College of Computer & Information Science — Fall 2010
Northeastern University — Handout 10
CS7800: Advanced Algorithms — 25 October 2010

Problems of the Week – 6 and 7

6. Robustness of a network

Define the *robustness* of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the robustness of a graph that is not connected is 0, that of a tree is 1, while that of a cycle is 2. Using network flows, design an algorithm to compute the edge connectivity of a given undirected graph. Analyze the efficiency of your algorithm, in terms of its worst-case running time.

7. Feasibility and optimality

Suppose you are given a *black box* algorithm that takes as input integers n, m, $m \times n$ matrix A with integer entries, and $m \times 1$ vector b with integer entries, and returns whether there exists a real $n \times 1$ vector x such that $Ax \geq b$ (i.e, the black box returns a yes or no answer).

You are faced with the following problem.

$$\text{Find } x \text{ that minimizes } c^T x \text{ subject to the constraint } A'x \geq b',$$

where c, A', and b' are $n' \times 1$ vector, $m' \times n'$ matrix, and $m' \times 1$ vector, all with integer entries, respectively. Show how to solve this problem by using the black box algorithm, where the number of calls you make is at most polynomial in n', m', and the sizes of A', b', and c.