POW 8–10

8. Using up all Scrabble letters

You have a set of n letter tiles, each tile having exactly one letter of the English alphabet. (Each letter may appear multiple times; thus you may have 5 of As, 2 of Bs, 7 of Cs, etc.) You also have a limited vocabulary of m English words. You want to determine whether you can make a collection of English words that use up all of the letter tiles, with each tile appearing exactly once in this collection, and each word coming from your vocabulary of m words. (A word may appear multiple times in your collection.)

Show that this problem is NP-complete.

9. Efficiently solvable 3-SAT instances

Consider the class of 3-SAT instances in which each of the n variables occurs in exactly 3 clauses (counting both positive and negated occurrences). Show that such 3-SAT instances are always satisfiable and a satisfying assignment can be solved in polynomial time.

10. Random walk on a directed graph

Consider the directed graph G over the vertex set $V = \{0, 1, \ldots, n\}$ and edge set given by

$$\{(i, i+1) : i < n\} \cup \{(i, 0) : i \leq n\} \cup \{(n, n)\}.$$

Compute the Markov chain corresponding to the standard random walk and calculate its stationary distribution. Compare with the stationary distribution one would get on the graph obtained by making all of the above edges undirected.