C transmits to B while B is receiving from A, a node that C cannot hear. If the A-B communication is preceded by an RTS-CTS mechanism, then C hears the CTS sent by B, which contains the length of the packet transmission, and waits until the completion of the data transmission to initiate its communication with $B$.

Packet length is important to include in CTS since that indicates to neighbors how long the channel is going to be occupied. For the length to be included in CTS, the sender needs to send the packet length in RTS.
2. For a digital modulation technique, the power efficiency is measured by the $E_{b} / N_{0}$ ratio needed to achieve a given bit-error-rate and the spectral efficiency is measured by the ratio of the information rate to the bandwidth. Explain why there is a tradeoff between spectral efficiency and power efficiency in digital modulation. In phase shift-keying (PSK), increasing the number of bits per symbol generally increases the spectral efficiency at the expense of power efficiency. Why?

Answer: A digital modulation technique is specified by a mapping of symbols to signal elements. A modulation technique with superior error control has higher power efficiency, but lower bandwidth efficiency since the different data symbols are encoded by signals that are well-separated in the signal space.

The greater the maximum distance between two signal points, the higher the power efficiency since the more immune is the encoding technique to noise. In PSK, increasing the number of bits per symbol increases the number of signal elements packed in the two-dimensional signal space. As a result, the spectral efficiency increases - this can also be seen in the PSK formulae on text - but power efficiency decreases. Thus, for example, QAM is more bandwidth-efficient than QPSK but less power-efficient. QPSK has the same power-efficiency as BPSK but better bandwidth-efficiency.
3. Consider a linear block code where each codeword consists of three data bits and one parity bit. Find all codewords in this code. How many bit errors can the code detect? How many bit errors can the code correct?

Answer: The codewords are 0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111.
It can detect 1 bit errors. It can correct no errors.
4. Give one reason why Mobile IP is difficult to implement with firewalls. Give one advantage and one disadvantage of the use of reverse tunneling (setting up a tunnel from the Foreign Agent to the Home Agent for transporting packets originating from the Mobile Node).

Answer: Mobile IP is difficult to implement with firewalls because many firewalls forward packets only with source address within an associated subnet. A mobile node in a foreign network sends packets with its original IP address as source, which is not part of the foreign subnet; hence a packet originating from this mobile station may be rejected by the firewall.

Reverse tunneling alleviates the above problem since every packet from the mobile node is sent to the foreign agent which then encapsulates the packet within an IP packet destined to the Home Agent, with the Foreign Agent as source. The Home Agent then decapsulates and sends it off to the intended destination. The main disadvantage with reverse tunneling is that it can hijacked by a third party, who pretends to be the Mobile Node and sends a packet to the foreign agent, which then forwards it through the Home Agent on to the Internet.

## Part II

## 5. (5 points) Antennas and propagation

Two antennas, each with a directivity gain of 5 dB , are separated by a distance of 10 km . If the transmitted power of a signal by one antenna on frequency 100 MHz is 1 W , what is the received power at the other antenna? Assume a free space path loss model and that the two antennas are aligned so that directive gains are achieved.
Answer: Each gain equals $10^{5 / 10}$, so the total gain is 10 . The received power equals

$$
\frac{10 \cdot 1 \cdot 9 \cdot 10^{16}}{(4 \pi)^{2} \cdot 10^{16} \cdot 10^{8}}=5.7 \times 10^{-9} \mathrm{~W}
$$

## 6. (6 points) Spread Spectrum

Sketch or describe the Direct Sequence Spread Spectrum signal $s(t) c(t)$ over two bit time periods assuming that $s(t)$ is BPSK modulated with carrier frequency 8 MHz and bit time period equal to $1 \mu \mathrm{~s}$. Assume that there are four chips per bit in the spreading code $c(t)$ and that the chips alternate between $\pm 1$, with the first chip equal to +1 . Assume that the first data bit is a 1 and the second data bit is a 0 .

Answer: The signal has 8 time periods in $1 \mu \mathrm{~s}$. So each chip should have 2 time periods of the signal. Since there are 4 chips per bit, we would have phase shifts every 2 time periods of the signal; at the $1 \mu s$ point, however, when we move from a 1 bit to a 0 bit, there will not be any phase shift since the chip change and bit change cancel each other.

## 7. (7 points) The DCF mode of 802.11

An infrastructure network using a $1 \mathrm{Mb} / \mathrm{s}$ IEEE 802.11 has 3 nodes N 1 , N 2 , N 3 , connecting to an access point AP. At time 0, N1 begins the transmission of a data packet to AP. There is no other ongoing transmission. At time $10 \mu \mathrm{~s}$, N2 has a packet to be sent to AP and at $30 \mu \mathrm{~s}$, N3 has a packet to be sent to AP. Assume that the random number generator (for backoff) will give the following values for $\mathrm{N} 1: 1,4, \ldots$; for $\mathrm{N} 2: 2,5, \ldots$ for $\mathrm{N} 3: 3,3, \ldots$.

Show the execution of the DCF mode of IEEE 802.11 for the above transmissions. Assume that SIFS is $10 \mu \mathrm{~s}$, DIFS $50 \mu \mathrm{~s}$, and slot time is $20 \mu \mathrm{~s}$. Assume that we don't use RTS/CTS or fragmentation, and that all data packets have the same length of 125 bytes and that the Ack packet has length 25 bytes. Furthermore the channel bit error rate is assumed to be 0 , and 802.11 provides the maximum possible throughput when there are no collisions.

