Problems of the Week – 6 and 7

6. Robustness of a network

Define the robustness of an undirected graph is the minimum number $k$ of edges that must be removed to disconnect the graph. For example, the robustness of a graph that is not connected is 0, that of a tree is 1, while that of a cycle is 2. Using network flows, design an algorithm to compute the edge connectivity of a given undirected graph. Analyze the efficiency of your algorithm, in terms of its worst-case running time.

7. Feasibility and optimality

Suppose you are given a black box algorithm that takes as input integers $n$, $m$, $m \times n$ matrix $A$ with integer entries, and $m \times 1$ vector $b$ with integer entries, and returns whether there exists a real $n \times 1$ vector $x$ such that $Ax \geq b$ (i.e., the black box returns a yes or no answer).

You are faced with the following problem.

Find $x$ that minimizes $c^T x$ subject to the constraint $A'x \geq b'$,

where $c$, $A'$, and $b'$ are $n' \times 1$ vector, $m' \times n'$ matrix, and $m' \times 1$ vector, all with integer entries, respectively. Show how to solve this problem by using the black box algorithm, where the number of calls you make is at most polynomial in $n'$, $m'$, and the sizes of $A'$, $b'$, and $c$. 