Answer: All times are in $\mu \mathrm{s}$.
1000: N1's packet completed
1010: SIFS completed
1210: ACK completed
1260: DIFS completed, backoff begins
1300: N2 transmits
2300: N2's packet completed
2310: SIFS completed
2510: ACK completed
2560: DIFS completed, backoff resumes
2580: N3 transmits
3580: N3 completes
3590: SIFS completed
3790: ACK completed.

## 8. $(2+2+3=7$ points) Capacity of a cellular network

Consider a cellular system with hexagonal cells of radius $R=1 \mathrm{~km}$. Suppose the minimum distance between cell centers using the same frequency must be $D=6 \mathrm{~km}$ to maintain the required signal-to-interference ratio.
(a) Find the required reuse factor $N$.

Answer: $N=(D / R)^{2} / 3=12$.
(b) If the total available bandwidth is 24 MHz and each user channel requires 20 KHz , find the number of user channels that can be assigned to each cell.
Answer: The number of user channels that can be assigned to each cell is $1200 / 12=100$.
(c) Suppose each subscriber generates an average of one call an hour, with a mean holding time of 100 seconds. If the cellular network operator wants to offer a grade of service of 0.02 , how many subscribers can the network support per cell? Please refer to the Erlang table given in the text, if needed.
Answer: For a GOS of 0.02, with 100 servers, the number of Erlangs that can be supported is 87.97. Traffic generated by a subscriber in holding time equals $1 / 36$ Erlangs. So the number of subscribers that can be supported in the cell is $87.97 \cdot 36 \approx 3167$.

## 9. $(4+2+3=9$ points) Path loss, frame error rates, and error correction

Two wireless nodes are communicating with each other through a wireless network using BPSK modulation and operating at a frequency band centered around 1 GHz . One node transmits at a power level of 100 mW and the noise density in the area is $10^{-16} \mathrm{~W} / \mathrm{Hz}$. Assume a free space path loss model, no other interference, and a frame length of 100 bits. Use the following formula for bit-error rate of BPSK.

$$
B E R=Q\left(\sqrt{\frac{2 E_{b}}{N_{0}}}\right)
$$

A table for the $Q$-function is given on the last page of this document, for reference. Please use the nearest values whenever you do not find the exact values you are looking for.
(a) Find the separation distance at which a transmission rate of 1 Mbps can be achieved with a frame error rate of $5 \%$. Assume no error-correction is used.

Answer: $\mathrm{FER}=100^{*}$ BER. So BER needs to be 0.0005 . From the Q -function table, $\sqrt{2 E_{b} / N_{0}}$ needs to be $Q^{-1}(0.0005)=3.3$, so $E_{b} / N_{0}$ has to be at least 5.445.

$$
\frac{E_{b}}{N_{0}}=\frac{10^{-1} \cdot 10^{16} \cdot 9 \cdot 10^{16}}{16 \pi^{2} \cdot 10^{6} \cdot 1^{2} \cdot 10^{18} d^{2}}=\frac{9 \cdot 10^{30}}{16 \pi^{2} \cdot 10^{24} d^{2}}=570510 / d^{2}
$$

Therefore, $d$ is at most 323.69 meters.
(b) At the separation distance that you obtain for (a), what is the frame error rate incurred if the transmission rate is set to 2 Mbps ? Assume no error-correction is used.

Answer: $E_{b} / N_{0}$ now becomes $5.445 / 2=2.722$, so $\sqrt{2 E_{b}} / N_{0}=2.33$. So the BER is between 0.01072 and 0.00820 . Interpolation gives an approximate value of 0.01 . For this BER, the FER is $1-.99^{100}=0.634$, a very high frame error rate.
(c) Suppose now that each packet is encoded using a $1 / 2$ rate encoding technique with a minimum distance of 11. The packet length, after encoding, is set to be the same as before. What is the frame error rate incurred if the encoded packets are transmitted at a rate of 2 Mbps ? Compare the results of this approach with that of part (a).
Answer: Given that BER equals 0.01, FER now equals the probability that there are at least six bit errors, which is $\binom{100}{6}(B E R)^{6} / 720=0.00119$. Thus, we obtain an effective throughput of 1 Mbps with a frame error rate of approximately $.1 \%$, much better than part (a).

| $z$ | $Q(z)$ | $z$ | $Q(z)$ |
| :---: | :---: | :---: | :---: |
| 0.0 | 0.50000 | 2.0 | 0.02275 |
| 0.1 | 0.46017 | 2.1 | 0.01786 |
| 0.2 | 0.42074 | 2.2 | 0.01390 |
| 0.3 | 0.38209 | 2.3 | 0.01072 |
| 0.4 | 0.34458 | 2.4 | 0.00820 |
| 0.5 | 0.30854 | 2.5 | 0.00621 |
| 0.6 | 0.27425 | 2.6 | 0.00466 |
| 0.7 | 0.24196 | 2.7 | 0.00347 |
| 0.8 | 0.21186 | 2.8 | 0.00256 |
| 0.9 | 0.18406 | 2.9 | 0.00187 |
| 1.0 | 0.15866 | 3.0 | 0.00135 |
| 1.1 | 0.13567 | 3.1 | 0.00097 |
| 1.2 | 0.11507 | 3.2 | 0.00069 |
| 1.3 | 0.09680 | 3.3 | 0.00048 |
| 1.4 | 0.08076 | 3.4 | 0.00034 |
| 1.5 | 0.06681 | 3.5 | 0.00023 |
| 1.6 | 0.05480 | 3.6 | 0.00016 |
| 1.7 | 0.04457 | 3.7 | 0.00011 |
| 1.8 | 0.03593 | 3.8 | 0.00007 |
| 1.9 | 0.02872 | 3.9 | 0.00005 |

